Geometrical branching model: Correlations and jets

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A geometrical model for multiparticle production at low as well as high p_T is discussed. Below the threshold of substantial production of jets, the model has geometrical scaling and Koba-Nielsen-Olesen scaling, the latter being a result of Furry branching in multiplicity distribution at each impact parameter. Above the threshold the production of jets is explicitly taken into account by use of perturbative QCD. The separation into soft and hard components is done in the eikonal formalism consistent with unitarity. Geometrical scaling defines the soft component of the eikonal function. The hard component is related to the jet-production cross section; the p_T cutoff is not chosen arbitrarily, but is to be determined by σ_{el} and σ_{tot}. Forward-backward multiplicity correlation can be calculated separately for the cases of no jets and with jets. The emphasis in this paper is on the formalism of the model. The procedure to determine the multiplicity distribution at all s is discussed.

I. INTRODUCTION

In a recent paper,[1] we have shown that a model for multiparticle production at low p_T based on Furry branching[2] and impact-parameter smearing can lead to Koba-Nielsen-Olesen (KNO) scaling[3] and can produce a multiplicity distribution P_n that fits well the experimental data in the CERN ISR range, $\sqrt{s} < 65$ GeV. In that model the violation of KNO scaling, as observed by the UA5 Collaboration[4] at the CERN $Sp\bar{p}S$ collider, is attributed to the production of low-E_T jets (called minijets), observed by the UA1 Collaboration.[5] We develop here the formalism to include jet production in the geometrical branching model consistent with unitarity and to describe the forward-backward multiplicity correlation in that model. At energies where jet production is unimportant, we have calculated the correlation parameter, which agrees well with the data from the ISR (Ref. 6). The result is relatively independent of the details of P_n. At higher energies where jet production could be important, we have at present only a crude estimate, which is in rough agreement with the data from the $Sp\bar{p}S$ (Ref. 7). An extensive phenomenological work on the subject is currently in progress.

It is generally agreed that at some value of the collision energy the production of jets should become an important part of the total cross section; however, there is some disagreement on whether or not the observation of minijets by UA1 should mark the beginning of such a regime, since the experimental criteria for the definition of a jet have been set rather arbitrarily. Our view on this issue is based more on the general observation that a number of features of high-energy data exhibit energy independence up to the top of the ISR energies. They are[8] (1) KNO scaling, (2) $\langle p_T \rangle \simeq 350$ MeV, (3) $\sigma_{el}/\sigma_{tot} \simeq 0.17$, and (4) $B/\sigma_{tot} \simeq 0.3$, where B is the slope of the diffraction peak. Features (3) and (4) are usually referred to as geometrical scaling. The fact that all these scaling properties are violated at $\sqrt{s} \geq 200$ GeV clearly suggests that a threshold exists somewhere between the top of the ISR and the bottom of the $Sp\bar{p}S$ collider energies, say, 100 GeV for definiteness. Above that threshold a new dynamical process makes a sufficient contribution to the total cross section as to make the scaling violation noticeable. It does not mean that for $\sqrt{s} < 100$ GeV such processes are nonexistent. Since important contributions from the hard scattering of quarks and gluons are not only expected theoretically but also observed experimentally, it seems reasonable to identify the new dynamical process with the production of jets and to regard 100 GeV as the approximate value of $\sqrt{s}$, above which such processes can no longer be ignored—whatever the criteria for the definition of a jet may be. The observation of minijets by UA1 should be regarded as a specific demonstration of the plausibility of this view. Conversely, we would regard any theoretical interpretation of the high-energy data that does not take into account the production of jets at $\sqrt{s} > 100$ GeV as missing something dynamically significant.

Although the study of long-range correlations was initiated a long time ago,[9] even before the ISR data on forward-backward (F-B) multiplicity correlation was published,[6] the subject laid dormant for many years until the CERN collider experiment[7] kindled renewed interest in it. A large number of papers[10–16] have reported on recent studies from various approaches, ranging from dual parton model based on specific dynamics to statistical models practically devoid of dynamical details. Nearly the only consensus is that the F-B correlation is as important as the multiplicity distribution in characterizing the nature of the dominant mechanism for particle production at low p_T. Our aim in this paper is to investigate how various dynamical and geometrical properties of our branching model without and with minijets affect the F-B multiplicity correlation.

The branching model is, relatively speaking, a late comer as a possible description of multiparticle production at low p_T (Refs. 17–20), despite the fact that Giovannini[21] long ago worked on QCD branching and its connection with jet calculus.[22] Its virtue lies in the dual property that on the one hand it agrees at the level of leading order with the results of perturbative calculations for hard-collision processes, while on the other hand its dynamical content can be extended to the realm of soft interaction, where no calculational method (perturbative or nonperturbative) in QCD exists, since no small parameter is assumed in the branching mechanism.[23] Multiparticle production in soft hadronic interaction is a many-body problem involving long distances (long in the sense that the relevant distance scale is $> m_\pi^{-1}$). Thus it is not only natural but sensible to make use of the powerful techniques of stochastic methods[23,24] to treat what is untractable in strong interactions. What has been lacking in the past was a convincing demonstration that the branching model provides an excellent phenomenological description of the data. Because of the complexity of the various strands of physics involved, we believe that the demonstration must come in stages.

The first step toward serious phenomenology in the branching model has been taken in Ref. 1. Regarding minijets as being responsible for the violation of KNO scaling, a subject that extends beyond the scope of branching anyway, we focused our attention on the issue of KNO scaling in the ISR energy range. We found that branching alone is insufficient: it must be supplemented by impact-parameter (IP) smearing. The importance of considering the geometrical aspect of extended objects in collision (such as hadrons and nuclei) has been known for a long time.[25-29] Various attempts have been made to smear narrow distributions (such as the multiplicity distribution for e^+e^- annihilation) to get broad distributions that scale at ISR (Refs. 25–28). Our success in amalgamating branching with IP smearing implies that, even at fixed impact parameter, the multiplicity distribution is broad and unlike those associated with jets produced in hard processes.[4] Above 100 GeV, minijets make a significant contribution to the total cross section[5] and their effect in further broadening the multiplicity distribution must separately be taken into account.

Further testing of the physical relevance of the geometrical branching model must now go beyond the multiplicity distribution, and the natural next step is to examine its implications on correlations. In the absence of a reliable prediction on the two-particle inclusive distribution at this stage, we concentrate on the long-range forward-backward multiplicity correlation. We shall show that a branching process by itself does not have *F-B* multiplicity correlation, just as back-to-back jets do not. However, IP smearing introduces correlation for branching, and smearing in the virtuality of the jets does the same for minijets. For these and other reasons the correlation parameter is nonzero at ISR and increases at the S$p\bar{p}$S collider.

