CP-violating lepton asymmetry due to B-$\bar{B}$ mixing

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As a result of B-$\bar{B}$ mixing, associated production of B-$\bar{B}$ pairs yields like-sign lepton pairs when both B's decay semileptonically. Formulas are given for the CP-violating charge asymmetry of these like-sign pairs. It is argued that previous calculations based on quark diagrams are unreliable and that the asymmetry might be considerably larger. It is concluded that a reasonable estimate of the asymmetry lies between 10^{-3} and 10^{-2}, but neither the sign nor the magnitude can be reliably calculated.

Evidence for B-$\bar{B}$ mixing has been found[1] from the observation of same-sign dileptons from a system originally containing a B-$\bar{B}$ pair. Neglecting errors this leads to a value

$$\frac{\Delta M}{\Gamma}=0.7 . \qquad (1)$$

It is possible to search for the CP-violating charge asymmetry given by[2]

$$a=\frac{N(++)-N(--)}{N(++)+N(--)}\simeq \mathrm{Im}\left[\frac{\Gamma_{12}}{M_{12}}\right] , \qquad (2)$$

where

$$\Gamma_{12}=\sum_n \langle B \mid H_w \mid n\rangle\langle n \mid H_w \mid \bar{B}\rangle 2\pi\delta(E_n-E_B) , \qquad (3)$$

$$M_{12}=M_0\xi_t^2 , \qquad (4)$$

$$\Delta M=2M_0\xi_t\xi_t^* . \qquad (5)$$

Equation (4) for the B-$\bar{B}$ mass matrix follows from the box diagram[3] considering only the intermediate top quarks. The box diagram involves the Kobayashi-Maskawa (KM) matrix factors

$$\xi_i=V_{ib}V_{id}^* \qquad (6)$$

which are subject to the unitarity constraint

$$\xi_u+\xi_c+\xi_t=0 . \qquad (7)$$

The approximation used in obtaining Eq. (2) is $|\Gamma_{12}| \ll |M_{12}|$.

The main problem is the calculation of Γ_{12}. Whereas M_{12} involves a sum over virtual intermediate states which is dominated by $t+\bar{t}$, Γ_{12} involves a sum over real states. The most detailed calculation is that by Hagelin[4] who assumes these states can be described in terms of quarks (either $\bar{q}q$ or $\bar{q}q\bar{q}q$). He calculates the "absorptive part" of the box diagram. In this paper we wish to look at an alternative approach.

We first look at the quark transitions that contribute to the transitions to the states $|n\rangle$ of Eq. (3). There are three classes:

$$(\mathrm{A})\quad b(\bar{d})\to c+\bar{c}+d+(\bar{d})\to \bar{b}d , \qquad (8a)$$

$$(\mathrm{B1})\quad b(\bar{d})\to c+\bar{u}+d+(\bar{d})\to \bar{b}d , \qquad (8b)$$

$$(\mathrm{B2})\quad b(\bar{d})\to u+\bar{c}+d+(\bar{d})\to \bar{b}d ,$$

$$(\mathrm{C})\quad b(\bar{d})\to u+\bar{u}+d+(\bar{d})\to \bar{b}d . \qquad (8c)$$

The $(\bar{d})$ represents the "spectator" in the initial transition; of course, the role of d and $\bar{d}$ is exchanged if we run the arrows from right to left. In addition to these "spectator" decays there are also exchange contribution such as

$$b+\bar{d}\to c+\bar{c}\to d+\bar{b} , \qquad (9a)$$

$$b+\bar{d}\to u+\bar{u}\to d+\bar{b} . \qquad (9b)$$

We shall emphasize the spectator graphs, which dominate the calculation of Hagelin, but the exchange graphs do not modify our general discussion. For each class of transition in Eq. (8) there is a characteristic combination of KM elements:

$$\mathrm{A},\ \xi_c^2;\quad \mathrm{B},\ \xi_u\xi_c;\quad \mathrm{C},\ \xi_u^2 .$$

We then write

$$\Gamma_{12}=\Gamma_0(\xi_c^2M_{\mathrm{A}}+2\xi_u\xi_cM_{\mathrm{B}}+\xi_u^2M_{\mathrm{C}}) . \qquad (10)$$

