Transformation of neutron polarization in polarized media and tests of T invariance

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A general analysis is given for the modification of neutron polarization resulting from passage through a polarized medium. The previously proposed search for precession (properly defined) of neutron spin about $\mathbf{k}\times\mathbf{S}$ remains a valid test of T invariance in the presence of other spin-dependent interactions which cause further changes. However, as noted by Bunakov and Gudkov, the development of transverse polarization along this direction is not an absolute test, but, if such polarization is found, the measurement of corresponding asymmetry in the transmission of polarized neutrons is shown to furnish an unambiguous test. A complete set of such tests is given.

I. INTRODUCTION

Some time ago[1,2] we suggested that the propagation of polarized neutrons through polarized media could be used to search for T-noninvariant interactions. Subsequently, Bunakov and Gudkov[3] pointed out the presence of other spin-dependent interactions of the neutron which also affect its polarization in the same way. This would make the search for T noninvariance through the suggested tests, if not "impossible," at least considerably more difficult. These arguments were elaborated further by Stodolsky.[4] In this note, we systematically examine the Bunakov-Gudkov mechanisms and show that, although there appears to be a simulating mechanism corresponding to every possible spin change which could be induced by T-noninvariant interactions, attention to the primary definition of T invariance permits a clear and unambiguous distinction between effects which are a consequence of T-invariant interactions alone and those which require the participation of T-noninvariant interactions.

Section II reviews the originally suggested tests of T invariance, and the difficulties noted by Bunakov and Gudkov. The Bunakov-Gudkov-Stodolsky effects are then fully analyzed, and the results are then simply related through the basic requirements of time-reversal invariance. Our conclusions are summarized in Sec. III.

II. THE ORIGINAL TESTS AND BUNAKOV-GUDKOV-STODOLSKY SIMULATIONS

The occurrence of a term f_T proportional[5] to $\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{S})$ in the forward-scattering amplitude for neutrons of momentum $\mathbf{k}$ incident on a target with spin $\mathbf{S}$, manifestly violates time-reversal invariance. When polarized neutrons pass through a polarized medium, the real part of f_T would cause the neutron spin to precess around $\hat{\mathbf{n}}\propto\mathbf{k}\times\mathbf{S}$ while the imaginary part of f_T would cause differential absorption of neutrons with spin up and down relative to $\hat{\mathbf{n}}$, and a corresponding change of transverse polarization along this direction. Bunakov and Gudkov[3] drew attention to the fact that, in addition to a possible term of the form f_T, the forward-scattering amplitude for neutrons contains additional spin-dependent terms whose combined effect produces similar consequences. In their view, this makes the proposed experiments "impossible," which we may liberally interpret as an expression of their opinion that detection of the proposed effects would be inconclusive as a test of T invariance, because of the existence of such masquerading phenomena. Stodolsky[4] examined and confirmed, for certain cases, the assertion of Bunakov and Gudkov that spin-dependent but T-*invariant* interactions can mimic the signals which were proposed as tests of T invariance: he therefore proposed other "true null experiments" which "cannot be faked by the other interactions."

We shall discuss below *all* possible changes of neutron polarization which occur when traversing a polarized medium and show that, for every such change, there is no difficulty, in principle, in distinguishing between a genuine T-noninvariant effect and a spurious Bunakov-Gudkov-Stodolsky counterfeit.

The (spin) wave function χ of the incoming neutron is transformed, after traversal of a finite target, into $M\chi$, where M can be written in general as

$$M = AI + B\sigma_x + C\sigma_z + D\sigma_y \ , \tag{1}$$

where A, B, C, and D are complex functions of all variables other than the neutron spin. We choose the direction of the neutron beam as the z axis and assume that the target polarization lies in the x-z plane. Then any term in M proportional to σ_y necessarily violates[6] T invariance, if the target has a form and composition such that a time-reversed neutron beam, viz., one incident from the opposite direction with reversed polarization, sees a target which is indistinguishable[7] from the original one, apart from a well-defined change of polarization. If we make a decomposition similar to (1) for the elementary forward-scattering amplitude,

$$f = f_0 I + f_M(\boldsymbol{\sigma}\cdot\mathbf{S}) + f_P(\boldsymbol{\sigma}\cdot\mathbf{k}) + f_T\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{S}) \ , \tag{2}$$

the various terms (in decreasing order of importance) have the following clear interpretation.