An important part of our approach is to demonstrate that jets can be introduced at higher energies consistent with unitarity and geometrical scaling of the soft component. Since the border line between an event with a jet and one without is somewhat fuzzy, our approach is actually the opposite of the experimental one. We use geometrical scaling to define the soft component and introduce jets in the hard component in such a way that their combination satisfies unitarity at all energies.

The plan of the paper is as follows. In Sec. II we review the geometrical scaling model without jets. It is then applied to the problem of *F-B* multiplicity correlation in Sec. III. In Sec. IV we extend the geometrical scaling model to include jet production in a way that is consistent with unitarity. Then in Sec. V we develop the formalism for calculating the correlation parameter when the collision energy is above the jet-production threshold. A rough estimate of the parameter is given in the Appendix.

II. THE GEOMETRICAL BRANCHING MODEL

We review in this section the key points of the geometrical branching model.[1,23] We give first the highlights of Furry branching, and then IP smearing.

In connection with the study of cosmic-ray showers, Furry[2] first considered the branching equation

$$\frac{d}{dt}F_n^k=(n-1)F_{n-1}^k-nF_n^k\ , \tag{2.1}$$

where $F_n^k(t)$ is the multiplicity distribution at time t due to branching; n is the number of particles at t, and k is the number of initial sources at $t=0$. The notion of time may be ill defined in multiparticle production at high energy, but its meaning in (2.1) can be unambiguously deduced by multiplying (2.1) by n and then summing over n, yielding (with $\sum_n F_n^k=1$)

$$t=\ln(\bar{n}/k)\ , \tag{2.2}$$

where $\bar{n}$ is the average multiplicity

$$\bar{n}=\sum_n nF_n^k\ . \tag{2.3}$$

Later we shall emphasize the dependences of $\bar{n}$ and k on s and the impact parameter b. For now, we need only regard (2.1) as the essence of a stochastic process describing branching, viz., for an incremental increase in t, or s, any one of the n particles can branch.

If we define the generating function $G_k(u,t)$ by

$$G_k(u,t)=\sum_{n=k}^{\infty}u^nF_n^k(t)\ , \tag{2.4}$$

it can be shown from (2.1) that[23]

$$G_k(u,t)=\left[\frac{pu}{1-qu}\right]^k\ , \tag{2.5}$$

where $p=e^{-t}$ and $q=1-p$. By negative-binomial expansion[30] of (2.5) we get

$$F_n^k=\begin{bmatrix}n-1\\ n-k\end{bmatrix}p^kq^{n-k}\ . \tag{2.6}$$

In the following we shall always take F_n^k to be zero for $n<k$. From $\partial G_k/\partial u$ and $\partial^2 G_k/\partial u^2$ evaluated at $u=1$, we further get

$$\bar{n}=k/p\ , \tag{2.7}$$

$$\gamma_2 \equiv (\overline{n^2} - \bar{n}^2)/\bar{n}^2 = \frac{1}{k} - \frac{1}{\bar{n}} \ . \tag{2.8}$$

Let us now define an evolution parameter w:

$$w = \bar{n}/k = e^t \ . \tag{2.9}$$

Then the Furry distribution in terms of w for any nonintegral k is

$$F_n^k(w) = \frac{\Gamma(n)}{\Gamma(k)\Gamma(n-k+1)} \left[\frac{1}{w}\right]^k \left[1-\frac{1}{w}\right]^{n-k} \ . \tag{2.10}$$

The factorizability of (2.5) implies an important property of the Furry distribution, since for $k = k_1 + k_2$ we have

$$G_k(s,t) = G_{k_1}(s,t) G_{k_2}(s,t) \ . \tag{2.11}$$

It then follows from (2.4) that

$$F_n^{k_1+k_2}(w) = \sum_{n_1 n_2} F_{n_1}^{k_1}(w) F_{n_2}^{k_2}(w) \delta_{n,n_1+n_2} \ , \tag{2.12}$$

a relationship of some significance in the following.

The Furry distribution (2.10) was applied directly to a phenomenological description of P_n in pp collision with only moderate success.[31] However, after IP smearing the result gave a spectacular fit of the KNO distribution in the ISR energy range.[1] The key point is that the branching process is a description of particle production for every collision between hadrons at any (random) impact parameter b. The number of initial sources of branching depends on b in a way that is related to the opacity of hadrons in the eikonal model. It is after the integration over b that the resultant multiplicity distribution should be compared to the data. For the purpose of facilitating subsequent discussions on correlation, let us recall the geometrical description of hadronic collisions.[25,29]

For $\sqrt{s} < 100$ GeV we assume geometrical scaling[29] so the impact parameter may be written in the form $b = Rb_0(s)$, where R is a dimensionless scaled radius, in terms of which the eikonal $\Omega(s,b)$ becomes simply $\Omega(R)$. The inelastic overlap function $g(s,b)$ becomes

$$g(R) = 1 - \exp[-2\Omega(R)] \ , \tag{2.13}$$

which satisfies $\int_0^\infty dR^2 g(R) = 1$. If, in general, $Q_n(s,R)$ denotes the multiplicity distribution at a given b, then IP smearing means

$$P_n = \int_0^\infty dR^2 g(R) Q_n(s,R) \ , \tag{2.14}$$

which we shall abbreviate by the notation

$$P_n = \{Q_n\} \ . \tag{2.15}$$

Average multiplicities, defined by

$$\langle n \rangle = \sum_n nP_n \quad \text{and} \quad \bar{n} = \sum_n nQ_n \tag{2.16}$$

are then related by

$$\langle n \rangle = \{\bar{n}\} \ . \tag{2.17}$$

Clearly, $\bar{n}$ depends on both s and R.

A variety of possibilities have been considered for $Q_n(s,R)$ in the past.[26,28] In the geometrical branching model we identify $Q_n(s,R)$ with the Furry distribution $F_n^k(w)$, and introduce the R dependence in a specific way.[1] Since both $\bar{n}$ and k depend, in general, on s and R, we assume that they have the same R dependence by factorizing as follows:

$$\bar{n}(s,R) = \langle n \rangle h(R) \ , \tag{2.18}$$

$$k(s,R) = \langle k \rangle h(R) \ . \tag{2.19}$$

Thus the evolution parameter w is a function of s only:

$$w(s) = \langle n \rangle / \langle k \rangle \ . \tag{2.20}$$

From (2.17), $h(R)$ is constrained by the normalization condition

$$\{h\} = 1 \ , \tag{2.21}$$

which, in turn, in conjunction with (2.19) defines $\langle k \rangle$ as $\{k(s,R)\}$, the average number of initial clusters.