To interpret Eq. (10) we focus first on type-A transitions. These arise from a term in H_w of the form

$$V_{cb}V_{cd}^*h_{\mathrm{A}}+\mathrm{H.c.} , \qquad (11)$$

where h_{A} is given explicitly in the Appendix. The total width for transitions of type A is

$$\Gamma_{\mathrm{A}}=|V_{cb}|^2|V_{cd}|^2\sum_n |\langle n \mid h_{\mathrm{A}} \mid \bar{B}\rangle|^2 2\pi\delta(E_n-E_B)$$

$$=|V_{cb}|^2|V_{cd}|^2\Gamma_0\rho_{\mathrm{A}} . \qquad (12)$$

Here Γ_0 yields the rate expected in the limit $m_c \to 0$ and ρ_A is the phase-space suppression factor, calculated[5] in the quark spectator model to be 0.12. The corresponding expression for the contribution of A-type transitions to Γ_{12} can be written

$$\Gamma_{12}(A)=(V_{cb}V_{cd}^*)^2\sum_n \langle B \mid h_A \mid n\rangle\langle n \mid h_A \mid \bar{B}\rangle \times 2\pi\delta(E-E_n)$$
$$=-\xi_c^2\sum_n \mid \langle n \mid h_A \mid \bar{B}\rangle \mid^2 (CP)_n 2\pi\delta(E-E_n)$$
$$=-\xi_c^2\Gamma_0\rho_A(\overline{CP})_A , \quad (13)$$

where we have used a set of intermediate states which are CP eigenstates with eigenvalues $(CP)_n$. We have used $(CP)h_A(CP)^{-1}=h_A^\dagger$ and the convention $\mid \bar{B}^0\rangle = \mid b\bar{d}\rangle$, $\mid B^0\rangle=\mid d\bar{b}\rangle$, with $CP \mid \bar{B}^0\rangle = -\mid B^0\rangle$, $CP\mid B^0\rangle = -\mid \bar{B}^0\rangle$. The quantity $(\overline{CP})_A$ is the average value of CP for the intermediate states $\mid n\rangle$ contributing to Γ_A. The same considerations hold for transitions of type C for which $\rho_C=0.99$. Comparing Eq. (10) with Eq. (13) we have

$$M_I=-\rho_I(\overline{CP})_I , \quad (14)$$

where $I=$A or C. For the case of B-type transitions the contributions to Γ_{12} involve only the interference between the allowed $(V_{cb}V_{ud}^*)$ and the doubly suppressed $(V_{ub}V_{cd}^*)$ transitions. We shall also use $(\overline{CP})_B$ for the factor $(-M_B/\rho_B)$ although it only relates to the CP values associated with this interference term.

To determine the value of Γ_0 we may relate it to the total width. The major contribution to Γ comes from the allowed B-type transitions:

$$\Gamma_B=\Gamma_0 \mid V_{cb}\mid^2 \mid V_{ud}\mid^2 \rho_B ,$$

where ρ_B is estimated to be 0.44. Estimating from the quark model the relative rates of other nonleptonic decays and using the measured value for semileptonic decays one estimates that about 55% of all decays are of type B so that

$$\Gamma_0 \simeq \frac{0.55\Gamma}{\mid V_{cb}\mid^2 \mid V_{ud}\mid^2 \rho_B} . \quad (15)$$

In the calculation of Hagelin the main intermediate states are four-quark $(q\bar{q}q\bar{q})$ states and the relative values of the M_I in Eq. (10) are determined by phase-space integrals

$$M_C=F, \quad M_B=F\left[1-\beta\frac{m_c^2}{m_b^2}\right], \quad M_A=F\left[1-2\beta\frac{m_c^2}{m_b^2}\right] , \quad (16)$$

where we have kept only the leading order in m_c^2/m_b^2. The value of β in Hagelin's calculation is $\frac{4}{3}$ ignoring QCD corrections and approximately unity if they are included. Comparing Hagelin's equations for Γ_{12} and Γ yields