(a) f_0 is the (presumably dominant) spin-independent part of the forward-scattering amplitude.

(b) f_M represents the interaction between the neutron spin and the spin of the target through magnetic and nuclear (pseudomagnetic) interactions, and is quite important[8] for neutron scattering.

(c) f_P is a parity-nonconserving term representing the difference in forward scattering for neutrons whose spins are pointed along or opposite the direction of propagation. While not quite as important as f_M, the existence of such a term is now well established,[9] and can be as large as a few percent of f_0 in particular cases.

(d) f_T is an as yet undetected term, whose occurrence requires T *non*invariance and, if there is no additional factor $(\mathbf{k}\cdot\mathbf{S})$ present,[10,11] also P noninvariance.

Bunakov and Gudkov noted, and Stodolsky confirmed by explicit calculation, that f_P and f_M acting in succession can simulate the effect of f_T, viz., change the neutron spin components in the plane perpendicular to $\hat{\mathbf{n}} \propto (\mathbf{k}\times\mathbf{S})$. Similarly, longitudinal polarization produced by the imaginary part of f_P would, as a consequence of f_M, precess about $\mathbf{S}$ creating transverse polarization along $\hat{\mathbf{y}}$. Consequently, both of the primary effects of f_T [transformation of z component of neutron polarization to x component and vice versa, and the acquisition (or diminution) of polarization along $\hat{\mathbf{y}}$] can be simulated by secondary effects of f_M and f_P.

A possible way to suppress these obscuring effects is to use composite materials for which the average values of f_M (Ref. 3) or f_P (Ref. 11) are made as small as possible. Another way to avoid these complications, proposed by Stodolsky,[4] is to compare pairs of transmission rates, for given initial and final spin orientations, whose difference authentically represents T noninvariance. We shall interpret and extend Stodolsky's tests as part of a complete analysis presented below.

A general examination of all possible spin transformations undergone by a neutron beam while traversing a polarized target [possessing the requisite symmetry described after Eq. (1)] shows that despite the occurrence of the Bunakov-Gudkov-Stodolsky disturbances, the originally proposed "signatures" for T noninvariance are still useful. Spin precession *about* $\hat{\mathbf{n}}$ remains an unambiguous indicator. Differential absorption of neutrons with spin up versus spin down (relative to $\hat{\mathbf{n}}$) and the associated change of polarization *along* $\hat{\mathbf{n}}$ are not definitive, but comparison of the two could provide conclusive evidence.[6]

The density matrix of the incident neutron beam is given by

$$\rho_i = \tfrac{1}{2}(I + \boldsymbol{\sigma}\cdot\mathbf{P}) , \qquad (3)$$

where $\mathbf{P}$ is the initial neutron polarization. The density matrix of the transmitted beam is then given by

$$\rho_f = M\rho_i M^\dagger . \qquad (4)$$

Replacing ρ_i by $\frac{1}{2}\sigma_\lambda P_\lambda$, where we have defined $\sigma_0 \equiv I$ and $P_0 \equiv 1$, ρ_f will be given by a corresponding expression with P_λ replaced by

$$P'_\lambda \equiv (P'_0 = 1, P'_x, P'_y, P'_z) ,$$

apart from a normalization factor. From Eqs. (3) and (4),

$$\rho_f = \gamma_{\mu\nu}\sigma_\mu P_\nu , \qquad (5)$$

where

$$\gamma_{\lambda\mu} = \tfrac{1}{4}\mathrm{Tr}(\sigma_\lambda M \sigma_\mu M^\dagger) . \qquad (6)$$

With M given by the general expression in Eq. (1), it is a simple matter to work out the elements of $\gamma_{\lambda\mu}$, which are shown in Table I. It will be seen that for the off-diagonal elements, which represent changes of neutron polarization, there is a possible D-independent (and thus T-invariant) contribution in every case. So the occurrence of any particular spin transition cannot by itself signify T noninvariance, and one must look more closely at the nature of these transitions.