A crucial part of the model is in specifying the R dependence of $h(R)$, which is the key link between branching and geometry, as evidenced by the variables of $F_n^k(w)$: only the superscript k depends on R through (2.19). That is, R specifies the initial condition of the collision process; $k(R)$ (with s omitted) specifies the number of initial clusters (or sources) at that R. What happens thereafter is specified by the branching process for each cluster in a factorizable way [cf. (2.12)]. Since the efficiency for particle production must be intimately connected with the opacity of hadrons, it is reasonable for us to assume the relationship

$$h(R) = h_0 \Omega^\gamma(R) \ , \tag{2.22}$$

where γ is an adjustable parameter in the model, but h_0 is fixed by (2.21). In Ref. 1 we have been able to render an excellent fit of P_n by choosing γ to be

$$\gamma = 0.3 \pm 0.05 \ . \tag{2.23}$$

One of the aims of this paper is to show that this value of γ is consistent with what is necessary to yield the observed F-B multiplicity correlation. In this sense of an overconstrained system we have therefore no more free parameters in the model.

In the following we shall need $\{h^2\}$. For brevity we denote it by

$$\mu \equiv \{h^2\} \ , \tag{2.24}$$

whose dependence on γ is shown by the dashed line in Fig. 1, on the basis that $\Omega(R)$ has the form $1.4e^{-1.6R^2}$ (Refs. 25 and 1). From (2.8) and (2.20) we get

$$w - 1 = (C_2 - \mu)\langle n \rangle \ , \tag{2.25}$$

where

$$C_2 = \langle n^2 \rangle / \langle n \rangle^2 \ . \tag{2.26}$$

We use the experimental value[4] of 1.2 for C_2 and the value of μ determined by the best value of γ to specify the s dependence of w, assuming that $\langle n \rangle$ is a known

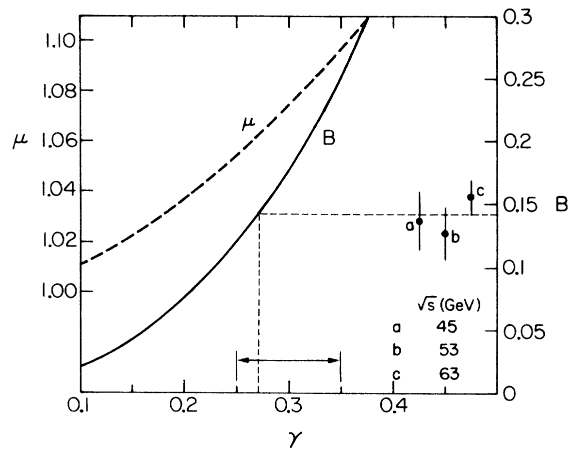

FIG. 1. Plot of μ and B vs γ. Data points are from Ref. 6. The bracketed range of γ is from fitting the multiplicity distribution in Ref. 1.

function of s. Note that this procedure yields the s dependence of $\langle k \rangle$. What has been used as empirical input is the value and constancy of C_2. The output of the model are the values and near constancies of the three higher moments C_3, C_4, and C_5.

III. FORWARD-BACKWARD CORRELATION WITHOUT MINIJETS

We now investigate the implications of the geometrical branching model on *F-B* multiplicity correlation. In this section we focus first on the energy range $\sqrt{s} < 100$ GeV, for which there is KNO scaling and our model is quite successful in producing the observed P_n without minijets.

The multiplicity correlation has been measured either for forward and backward hemispheres without a rapidity gap, or with a gap of $\Delta\eta = 2$ at $\eta = 0$, where η is pseudorapidity.[6] We shall concentrate on the latter because it eliminates any contaminating effects due to short-range correlation, which is usually regarded as having a range of $\Delta\eta < 2$. Let us use n_1 and n_2 to denote the particle multiplicities measured in two detector windows separated in η and having arbitrary η widths. For definiteness, we may associate n_1 with a window in the forward hemisphere, and n_2 backward, with an η gap of at least 2 units between them.

In terms of the joint multiplicity distribution $P_{n_1 n_2}$ the average n_2 for fixed n_1 is defined by

$$\langle n_2(n_1) \rangle = \sum_{n_2} n_2 P_{n_1 n_2} / P_{n_1} , \tag{3.1}$$

where $P_{n_1} = \sum_{n_2} P_{n_1 n_2}$. If $\langle n_2(n_1) \rangle$ depends linearly on n_1, i.e.,

$$\langle n_2(n_1) \rangle = A + B n_1 , \tag{3.2}$$

as has been observed,[6] then it can be shown that[7,11,12]

$$B = \frac{\langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle}{\langle n_1^2 \rangle - \langle n_1 \rangle^2} . \tag{3.3}$$

We want to establish here the relationship between this correlation parameter B and the parameter γ discussed in the previous section.

In the geometrical branching model the number of initial sources of branching is $k(R)$ at R. How these $k(R)$ sources are distributed in rapidity is a subject that has not yet been investigated in detail. It is reasonable to assume at this point that they are uniformly distributed throughout the rapidity range that excludes the fragmentation regions, since we regard the branching model to be applicable to only the particle production process in the central region due to the overlap of the incident hadrons in the transverse plane, while the spectatorial portions are mainly responsible for the production of particles in the fragmentation regions. In this picture of uniform distribution of the initial sources in the rapidity space, we further assume that the sources in the forward (backward) hemisphere give rise to particles through branching that stay mainly in the forward (backward) hemisphere after the hadronization process is complete. Some crossover is unavoidable, but the particles that do are limited in rapidity range and would be excluded from a consideration of the long-range multiplicity correlation, if the separation between the forward and backward detector windows has a pseudorapidity gap $\Delta\eta \geq 2$. The basis for this view is grounded in the observation that phenomenologically the extent of branching, as measured by w, is not great:[1] since $C_2 = 1.2$ and μ is between 1.05 and 1.1, we see from (2.25) that w is less than 3. That is, each source branches, on the average, into less than three charged particles. Considerable crossing or interlacing of the branches is therefore not expected.