$$F=-8\pi^2 f_B^2\frac{m_B}{m_b^3}=-0.06 , \quad (17)$$

where we have used $f_B=140$ MeV, $m_b=5.1$ GeV. The quantities $(-M_I/\rho_I)$ which we have interpreted in Eq. (14) as $(\overline{CP})_I$, then have the values

$$(\overline{CP})_A=0.39, \quad (\overline{CP})_B=0.12, \quad (\overline{CP})_C=0.06 , \quad (18)$$

where we have set $\beta=1$ and $m_c/m_b=\frac{1}{3}$. Substituting Eqs. (16) into Eq. (10) and using the unitarity relation (7), we obtained the Hagelin result

$$\Gamma_{12}=\Gamma_0 F\left[\xi_t^2+2\beta\left[\frac{m_c}{m_b}\right]^2\xi_c\xi_t\right] . \quad (19)$$

A point emphasized by Hagelin is that the "leading term" in Γ_{12} is proportional to ξ_t^2 with the result that it does not contribute to the asymmetry Eq. (2) since M_{12} is also proportional to ξ_t^2 and so has the same phase. Thus the asymmetry is suppressed by a factor $(m_c/m_b)^2$. This conforms to the expectation that in the limit $m_c \to 0$, or more rigorously, $m_c \to m_u$, any CP-violating observable like the asymmetry must vanish in the KM model.

To analyze this limit we rewrite the asymmetry using Eqs. (2), (4), (7), (10), and (14):

$$a=\frac{-\Gamma_0}{M_0}\left[[\rho_A(\overline{CP})_A-\rho_B(\overline{CP})_B]\mathrm{Im}\left[\frac{\xi_c}{\xi_t}\right]^2 + [\rho_C(\overline{CP})_C-\rho_B(\overline{CP})_B]\mathrm{Im}\left[\frac{\xi_u}{\xi_t}\right]^2\right] . \quad (20)$$

In the limit $m_c \to 0$ we have $\rho_I \to 1$ and all $(\overline{CP})_I$ become equal so that $a=0$. In Hagelin's calculation the cancellation between the terms in the square brackets is very large

$$\frac{\rho_A(\overline{CP})_A-\rho_B(\overline{CP})_B}{\rho_C(\overline{CP})_C}=-\frac{\rho_C(\overline{CP})_C-\rho_B(\overline{CP})_B}{\rho_C(\overline{CP})_C} \simeq -\beta\left[\frac{m_c}{m_b}\right]^2 \simeq -0.11 . \quad (21)$$

However, we are really very far from the limit $m_c=0$ as indicated by the order-of-magnitude difference between ρ_A $(\simeq 0.12)$ and unity and the corresponding range of values required for $(\overline{CP})_I$ in Eq. (18). Thus, from our point of view, the large amount of cancellation in Eq. (20) requires very accurate values for the quantities $(\overline{CP})_I$.

We believe the evaluation of $(\overline{CP})_I$ using quark diagrams is not sufficiently accurate even though the final result of Hagelin may give a reasonable order of magnitude. At the quark level the factors $(\overline{CP})_I$ represent the degree of mismatch in phase space of the quark configuration emergent from b decay (plus $\bar{d}$ spectator) with that of $\bar{b}$ decay (plus spectator d). To get efficient overlap, both the d and $\bar{d}$ quarks must have bounded momentum in the b rest frame; thus the overall quark configuration is collinear. From this point of view $(\overline{CP})_I$ may be roughly viewed as the fraction of final-state phase space leading to collinear configurations. However, the physical final states contain several mesons and it is unclear that their average CP is accurately represented by the quark model picture.

To be specific, consider the two-meson state D^+D^- and $D^{*+}D^{*-}$ for class A transitions. Given the limited phase space, these states, which are primarily CP even, may play a major role. The corresponding states for class C, $\pi\pi$ and $\rho\rho$, are likely to be extremely rare because of the large phase space available for extra pions. The quark model, which appears to give a one-to-one correspondence between SU(4)-related final states, would seem to imply that the ratio of the rates for these two-meson states is determined by phase space only. Thus we are inclined to distrust the Hagelin relative values for $(\overline{CP})_A$ and $(\overline{CP})_C$.