Dividing by the normalization factor γ_{00}, γ_{j0}/γ_{00} gives the j component P_j of the polarization of the transmitted beam when the initial beam is unpolarized. Similarly, γ_{0j}/γ_{00} is the polarization-dependent fraction A_j of the transmitted beam when the incident beam is polarized along $\hat{\mathbf{j}}$. Inspection of Table I shows that the terms which remain when we set $D=0$ (as required by T invariance) obey the restrictions $\gamma_{01}=\gamma_{10}$ and $\gamma_{03}=\gamma_{30}$, while $\gamma_{02}=-\gamma_{20}$. In general, the transverse polarization P_y generated in an unpolarized beam, passing through a medium polarized in the (x-z) plane, would be unrelated to the asymmetry A_y. But if the effects are caused by purely T-invariant interactions, we must have $P_y = -A_y$. On the other hand, if they arise solely from the (T-noninvariant) differential absorption of neutrons with spin up and down with respect to $\hat{\mathbf{n}} \equiv \hat{\mathbf{y}}$, we must have $P_y = A_y$. Thus, one can clearly distinguish between the two alternatives, if P_y and A_y are nonzero.

Any departure from $P_y + A_y = 0$ unambiguously indicates[6] the presence of T-noninvariant interactions. If we inspect the remaining off-diagonal elements in Table I, we find that T invariance (i.e., $D=0$) requires γ_{ij} to be symmetric if both $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are orthogonal to $\hat{\mathbf{y}}$, viz., if both are in the (x,z) plane, while γ_{ij} should be antisymmetric when either $\hat{\mathbf{i}}$ or $\hat{\mathbf{j}}$ is along $\hat{\mathbf{y}}$. According to Eq. (5), γ_{jk} measures the part of the excess of spin-up (along $\hat{\mathbf{j}}$) neutrons in the transmitted beam, which is proportional to the polarization along $\hat{\mathbf{k}}$ of the incident. Since a rotation is represented by an antisymmetric submatrix, this means that rotation (precession) of the neutron spin about the $\hat{\mathbf{x}}$ or $\hat{\mathbf{z}}$ axes is perfectly consistent with T invariance, whereas precession about $\hat{\mathbf{y}}$ is not. This confirms the assertion made in Ref. 1 that observation of neutron spin *precession* about $\hat{\mathbf{y}}$ is an unambiguous indicator of T noninvariance. The transformations of neutron spin in the (x,z) plane arising from Bunakov-Gudkov-Stodolsky simulations have a qualitatively different character. Table I shows that the excess of x-polarized neutrons in a beam initially polarized along $\hat{\mathbf{z}}$, from T-invariant interactions [proportional to $\mathrm{Re}(B^*C)$] is exactly the same as the excess of z-polarized neutrons developed in the

TABLE I. Matrix elements of $\gamma_{\alpha\beta}=\frac{1}{4}\mathrm{Tr}(\sigma_\alpha M\sigma_\beta M^\dagger)$. Diagonal elements which have not been mentioned in the text are omitted for simplicity.

$\alpha \backslash \beta$	0	$1=x$	$2=y$	$3=z$
0	$\frac{1}{2}(\|A\|^2+\|B\|^2\|C\|^2+\|D\|^2)$	$\mathrm{Re}(A^*B)+\mathrm{Im}(C^*D)$	$\mathrm{Re}(A^*D)+\mathrm{Im}(B^*C)$	$\mathrm{Re}(A^*C)+\mathrm{Im}(D^*B)$
1 (x)	$\mathrm{Re}(B^*A)+\mathrm{Im}(CD^*)$		$\mathrm{Re}(B^*D)+\mathrm{Im}(CA^*)$	$\mathrm{Re}(B^*C)+\mathrm{Im}(AD^*)$
2 (y)	$\mathrm{Re}(D^*A)+\mathrm{Im}(BC^*)$	$\mathrm{Re}(D^*B)+\mathrm{Im}(AC^*)$		$\mathrm{Re}(D^*C)+\mathrm{Im}(A^*B)$
3 (z)	$\mathrm{Re}(C^*A)+\mathrm{Im}(DB^*)$	$\mathrm{Re}(C^*B)+\mathrm{Im}(A^*D)$	$\mathrm{Re}(C^*D)+\mathrm{Im}(AB^*)$	

same way in a neutron beam initially polarized along $\hat{\mathbf{x}}$. On the other hand, true precession about $\hat{\mathbf{y}}$ would require the amount of z polarization arising from initially x-polarized neutrons to be equal in magnitude but *opposite in sign* to the x polarization resulting from initial z polarization, as is seen to be the case for the terms proportional to D.