On the basis of this general picture we now make the specific assumption that there are k_1 initial sources somewhere in the forward hemisphere that gives rise to the n_1 particles, and k_2 in the backward hemisphere giving rise to n_2, provided that the n_1 and n_2 particles are well separated by a rapidity gap. This assumption is obviously well justified if the gap is very large; in our present case it amounts to the assumption that $\Delta\eta = 2$ is wide enough. Because of the factorizable property of the Furry distribution,

$$F_{n_1 n_2}^{k_1 k_2}(w) = F_{n_1}^{k_1}(w) F_{n_2}^{k_2}(w) , \tag{3.4}$$

which follows from (2.11) and (2.12), we therefore have

$$P_{n_1 n_2}(w) = \int dR^2 g(R) F_{n_1}^{k_1(R)}(w) F_{n_2}^{k_2(R)}(w) . \tag{3.5}$$

The R dependences are exhibited explicitly here to make transparent the origin of long-range multiplicity correlations. Note that if it were not for IP smearing, $P_{n_1 n_2}$ would have been factorizable, and correlations would be totally absent.

There is a tacit assumption in (3.5) that should be mentioned. The detector windows for n_1 and n_2 should not only be separated, but also be wide enough themselves so that all the particles produced by branching from k_1 and k_2 are included. Otherwise, the evolution parameter w on the right-hand side of (3.5) must be reduced to reflect the appropriate amount of branching necessary; concurrently, k_1 and k_2 would not be the sources at $t=0$, but at some later t. The experimental conditions of Ref. 6 satisfy our criteria for the validity of (3.5).

For the observables related to the n_1 window, we have

$$P_{n_1}=\{F_{n_1}^{k_1}\}\ , \tag{3.6}$$

$$\overline{n_1^l}=\sum_{n_1} n_1^l F_{n_1}^{k_1}\ , \tag{3.7}$$

$$\bar{n}_1/k_1=w\ , \tag{3.8}$$

$$\overline{n_1^2}=\bar{n}_1^2+(w-1)\bar{n}_1\ , \tag{3.9}$$

the last equation being a consequence of (2.8) and (3.8). In performing the IP smearing of (3.9) we assume the same R dependence for $k_1(R)$ as for $k(R)$, i.e.,

$$k_1(R)=\langle k_1\rangle h(R)\ . \tag{3.10}$$

This is reasonable so long as R is not too large for which there may be so few particles produced that kinematical constraints in rapidity may lead to nonuniform η distribution of the initial sources. Since the effect of large-R collisions on average quantities is negligible, we shall regard (3.10) as valid for all R. Thus we obtain, from (3.9),

$$\langle n_1^2\rangle=\mu\langle n_1\rangle^2+(w-1)\langle n_1\rangle\ . \tag{3.11}$$

Similar results follow for n_2. For multiplicities in both windows, we have

$$\overline{n_1n_2}=\sum_{n_1n_2} n_1n_2F_{n_1n_2}^{k_1k_2}=\bar{n}_1\bar{n}_2\ , \tag{3.12}$$

$$\langle n_1n_2\rangle=\{\overline{n_1n_2}\}=\mu\langle n_1\rangle\langle n_2\rangle\ . \tag{3.13}$$

Substituting these results in (3.3) we obtain

$$B=\frac{(\mu-1)\langle n_1\rangle\langle n_2\rangle}{(\mu-1)\langle n_1\rangle^2+(w-1)\langle n_1\rangle}\ . \tag{3.14}$$

Notice immediately that without IP smearing we would have factorizability in (3.13) with $\mu=1$, and there would be no multiplicity correlation. Now, consider the common case where the two windows are of equal size, symmetrically placed on opposite sides of $\eta=0$. Define

$$r=\frac{\langle n_1\rangle}{\langle n\rangle}=\frac{\langle n_2\rangle}{\langle n\rangle}\ . \tag{3.15}$$

Then it follows from (2.25) and (3.14) that

$$B=\left[1+\frac{C_2-\mu}{(\mu-1)r}\right]^{-1}\ . \tag{3.16}$$

The experimental value of C_2 in the ISR range is 1.2. The dependence of μ on γ is shown by the dashed curve in Fig. 1. It has been shown in Ref. 1 that the value $\gamma=0.3\pm0.05$ is consistent with $C_2=1.2$ and gives a good fit of P_n throughout the ISR energy range. Now, the value of r depends on the correlation experiment. In Ref. 6 the pseudorapidity cut is for $|\eta|\geq1$, but the acceptance is for $|\eta|\leq3.6$. Thus, assuming flat distribution, we get

$$r=2.6/7.2=0.36\ . \tag{3.17}$$

The resultant values of B as a function of γ are shown by the solid curve in Fig. 1.

The experimental values of B at the three upper ISR energies are also shown in Fig. 1. Their average value is 0.14, which corresponds to $\gamma=0.27$, well within the range $\gamma=0.3\pm0.05$ required by P_n. We regard this as a very satisfactory demonstration that the geometrical branching model is consistent with the most prominent features of soft production of particles at the ISR energies.

It should be remarked that at the two lower ISR energies (24 and 31 GeV) the values of B are 0.032 ± 0.015 and 0.063 ± 0.016, respectively.[6] At those energies the central rapidity plateau extends out only to $|\eta|=2$ or less. Thus the value of r would be smaller, resulting in lower values for B. However, we feel that there is no quantitatively reliable way to determine the precise value of B, since there are numerous uncertainties at lower energies, not the least of which is the greater importance of the effect of diffractive production of particles (which has no F-B correlation) for the rapidity cut and acceptance of the experiment. We have therefore left those two data points out from Fig. 1.

IV. GEOMETRICAL SCALING, HARD PROCESSES, AND UNITARITY

As discussed in Sec. I we regard the production of jets through hard collisions of partons as an important part of the total cross section that cannot be neglected when $\sqrt{s}>100$ GeV. Various attempts have been made to associate the rising total cross section with jet production.[32,33] The simple addition of the soft and hard components of σ_{tot} has been criticized[34,35] on the grounds of unitarity, since each of those components is affected by the other by unitarity correction. However, experimentally there is no ambiguity in identifying an inelastic event as being either one with jets (hard), or one without any jets (soft), once the criteria for a jet have been adopted. Those events are unquestionably additive to make up the total inelastic contribution to σ_{tot}. The crux of the problem is not whether or not there are two components that are additive, but rather whether or not there exist components that can more easily be calculated and can be continued from lower to higher energies. For, in assuming a soft component σ_{soft}, which may or may not be constant, one is also tacitly assuming a simple energy dependence. It is that energy dependence of σ_{soft} that is subject to unitarity correction when the hard component σ_{hard} is nonzero.