As one goes beyond two-meson states one adds to the sum in Eq. (3) states with the opposite value of CP. Indeed all one has to do to change CP is to add a soft π^0. For states of class C the sum includes many terms of opposite sign. From general ideas of duality we expect the quark model calculation to give a reasonable estimate of this sum. On the other hand, because of long-distance effects we do not expect an accuracy as good as 10% and so believe the cancellation in Eq. (21) is not trustworthy.

We now turn to numerical estimates of the asymmetry a. Using Eqs. (4), (5), and (15),

$$\frac{\Gamma_0}{M_0}=2.6\left[\frac{\Gamma}{\Delta M}\right]\left|\frac{\xi_t}{V_{cb}}\right|^2=3.8\left|\frac{\xi_t}{V_{cb}}\right|^2, \tag{22}$$

where we have used the experimental results of Eq. (1) in the last equality. The explicit KM factors in Eq. (20) can be expressed in terms of the CP-odd phase invariant J of Jarlskog, Wu, and Greenberg:[6]

$$\begin{aligned} J&=\mathrm{Im}\xi_c\xi_t^*=-\mathrm{Im}\xi_u\xi_t^* ,\\ \mathrm{Im}\left[\frac{\xi_c}{\xi_t}\right]^2&=\frac{2J}{|\xi_t|^2}\mathrm{Re}\left[\frac{\xi_c}{\xi_t}\right],\\ \mathrm{Im}\left[\frac{\xi_u}{\xi_t}\right]^2&=-\frac{2J}{|\xi_t|^2}\mathrm{Re}\left[\frac{\xi_u}{\xi_t}\right]. \end{aligned} \tag{23}$$

Substituting Eqs. (22) and (23) in Eq. (20) we find

$$a=-7.6\frac{J}{|V_{cb}|^2}\left\{[\rho_A(\overline{CP})_A-\rho_B(\overline{CP})_B]\mathrm{Re}\frac{\xi_c}{\xi_t} -[\rho_C(\overline{CP})_C-\rho_B(\overline{CP})_B]\mathrm{Re}\frac{\xi_u}{\xi_t}\right\}. \tag{24}$$

If we use the notation[7]

$$V_{cb}=A\lambda^2,\quad V_{ub}=A\lambda^3(\rho-i\eta),$$
$$V_{td}=A\lambda^3(1-\rho-i\eta)$$

with $\lambda=0.22$, then $J=A^2\lambda^6\eta$ and

$$\begin{aligned} a&=0.37\eta\{[\rho_A(\overline{CP})_A-\rho_B(\overline{CP})_B]K\\ &\quad+[\rho_C(\overline{CP})_C-\rho_B(\overline{CP})_B](K-1)\},\\ K&=\frac{1-\rho}{(1-\rho)^2+\eta^2}. \end{aligned} \tag{25}$$

The only uncertainty in the numerical coefficient comes from the use of Eq. (1). The value of a varies inversely as $(\Delta M/\Gamma)$; thus the relatively large value from the recent experiment has the consequence of decreasing the value of a relative to earlier evaluations.

If we use the result of Hagelin, substituting Eq. (18) into Eq. (24) or (25) we find

$$a=-2.5\times10^{-3}\eta \tag{26}$$

independent of the value of ρ. To fit the observed values of ϵ and ϵ' we need a value of η of about 0.4 within a factor of 2. Thus Eq. (26) gives an asymmetry around 10^{-3}.

To obtain an alternative estimate we look only at the contribution to Γ_{12} from class A intermediate states. This should give a reasonable upper limit since it corresponds to completely eliminating the cancellations in the Hagelin calculation. To estimate $(\overline{CP})_A$ we have calculated in the Appendix the contributions of the intermediate states D^+D^-, $D^{*+}D^-$, D^+D^{*-}, and $D^{*+}D^{*-}$. The calculation is carried out using the Stech[8] factorization approximation which gives reasonable results for such measured exclusive decays as $B\to D\pi$. If we assume all other states cancel this gives