The T-invariance restrictions on the matrix $\gamma_{\alpha\beta}$ can be synthesized into

$$\gamma_{\beta\alpha}=\eta_\alpha\eta_\beta\gamma_{\alpha\beta}\ , \tag{7}$$

where $\eta_\lambda=-1$ for $\lambda=2$ and $\eta_\lambda=+1$ otherwise. When one of the indices is zero, Eq. (7) is a statement of the general polarization-asymmetry requirement[6] of T invariance:

$$P_j=\eta_j A_j\ . \tag{8}$$

The symmetries required by Eq. (7) for the remaining off-diagonal elements, when neither index is zero, can be deduced as follows. Replacing B, D, and C in Eq. (1) by h_1, h_2, and h_3, respectively, we find the following by direct substitution in Eq. (6):

$$\gamma_{jk}=\tfrac{1}{2}(\ |\ A\ |^2-h_m h_m^*)\delta_{jk}+\mathrm{Re}(h_j h_k^*)-\epsilon_{jkm}\mathrm{Im}(Ah_m^*)\ . \tag{6$'$}$$

Equation (6$'$) shows that if $h_y\equiv D$ is zero, as required by time-reversal invariance, γ_{jk} must be symmetric for any two axes $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ perpendicular to $\hat{\mathbf{y}}$, and antisymmetric (for $j\neq k$) when either $\hat{\mathbf{j}}$ or $\hat{\mathbf{k}}$ is chosen along $\hat{\mathbf{y}}$.

Finally, we show how the various tests all relate to the basic condition of reciprocity,

$$C_\alpha(\beta)=C_{\tilde\beta}(\tilde\alpha)\ , \tag{9}$$

required by T invariance. $C_\alpha(\beta)$ is the counting rate for transmission of neutrons with spin along $\hat{\boldsymbol{\beta}}$ when the incident beam is fully polarized along $\hat{\boldsymbol{\alpha}}$. $\tilde\alpha$ is the direction of spin corresponding to $\hat{\boldsymbol{\alpha}}$ in the time-reversed system. Under the action of time reversal T (more accurately, motion reversal if we adopt an active interpretation), $\mathbf{k}$, $\mathbf{S}$, and $\boldsymbol{\sigma}$ are all reversed. A further 180° rotation R about $\hat{\mathbf{y}}\propto(\mathbf{k}\times\mathbf{S})$ brings $\mathbf{k}$, $\mathbf{S}$, and the (x,z) components of $\boldsymbol{\sigma}$ back to their original values but σ_y remains reversed. We may express this formally by saying that under RT, σ_j changes to $\eta_j\sigma_j$, where η_j has been defined after Eq (7). Under the assumption of rotational invariance, Eq. (9) must also hold when $\tilde\alpha$ and $\tilde\beta$ represent the RT transforms of α and β, respectively. We distinguish between three possible cases.

(a) $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ are both chosen along the $\hat{\mathbf{n}}\equiv\hat{\mathbf{y}}$ direction. Then both will reverse sign under RT, and if we set $\hat{\boldsymbol{\beta}}=\hat{\boldsymbol{\alpha}}$ in Eq. (9) we obtain

$$C_n(n)=C_{-n}(-n)\ , \tag{10}$$

which is the first of the "novel" tests of T invariance proposed by Stodolsky.[4] The choice $\hat{\boldsymbol{\beta}}=-\hat{\boldsymbol{\alpha}}$ yields an identity.

(b) Both $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ lie in the (x,z) plane. Then both remain unchanged under RT, and the choice $\hat{\boldsymbol{\alpha}}=\hat{\boldsymbol{\beta}}$ now gives an identity, while an interesting relation follows from $\hat{\boldsymbol{\alpha}}=-\hat{\boldsymbol{\beta}}$:

$$C_l(-l)=C_{-l}(l)\ , \tag{11}$$

where $\hat{\mathbf{l}}$ denotes *any* unit vector in the (x,z) plane. Two further tests proposed explicitly by Stodolsky correspond respectively to the choices $\hat{\mathbf{l}}=\hat{\mathbf{z}}$ and $\hat{\mathbf{l}}=\hat{\mathbf{x}}$ in Eq. (11). Since $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ do not have to be chosen in the same direction, we obtain, more generally,

$$C_l(l')=C_{l'}(l)\ , \tag{12}$$

where $\hat{\mathbf{l}}$ and $\hat{\mathbf{l}}'$ are any two unit vectors orthogonal to $\hat{\mathbf{n}}$, a relation also mentioned by Stodolsky. It will be seen that Eq. (11) is a particular case of Eq. (12).