In the geometrical branching model the empirical law of geometrical scaling is adopted as an essential property of the model for $\sqrt{s}<100$ GeV. In extending to higher energies we seek to combine a component that persists to be geometrical scaling with a component that involves hard scattering of partons. Since the latter is not restricted to the energy regime $\sqrt{s} > 100$ GeV, the issue is really whether or not jets can be defined in such a way that their production noticeably breaks geometrical scaling at $\sqrt{s}$ around 100 GeV. We recall that because of the masslessness of the partons a cutoff in transverse momentum is unavoidable, and therein lies the ambiguity of what characterizes a jet. Our physical reasoning for using geometrical scaling of the soft component as a means to define jets of the hard component is based on the conventional understanding that the increase of σ_{tot} with energy is due to (a) hadron size getting larger and (b) hadrons getting more opaque. Since hard scattering of partons has the effect of rendering the host hadrons more absorptive, it should increase the hadron opacity, described by the eikonal function Ω, without changing the size. The issue of hadron size is a matter of geometry, and as such geometrical scaling should therefore continue to be a property of the soft interaction even when hard interaction begins to enhance the opaqueness. In short, we regard geometry and opaqueness due to soft interaction as being independent from the additional opaqueness due to hard interaction, at least until unitarity mixes them up. A thorough demonstration of whether or not this view is realistic requires extensive computational work which is to be reported elsewhere.[36]

The simplest way to impose unitarity is by means of the eikonal formalism, in which one has

$$\sigma_{el}=\pi \int_0^{\pi} db^2(1-e^{-\Omega})^2 , \tag{4.1}$$

$$\sigma_{in}=\pi \int_0^{\infty} db^2(1-e^{-2\Omega}) , \tag{4.2}$$

$$\sigma_{tot}=2\pi \int_0^{\infty} db^2(1-e^{-\Omega}) , \tag{4.3}$$

in the approximation that the real part of the elastic scattering amplitude is zero. For $\sqrt{s} < 100$ GeV, the ratio σ_{el}/σ_{tot} has been found experimentally[37] to be approximately constant at the value 0.175 ± 0.005, the property of geometrical scaling. At $\sqrt{s} = 540$ GeV, the ratio is found[38] to have increased to 0.215 ± 0.005. From elementary scattering theory this means that the absorption has increased. We therefore write the eikonal $\Omega(s,b)$ as a sum of two components,

$$\Omega(s,b)=\Omega_0(s,b)+\Omega_1(s,b) , \tag{4.4}$$

where $\Omega_0(s,b)$, the soft component that possesses the properties of geometrical scaling, is the same as for $\sqrt{s} < 100$ GeV, i.e.,

$$\Omega_0(s,b)=\Omega_0(R) , \tag{4.5}$$

where $b=b_0(s)R$. The hard component, $\Omega_1(s,b)$ is zero for $\sqrt{s} < 100$ GeV, and represents the effects of added absorption for $\sqrt{s} > 100$ GeV.

Substituting (4.4) and (4.2), we can separate the inelastic cross section into the soft and hard components in the conventional way:[34]

$$\sigma_{in}=\sigma^s+\sigma^h , \tag{4.6}$$

$$\sigma^s=\pi \int_0^{\infty} db^2(1-e^{-2\Omega_0})e^{-2\Omega_1} , \tag{4.7}$$

$$\sigma^h=\pi \int_0^{\infty} db^2(1-e^{-2\Omega_1}) . \tag{4.8}$$

The interpretation of (4.8) is that, at every b, $1-\exp(-2\Omega_1)$ is the probability of a hard scattering, whether or not it is accompanied by soft production. By a mean-free path argument,[34] $\Omega_1(s,b)$ is then half the average number of hard scattering per collision at s and b, and can therefore be written as

$$\Omega_1(s,b)=\tfrac{1}{2}A(b)\sigma_{jet}(s) , \tag{4.9}$$

where

$$A(b)=\int d^2\mathbf{r}\,\rho(\mathbf{r})\rho(\mathbf{b}-\mathbf{r}) . \tag{4.10}$$

$\rho(r)$ is the longitudinally integrated matter distribution of one of the incident hadrons at transverse coordinate $\mathbf{r}$ normalized by $\int d^2\mathbf{r}\,\rho(\mathbf{r})=1$. Since $\Omega_0(s,b)$ is proportional to $A(b)$ (Refs. 25 and 39), we may write

$$A(b)=\frac{\Omega_0(s,b)}{\int d^2b\,\Omega_0(s,b)} . \tag{4.11}$$

In a geometrical scaling model $A(b)$ is not energy independent but is

$$A(s,b)=\frac{1}{\sigma_0(s)}\frac{\Omega_0(R)}{\int dR^2\Omega_0(R)} , \tag{4.12}$$

where

$$\sigma_0(s)=\pi b_0^2(s) . \tag{4.13}$$

It then follows that we can write

$$\Omega_1(s,b)=\frac{\sigma_{jet}(s)}{2\sigma_0(s)}\frac{\Omega_0(R)}{\int dR^2\Omega_0(R)} . \tag{4.14}$$

It is important to stress that σ_{jet} has been introduced in (4.9) without the specification of the criteria for a jet. Whatever its definition, a hard scattering takes the partons out of the incident beams and therefore makes a direct contribution to Ω_1.

Using (4.5), (4.13), and (4.14), we can express $\sigma_{el}(s)$ and $\sigma_{tot}(s)$ in terms of $\sigma_0(s)$ and $\sigma_{jet}(s)$, with $\Omega_0(R)$ being regarded as a known function from $d\sigma/dt$ (Ref. 25). From the data on σ_{el} and σ_{tot} at $\sqrt{s} = 540$ GeV, σ_{jet} can be determined. The model would be a success, if the corresponding p_T cutoff suffices for all other values of s, and would be especially attractive if all other scaling violation effects can naturally be explained in terms of the production of jets so defined. The phenomenological work will be reported elsewhere.[36]

Regarding Ω_0 and Ω_1 as known, we return to (4.6)–(4.8) to discuss the topological cross section σ_n for the production of n particles. According to (4.7), σ^s is the soft component of the inelastic cross section, since the integrand describes the probability of soft interaction multiplied by the probability of no hard scattering. On the other hand, the integrand of (4.8) does not rule out soft interaction accompanying a hard scattering. Thus we have

$$\sigma_n \equiv \sigma_{\rm in} P_n = \sigma^s P_n^s + \sigma^h P_n^h \ , \tag{4.15}$$

$$\sigma_n^s \equiv \sigma^s P_n^s = \sigma_0 \int_0^\infty dR^2 (1 - e^{-2\Omega_0(R)}) \times e^{-2\Omega_1(s,R)} F_n^{k(R)}(w) \ , \tag{4.16}$$

$$\sigma_n^h \equiv \sigma^h P_n^h = \sigma_0 \int_0^\infty dR^2 (1 - e^{-2\Omega_1(s,R)}) \times H_n(s,R) \ , \tag{4.17}$$

where $H_n(s,R)$ is the multiplicity distribution associated with jet production. Note that in (4.16) we have used the Furry distribution soft production, as in Sec. II, but unlike the situation below the jet threshold the dependence on $\Omega_1(s,R)$ breaks the KNO scaling property of P_n^s above jet threshold. Nevertheless, the soft interaction is specified by the same quantities $\Omega_0(R)$ and $F_n^{k(R)}(w)$; in this sense the soft component is defined by a continuation of the mechanism that gives rise to geometrical scaling. The breaking of geometrical scaling is a direct and unavoidable consequence of the nonvanishing probability for hard scattering. These considerations are independent of the behavior of $\sigma_0(s)$, which has never been relevant even below jet threshold, since the multiplicative factors cancel in the ratio $\sigma_{\rm el}/\sigma_{\rm tot}$.