$$(\overline{CP})_A\simeq0.25 \tag{27}$$

and, from Eq. (25),

$$\begin{aligned} a&=1.1\times10^{-2}\frac{\eta(1-\rho)}{(1-\rho)^2+\eta^2}\\ &=-5.6\times10^{-3}\sin2\theta_{td}, \end{aligned} \tag{28}$$

where θ_{td} is the phase of V_{td} in our convention. Since $\eta>0$ and $\rho<1$, Eq. (28) gives a positive value for a. Note that if we had accepted the value of $(\overline{CP})_A$ from Eq. (18) the answer would be 1.6 times as large. Thus we feel it is possible but very unlikely that the asymmetry could be as large as 10^{-2}. Equation (28) is a reasonable order-of-magnitude estimate. Fits to the KM matrix[9] based on the value of ϵ and B-$\bar{B}$ mixing tend to require $|\sin2\theta_{td}|<\frac{1}{2}$. Thus we are led to estimates not much bigger than 10^{-3}.

Comparing Eqs. (26) and (28) we see that even the sign of the asymmetry is uncertain. Thus we cannot rule out a value even closer to zero than that of Hagelin. Our conclusion is that a reasonable estimate of the asymmetry lies between 10^{-3} and 10^{-2} but that neither the sign nor the magnitude can be reliably calculated.

It has been suggested by some authors that the asymmetry might be increased as a result of new physics. The most likely place for new physics to come in is in contributions to M_{12}. For example, it is possible that the large value of ΔM might be mainly due to new physics. Once one uses, as we do here, the empirical value of ΔM, the only effect is in changing the phase of M_{12}. In the Hagelin analysis the low value of a is in part due to the fact that Γ_{12} and M_{12} have the same phase. However we have argued that this feature of the Hagelin analysis is unreliable. Thus while a change in the phase of M_{12} will certainly change the asymmetry, we cannot tell in which direction the change will be and we do not expect any change in our order-of-magnitude estimate.

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APPENDIX

In this appendix we consider the contributions of the lowest-lying two-meson states to $\Gamma_{12}(\mathrm{A})$. In particular, we give estimates for D^+D^-, $D^{*+}D^-$, D^+D^{*-}, and $D^{*+}D^{*-}$.

The part of the QCD-corrected effective weak Hamiltonian which gives type-A contributions can be written

$$H_w^{\mathrm{A}}=\xi_c h_{\mathrm{A}}+\xi_c^* h_{\mathrm{A}}^\dagger ,$$

$$h_{\mathrm{A}}=\frac{G}{\sqrt{2}}[f_1\bar{c}^{\,\alpha}\gamma_\mu(1-\gamma_5)b^\alpha\bar{d}^{\,\beta}\gamma^\mu(1-\gamma_5)c^\beta + f_2\bar{c}^{\,\alpha}\gamma_\mu(1-\gamma_5)c^\alpha\bar{d}^{\,\beta}\gamma^\mu(1-\gamma_5)b^\beta] , \quad \text{(A1)}$$

where α,β are color indices and $f_{1,2}$ are QCD-correction coefficients. In the leading-logarithmic approximation,[10] choosing the scale $\mu\simeq 5$ GeV and $\Lambda_{\mathrm{QCD}}\simeq 0.25$ GeV, one finds

$$f_1\simeq 1.14, \quad f_2\simeq -0.315 . \quad \text{(A2)}$$

The f_2 term is a QCD-induced, effective flavor-changing neutral-current term. Of course, this term disappears in the limit where QCD corrections are small ($f_1\to 1$, $f_2\to 0$). Since $(CP)h_{\mathrm{A}}(CP)^{-1}=h_{\mathrm{A}}^\dagger$ and $CP(D^+D^-)=+1$,

$$\Gamma_{12}(D^+D^-)=-\xi_c^2\,|\langle D^+D^-|h_{\mathrm{A}}|\bar{B}\rangle|^2\rho_{DD} , \quad \text{(A3)}$$

where ρ_{DD} is the two-body phase-space factor. Similarly, we find

$$\Gamma_{12}(D^{*+}D^{*-}) =(-1)^{L+1}\xi_c^2\,|\langle D^{*+}D^{*-}|h_{\mathrm{A}}|\bar{B}\rangle|^2\rho_{D^*D^*} \quad \text{(A4)}$$