(c) $\hat{\boldsymbol{\alpha}}$ is chosen in the (x,z) plane while $\hat{\boldsymbol{\beta}}$ is along $\hat{\mathbf{n}}$. Then

$$C_l(n)=C_{-n}(l)\ . \tag{13}$$

Particular cases of Eq. (13) are

$$C_x(y)=C_{-y}(x)\ \text{ and }\ C_z(y)=C_{-y}(z)\ . \tag{14}$$

We have shown elsewhere[6] how the polarization-asymmetry relation, Eq. (8), follows directly from the reciprocity relations (9). Now we show how the remaining relations also follow directly from reciprocity. From the defining equation (5), the matrix elements γ_{jk} (when the latin indices run over the spatial components from 1 to 3) are directly related to the counting rates through

$$\gamma_{jk}/\gamma_{00}=\frac{C_k(j)-C_k(-j)-C_{-k}(j)+C_{-k}(-j)}{C_k(j)+C_k(-j)+C_{-k}(j)+C_{-k}(-j)}\ . \tag{15}$$

For any two directions $\hat{\mathbf{l}}$ and $\hat{\mathbf{l}}'$, orthogonal to $\hat{\mathbf{y}}$, RT invariance requires Eq. (12) to hold. When we make the corresponding substitutions in Eq. (15), for $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ in the $(x$-$z)$ plane, we obtain an expression which coincides with the definition of γ_{kj}. Similarly, if $\hat{\mathbf{j}}$ is chosen as $\hat{\mathbf{n}}$ and $\hat{\mathbf{k}}$

along $\hat{l}$, we may replace the individual terms in the expression for γ_{nl} according to Eq. (13), which applies in this case, and obtain an expression which is identical with that which defines $-\gamma_{ln}$. Thus we have proved that the symmetries imposed on γ_{jk} by the condition of T invariance arise directly from the primitive requirements of reciprocity.

Any of the six distinct cases of Eq. (7) serves as a test of T invariance. The three equations in which one index on each side is zero require measurement of neutron spin before *or* after transmission through the medium. Test of the other three equations, in which no index is zero, requires the spin to be measured both before *and* after transmission. With available experimental techniques, the method would be sensitive to interactions which would not have been detected in previous experiments.

III. SUMMARY

The main conclusion of this paper is that the changes of neutron polarization in passing through a polarized medium provide several possible tests of T invariance. Spin-dependent (T-invariant) interactions beside the sought-for T-noninvariant interaction complicate the search by causing some of the effects to be expected from a T-noninvariant interaction. Nevertheless, there is no difficulty of principle in distinguishing between T-invariant and T-noninvariant effects. All the proposed tests are deducible directly from the primitive requirement of reciprocity, permitting no ambiguity about their validity as tests of T invariance. Explicitly, the tests are (a) comparison of polarization P_j produced in an initially unpolarized beam, with the asymmetry A_j for transmission of neutrons polarized in the same direction, and (b) comparison of the detection rate $C_\alpha(\beta)$, for neutrons with spin along $\hat{\beta}$ when the incident neutrons are polarized along $\hat{\alpha}$, with an appropriately related $C_{\tilde{\beta}}(\tilde{\alpha})$. A combination of these rates can be interpreted in terms of spin precession.

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[5]The discussion which follows applies both to the displayed term of the form originally considered in Refs. 1 and 2 as well as to the variant containing an additional factor of $\mathbf{k}\cdot\mathbf{S}$ suggested by H. S. Mani (unpublished), discussed in Ref. 11.

[6]P. K. Kabir, Phys. Rev. Lett. **60**, 686 (1988). In Ref. 4, Stodolsky showed that, for a homogeneous medium described by a refractive index which is linearly related to the amplitude for forward scattering of neutrons by the individual scatterers, such a term cannot arise if the individual forward-scattering amplitudes themselves are free of such terms.

[7]One could check this in practice by seeing whether any result is changed when the target is turned back to front or rotated about the beam axis.

[8]See, e.g., I. Gurevich and V. Tarasov, *Low Energy Neutron Physics* (North-Holland, Amsterdam, 1968).

[9]See, e.g., B. Heckel *et al.*, J. Phys. (Paris) Colloq. **45**, C3-89 (1984).

[10]Such a factor is possible only for target spin $S \geq 1$, Ref. 11.

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