The normalization requirements

$$\sum_n F_n^{k(R)}(w) = 1 \ , \tag{4.18}$$

$$\sum_n H_n(s,R) = 1 \tag{4.19}$$

guarantee that the multiplicity distributions P_n^s (soft), P_n^h (hard) and P_n (overall) are all normalized similarly:

$$\sum_n P_n^s = \sum_n P_n^h = \sum_n P_n = 1 \ . \tag{4.20}$$

An n-particle event with jets has many contributions to its multiplicity. Let j denote the multiplicity of hadrons with large p_T belonging to (back-to-back) jets, m the multiplicity of hadrons with small p_T belonging to the initial-state bremsstrahlung before the hard scattering between partons, and l the multiplicity of low-p_T hadrons due to soft interaction of the residual hadronic system. Their total is n:

$$n = l + j + m \ . \tag{4.21}$$

If v is the virtuality associated with a hard-scattering process whose probability of occurrence is $f(s,v)$, then we have

$$H_n(s,R) = \sum_{ljm} \delta_{n,l+j+m} \int_0^s dv\, f(s,v) F_l^{k(R)}(w) \times \Phi_j(v) \Psi_m(v) \ , \tag{4.22}$$

where $\Phi_j(v)$ is the multiplicity distribution of a jet with virtuality v, and $\Psi_m(v)$ is the multiplicity distribution of particles associated with initial-state bremsstrahlung. For the soft production of particles associated with a hard scattering, we assume that the Furry distribution $F_l^k(w)$ continues to describe the multiplicity distribution of l low-p_T particles. The parameter w is still specified by (2.25) with $C_2 = 1.2$ and $\mu = 1.03$ (see Fig. 1), but with $\langle n \rangle$ modified to represent the low-p_T component of the average multiplicity evaluated at the residual energy that remains after the energy expended for the hard process is subtracted. The use of $F_l^k(w)$, is not only the natural assumption but also the objective for carrying out the detailed analysis of the soft process below the jet threshold in Ref. 1.

$H_n(s,r)$ is defined in such a manner that the normalizations

$$\sum_j \Phi_j(v) = 1 \ , \tag{4.23}$$

$$\sum_m \Psi_m(v) = 1 \tag{4.24}$$

ensure the validity of (4.19). Despite the range of integration in (4.22) being from 0 to s, the definition of a jet is contained in $f(s,v)$, which, according to the usual procedure in the QCD parton model,[40-42] is

$$f(s,v) = \frac{1}{\sigma_{\rm jet}} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F(x_1,v) F(x_2,v) \times S(s,x_1,x_2,v) \ , \tag{4.25}$$

where $F(x_i,v)$ is the parton distribution of the incident hadrons, and

$$S(s,x_1,x_2,v) = \tfrac{1}{2} \int_{-z_0}^{z_0} dz \frac{d\hat{\sigma}}{dz} \delta(v - \Theta x_1 x_2 s) \ , \tag{4.26}$$

z being the cosine of the parton-parton scattering angle in the c.m. system of the partons and $d\hat{\sigma}/dz$ the corresponding differential hard-scattering cross section. There is some ambiguity in the best value for v, since it depends how the higher-order corrections are dealt with. A favored choice is $v = k_T^2/4$, where k_T is the transverse momentum of the scattered parton.[43] Another choice that is phenomenologically motivated is $v = p_{\rm cut}^2/4$, where $p_{\rm cut} \simeq 3$ GeV, a value somewhat lower than $\sum E_T$ of 5 GeV used by the jet-finding algorithm of UA1 due to the underlying event.[44] We shall use

$$v = k_T^2/4 = \tfrac{1}{16} x_1 x_2 s (1 - \cos^2\theta) \ , \tag{4.27}$$

as a consequence of which Θ in (4.26) is $(1 - z^2)/16$. The minimum value of v for which a hard scattering is to be identified by a jet is v_0, a parameter to be adjusted to fit the total cross section above jet threshold. In terms of v_0 the limits of integration of z in (4.26) are $\pm z_0$, where

$$z_0 = \left[1 - \frac{16 v_0}{x_1 x_2 s} \right]^{1/2} \ . \tag{4.28}$$

This cutoff in z guarantees the finiteness of the otherwise divergent integral in (4.26) if v is first integrated over.

Indeed, $\sigma_{\rm jet}$ in (4.25) is defined by

$$\int_0^s dv\, f(s,v)=1\ , \tag{4.29}$$

which is necessary for the proper normalization of H_n; thus

$$\sigma_{\rm jet}(s)=\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\int_{-z_0}^{z_0}\frac{dz}{2}F(x_1)F(x_2)\frac{d\hat{\sigma}}{dz}\ , \tag{4.30}$$

in which the virtuality is as specified in (4.27).

In general, the parton distribution $F(x_i,v)$ in (4.25) should depend also on the transverse spatial coordinates.[45,46] However, since the dominant contribution to (4.25) comes from small x_i, that dependence is not important and we have factored out the b dependence from (4.25) allowing it to appear in (4.22) only through the k parameter in $F_l^k(w)$ in a way already described in Sec. II.

V. FORWARD-BACKWARD CORRELATION WITH MINIJETS

We have seen in Sec. III that *F-B* correlation below jet threshold is due to IP smearing. Since there is no multiplicity correlation between the two jets produced in e^+e^- annihilations,[47] one might naively expect that the *F-B* correlation in hadron-hadron collisions above jet threshold would remain the same as below. This is, however, not true in reality, as we shall now show.