because for total angular momentum zero,

$$CP(D^{*+}D^{*-})=(-1)^L$$

with L being the relative orbital angular momentum of the state. Since $L=0,1,2$, both CP even and CP odd are possible. In our calculation we find that $D^{*+}D^{*-}$ is predominantly CP even. Finally, since D^*D must have $L=1$, we find that

$$CP\,|D^{*+}D^-\rangle=|D^+D^{*-}\rangle$$

and

$$\begin{aligned}\Gamma_{12}(D^{*+}D^-)&=\Gamma_{12}(D^+D^{*-})\\ &\equiv\tfrac{1}{2}\Gamma_{12}(DD^*)\\ &=-\xi_c^2\langle\bar{B}|h_{\mathrm{A}}^\dagger|D^+D^{*-}\rangle\\ &\quad\times\langle D^{*+}D^-|h_{\mathrm{A}}|\bar{B}\rangle\rho_{D^*D}\end{aligned} \quad \text{(A5)}$$

which, except for ξ_c^2, is real by time-reversal symmetry. Because these are not CP eigenstates, the sign is not determined by CP. Therefore, one must be careful to be consistent in phase convention in order to calculate the sign. As we will show, we find the sign to be negative.

To estimate the matrix elements in Eqs. (A3)–(A5), we use the factorization approach of Stech.[8] Specifically, for $\bar{B}\to D^+D^-$ this yields

$$\langle D^+D^-|h_{\mathrm{A}}|\bar{B}\rangle=a_1\frac{G}{\sqrt{2}}\langle D^-|J_\mu^{\dagger cd}|0\rangle\langle D^+|J_{cb}^\mu|\bar{B}\rangle , \quad \text{(A6)}$$

where

$$J_{cb}^\mu\equiv\bar{c}^{\,\alpha}\gamma^\mu(1-\gamma_5)b^\alpha, \quad \text{etc.}$$

The factor a_1 is found by combining the direct contribution from the f_1 term in Eq. (A1) with that from doing a Fierz rearrangement of the f_2 term. This yields

$$a_1=f_1+\tfrac{1}{3}f_2\simeq 1.04 . \quad \text{(A7)}$$

The matrix elements in (A4) and (A5) factorize in a completely analogous way.

Using Lorentz covariance and parity, the most general forms of the needed matrix elements can be written

$$\langle D^-(q)|A_\mu^{\dagger cd}|0\rangle=if_D q_\mu , \quad \text{(A8a)}$$

$$\langle D^{*-}(q,\epsilon)|V_\mu^{\dagger cd}|0\rangle=f_{D^*}m_{D^*}\epsilon_\mu^*(q) , \quad \text{(A8b)}$$

$$\langle D^+(k)|V_\mu^{cb}|\bar{B}(p)\rangle=f_+(p+k)_\mu+f_-(p-k)_\mu , \quad \text{(A9)}$$

$$\langle D^{*+}(k,\epsilon)|A_\mu^{cb}|\bar{B}(p)\rangle=-if\epsilon_\mu^*(k)-ia_+(\epsilon^*\cdot p)(p+k)_\mu -ia_-(\epsilon^*\cdot p)(p-k)_\mu , \quad \text{(A10a)}$$

$$\langle D^{*+}(k,\epsilon)|V_\mu^{cb}|\bar{B}(p)\rangle=g\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(k)p^\alpha k^\beta , \quad \text{(A10b)}$$

where[11,12] $f_\pm$, f, $a_\pm$, and g are real, Lorentz-invariant form factors which depend on the kinematic scalar $q^2=(p-k)^2$. The decay constants f_D and f_{D^*} are also real. The relative phase between the matrix elements in Eqs. (A10) is the result of time-reversal symmetry. The remaining phases are the result of a consistent choice of phases for pseudoscalar and vector states.