We begin by generalizing the jet-production formalism to allow for forward and backward windows separated by a pseudorapidity gap as in Sec. III. Let j_1 (j_2) denote the multiplicity of hadrons in a jet produced in the forward (backward) window, and let m_1 and m_2 have similar meaning related to the initial-state bremsstrahlung. Denote further the jet-associated multiplicities of low-p_T hadrons produced by soft interaction in the two windows by l_1 and l_2. Then the joint distribution of n_1 and n_2 hadrons in those windows associated with hard processes is, following (4.22),

$$H_{n_1n_2}=\sum_{\substack{l_1j_1m_1\\ l_2j_2m_2}} F_{l_1l_2}^{k_1k_2}J_{j_1m_1j_2m_2}\delta_{n_1,l_1+j_1+m_1}\times\delta_{n_2,l_2+j_2+m_2}\ , \tag{5.1}$$

where

$$J_{j_1m_1j_1m_2}(s)=\int dv\, f(s,v)\Phi_{j_1}^{(1)}(v)\Psi_{m_1}^{(1)}(v)\times\Phi_{j_2}^{(2)}(v)\Psi_{m_2}^{(2)}(v)\ , \tag{5.2}$$

where for simplicity we have neglected the energy expenditure for hard scattering and taken the F term outside the integral over v. The superscripts on the multiplicity distribution Φ and Ψ signify the dependences on the window size and position. If the detector windows are such that n_1+n_2 exhausts the total multiplicity of any event, then, with

$$H_n=\sum_{n_1n_2}H_{n_1n_2}\delta_{n,n_1+n_2}\ , \tag{5.3}$$

(4.22) can be recovered. The dependence of $H_{n_1n_2}$ on the impact parameter, as it is with H_n in (4.22), is through $k_1(R)$ and $k_2(R)$ in (5.1). The hard component of the joint distribution with IP smearing is, similar to (4.17),

$$\sigma^hP_{n_1n_2}^h=\sigma_0\int_0^\infty dR^2(1-e^{-2\Omega_1(s,R)})H_{n_1n_2}(s,R)\ . \tag{5.4}$$

Let λ denote the ratio

$$\lambda=\sigma^s/\sigma_{\rm in}\ . \tag{5.5}$$

Then from (4.15) we have

$$P_n=\lambda P_n^s+(1-\lambda)P_n^h\ , \tag{5.6}$$

$$P_{n_1n_2}=\lambda P_{n_1n_2}^s+(1-\lambda)P_{n_1n_2}^h\ , \tag{5.7}$$

from which follow

$$\langle n_i\rangle=\lambda\langle n_i\rangle_s+(1-\lambda)\langle n_i\rangle_h\ , \tag{5.8}$$

$$\langle n_1n_2\rangle=\lambda\langle n_1n_2\rangle_s+(1-\lambda)\langle n_1n_2\rangle_h\ , \tag{5.9}$$

with obvious notation. Substituting these equations in (3.3) yields

$$B=\frac{B_s+(1-\lambda)(K_1K_2)^{1/2}+(\lambda^{-1}-1)LB_h}{1+(1-\lambda)K_1+(\lambda^{-1}-1)L}\ , \tag{5.10}$$

$$B_\alpha=\frac{\langle n_1n_2\rangle_\alpha-\langle n_1\rangle_\alpha\langle n_2\rangle_\alpha}{\langle n_1^2\rangle_\alpha-\langle n_1\rangle_\alpha^2},\quad \alpha=s,h\ , \tag{5.11}$$

$$K_i=\frac{(\langle n_i\rangle_h-\langle n_i\rangle_s)^2}{\langle n_1^2\rangle_s-\langle n_1\rangle_s^2},\quad i=1,2\ , \tag{5.12}$$

$$L=\frac{\langle n_1^2\rangle_h-\langle n_1\rangle_h^2}{\langle n_1^2\rangle_s-\langle n_1\rangle_s^2}\ . \tag{5.13}$$

In the symmetric case, i.e., $\langle n_1\rangle=\langle n_2\rangle$, we have $K_1=K_2$. We shall for simplicity consider this case only in the following.

Let r and ρ be defined as

$$r=\frac{\langle n_1\rangle_s}{\langle n\rangle_s},\quad \rho=\frac{\langle n_1\rangle_h}{\langle n_1\rangle_s}\ . \tag{5.14}$$

Then from (3.11) and (2.25) we get

$$\langle n_1^2\rangle_s-\langle n_1\rangle_s^2=(\mu-1)\langle n_1\rangle_s^2+(w-1)\langle n_1\rangle_s=[(\mu-1)r^2+(C_2-\mu)r]\langle n\rangle_s^2\ . \tag{5.15}$$

It then follows from (5.12) that

$$K\equiv K_1=K_2=\frac{(\rho-1)^2}{\mu-1+(C_2-\mu)/r}\ . \tag{5.16}$$

For the contribution from hard scattering we first note that the average $\langle n_1 \rangle_h$ is

$$\langle n_1 \rangle_h = \sum_{n_1 n_2} n_1 P^h_{n_1 n_2} = \{\bar{n}_1\}_h \ , \tag{5.17}$$

where

$$\bar{n}_1 = \sum_{n_1 n_2} n_1 H_{n_1 n_2}$$
$$= \sum_{n_1} n_1 \sum_{l_1 j_1 m_1} \bar{F}^{k_1}_{l_1} \bar{J}_{j_1 m_1} \delta_{n_1, l_1 + j_1 + m_1} \ , \tag{5.18}$$

$$\bar{F}^{k_1}_{l_1} \bar{J}_{j_1 m_1} = \sum_{l_2 j_2 m_2} F^{k_1 k_2}_{l_1 l_2} J_{j_1 m_1 j_2 m_2} \ , \tag{5.19}$$

$$\{ \cdots \}_h = (\sigma^h)^{-1} \sigma_0 \times \int_0^\infty dR^2 (1 - e^{-2\Omega_1(s,R)}) (\cdots) \ . \tag{5.20}$$

From (5.18) we have

$$\bar{n}_1 = \bar{l}_1 + \bar{j}_1 + \bar{m}_1 \ , \tag{5.21}$$

where

$$\bar{l}_1 = \sum_{l_1} l_1 \bar{F}^{k_1}_{l_1} \ , \tag{5.22}$$

$$\bar{j}_1 = \sum_{j_1 m_1} j_1 \bar{J}_{j_1 m_1} \ , \tag{5.23}$$

$$\bar{m}_1 = \sum_{j_1 m_1} m_1 \bar{J}_{j_1 m_1} \ . \tag{5.24}$$

In a similar way we have

$$\overline{n_1^2} = \overline{l_1^2} + \overline{(j_1 + m_1)^2} + 2\bar{l}_1 \times \overline{(j_1 + m_1)} \ . \tag{5.25}$$