In the nonrelativistic quark model, by comparing[13] to other decay constants which have been measured, one can estimate

$$f_D\simeq\left[\frac{m_{D^*}}{m_D}\right]^{1/2}f_{D^*}\simeq 175\text{ MeV} , \quad \text{(A11)}$$

where $f_{D,D^*}>0$ in our convention.

The form factors in (A9) and (A10) are calculated in the quark model in Ref. 12 by using flavor independence at zero recoil ($\mathbf{p}=0$, $\mathbf{k}=0$) and assuming a common q^2 dependence, $F(q^2)$, based on dominance of a B_c^* pole. Here, of course, q^2 is fixed for the two-body decays and $F(q^2)$ suppresses the matrix elements. As discussed in Ref. 12, a_+ and a_- cannot be determined separately in this method. However, the lower limit of $fa_+ = 0.35$, derived in Ref. 12 from the measurement of the D^* polarization in $B \to D^* e\bar{\nu}$, is used here.

Since $CPJ^\mu(CP)^{-1} = -J_\mu^\dagger$, in this convention only the matrix elements in (A9) and (A10a) change sign under $B \leftrightarrow \bar{B}$, $D^+ \leftrightarrow D^-$. Therefore,

$$\Gamma_{12}(D^+D^-) \simeq -\xi_c^2 \tfrac{1}{2} a_1^2 G^2 f_D^2 [f_+(m_B^2 - m_D^2) + f_- m_D^2]^2 \rho_{DD} \simeq -3.6 |V_{cb}|^2 \Gamma_0 \xi_c^2 . \quad (A12)$$

Also, since (A10b) does not contribute (by symmetry), we find

$$\Gamma_{12}(DD^*) \simeq -\xi_c^2 2 a_1^2 G^2 f_D f_{D^*} \frac{m_B^2 |\mathbf{k}|^2}{m_{D^*}} f_+ \times [f + a_+(m_B^2 - m_{D^*}^2) + a_- m_D^2] \rho_{DD^*} \simeq -9.5 |V_{cb}|^2 \Gamma_0 \xi_c^2 . \quad (A13)$$

Note that this term contributes with the same sign as the CP-even states. Finally, summing over all polarization states,

$$\Gamma_{12}(D^{*+}D^{*-}) \simeq -\xi_c^2 \tfrac{1}{2} a_1^2 G^2 f_{D^*}^2 m_{D^*}^2 \left[f^2 \left[3 - r + \frac{r^2}{4}\right] + a_+^2 m_B^4 \left[4 - 2r + \frac{r^2}{4}\right] + f a_+ m_B^2 \left[4 - 3r + \frac{r^2}{2}\right] - \tfrac{1}{2} g^2 m_B^4 (1 - 4r) \right] \rho_{D^*D^*} \simeq -7.8 |V_{cb}|^2 \Gamma_0 \xi_c^2 , \quad (A14)$$

where $r \equiv (m_B/m_{D^*})^2$. The VV term (g^2) gives a positive contribution because (A10b) behaves differently under CP. This term gives the $L=1$ contribution and is indeed small ($\sim 5\%$).

Therefore, if we only consider these DD-type states, we get

$$(\overline{CP})_A \simeq 0.25 , \quad (A15)$$

where the value $|V_{cb}| = 0.038$, found in Ref. 12 from semileptonic decays with $fa_+ = 0.35$, has been used. Varying fa_+ from 0.35 to -0.96 in the analysis of Ref. 12 has the consequence of increasing the value of $|V_{cb}|$ from 0.038 to 0.052. However, there is a compensating decrease in the numerical coefficients of Eqs. (A13) and (A14) so that the value of $(\overline{CP})_A$ changes very little.

We have also estimated the contributions from the $\psi\pi$ and $\psi\rho$ intermediate states. These arise directly from the f_2 term in Eq. (A1), but including the Fierz rearrangement they are proportional to a_2 where

$$a_2 = f_2 + \tfrac{1}{3} f_1 \simeq 0.065 . \quad (A16)$$

While we do not trust this very small value of a_2, it is probable that these states are indeed suppressed relative to the others we have considered. Our very uncertain estimate is that the contribution of these states might increase the magnitude of Γ_{12}, and thus $(\overline{CP})_A$, by 20%.

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