In the absence of any knowledge of the analytical form for $\Psi_{m_1}(v)$, we shall, for simplicity, assume

$$\Psi_{m_1}(v) = \Phi_{j_1}(v), \quad m_1 = j_1 \ , \tag{5.26}$$

so that

$$\bar{j}_1 = \bar{m}_1 \ , \tag{5.27}$$

$$\overline{(j_1 + m_1)^2} = 2(\overline{j_1^2} + \bar{j}_1^{\,2}) \ . \tag{5.28}$$

It then follows from (5.17), (5.21), and (5.25) that

$$\langle n_1^2 \rangle_h - \langle n_1 \rangle_h^2 = \langle l_1^2 \rangle_h - \langle l_1 \rangle_h^2 + 2(\langle j_1^2 \rangle_h - \langle j_1 \rangle_h^2) \ . \tag{5.29}$$

If we introduce some more symbols for abbreviations sake,

$$\rho_l = \frac{\langle l_1 \rangle_h}{\langle n_1 \rangle_s}, \quad \rho_j = \frac{\langle j_1 \rangle_h}{\langle n_1 \rangle_s}, \quad \rho = \rho_l + 2\rho_j \ , \tag{5.30}$$

$$\gamma_2^l = \frac{\langle l_1^2 \rangle_h}{\langle l_1 \rangle_h^2} - 1, \quad \gamma_2^j = \frac{\langle j_1^2 \rangle_h}{\langle j_1 \rangle_h^2} - 1 \ , \tag{5.31}$$

we then have, from (5.13),

$$L = \frac{\gamma_2^l \rho_l^2 + 2\gamma_2^j \rho_j^2}{\mu - 1 + (C_2 - \mu)/r} \ . \tag{5.32}$$

Finally, for B_h, we come to an important point in *F-B* correlation for jets. First of all, we have

$$\langle n_1 n_2 \rangle_h = \langle l_1 l_2 \rangle_h + 4\langle l_1 j_2 \rangle_h + 4\langle j_1 j_2 \rangle_h \ . \tag{5.33}$$

The jet correlation part is

$$\langle j_1 j_2 \rangle_h = \{\overline{j_1 j_2}\}_h \ , \tag{5.34}$$

where

$$\overline{j_1 j_2} = \sum_{\substack{j_1 m_1 \\ j_2 m_2}} j_1 j_2 J_{j_1 m_1 j_2 m_2}$$
$$= \int dv\, f(v) \tilde{j}_1^{\,2}(v) \ , \tag{5.35}$$

$$\tilde{j}_1(v) = \sum_{j_1} j_1 \Phi_{j_1}(v) \ . \tag{5.36}$$

It is the integration over virtuality that makes it different from

$$\bar{j}_1 \times \bar{j}_2 = \left[\int dv\, f(v) \tilde{j}_1(v) \right]^2 \ . \tag{5.37}$$

Otherwise, the factorizability of the back-to-back jet would imply $\overline{j_1 j_2} = \bar{j}_1 \times \bar{j}_2$. Thus, the origin of jet correlation here due to virtuality smearing is similar to that in soft interaction due to IP smearing, as described in Sec. III.

The correlation $\langle l_1 l_2 \rangle_h$ is not the same as in the pure soft case considered in (3.13) because the integral over the impact parameter b is different. We have

$$\langle l_1 l_2 \rangle_h = \{\overline{l_1 l_2}\}_h = \mu_h \langle l_1 \rangle_h^2 \ , \tag{5.38}$$

$$\mu_h = \{h^2(R)\}_h \ . \tag{5.39}$$

Thus from (5.33) we obtain

$$\langle n_1 n_2 \rangle_h - \langle n_1 \rangle_h^2 = (\mu_h - 1)\langle l_1 \rangle_h^2 + 4d_{jj}^2 + 4d_{lj}^2 \ , \tag{5.40}$$

where

$$d_{jj}^2 = \{\overline{j_1 j_2}\}_h - \{\bar{j}_1\}_h \{\bar{j}_2\}_h \ , \tag{5.41a}$$

$$d_{lj}^2 = \{\bar{l}_1 \times \bar{j}_2\}_h - \{\bar{l}_1\}_h \{\bar{j}_2\}_h \ . \tag{5.41b}$$

If we assume, as discussed after (4.30), that the only dependence on b in (5.1) is in the soft part, $F^{k_1 k_2}_{l_1 l_2}$, and not in the hard part, $J_{j_1 m_1 j_2 m_2}$, then the operation $\{ \cdots \}_h$ in (5.41) does nothing, i.e.,

$$d_{jj}^2 = \overline{j_1 j_2} - \bar{j}_1^{\,2} \tag{5.42}$$

and $d_{lj}^2 = 0$. Substituting these results into (5.11) for B_h, we get

$$B_h = \frac{(\mu_h - 1)\rho_l^2 + 4\gamma_{12}^j \rho_j^2}{\gamma_2^l \rho_l^2 + 2\gamma_2^j \rho_j^2} \ , \tag{5.43}$$

where

$$\gamma_{12}^{j}=d_{jj}^{2}/\bar{j}_1^{\,2}=(\overline{j_1 j_2}/\bar{j}_1^{\,2})-1 \ . \tag{5.44}$$

We note that γ_{12}^{j} is nonvanishing because of virtuality smearing, and $\mu_h \neq 1$ because of b smearing. A crude estimate indicated that the two terms in the numerator of (5.43) are of comparable sizes, so the two types of smearing are both relevant.

The soft contribution B_s is as given before in (3.16). Assuming scaling for the soft component, as we have formulated the problem, B_s would remain at around 0.17 above jet threshold. For the symmetric-window case, (5.10) becomes [cf. (5.16)]

$$B=\frac{B_s+(1-\lambda)K+(\lambda^{-1}-1)LB_h}{1+(1-\lambda)K+(\lambda^{-1}-1)L} \ . \tag{5.45}$$

B is necessarily larger than B_S, if $B_h \geq 1$, irrespective of what values the other parameters have. (K and L must, of course, be positive and $\lambda<1$.) As it turns out, B_h is slightly less than 1, but large enough to render the second and third terms in the numerator of (5.45) to be roughly the same as the corresponding terms in the denominator, thus ensuring B to be significantly greater than B_s.

The physical reason why B can increase above B_s is that hard scattering increases the opacity of hadrons from Ω_0 to Ω, so the increase of multiplicity due to jets in the forward hemisphere is accompanied by an increase in the backward hemisphere. This is the dominant effect over and above the effect due to virtuality smearing. At very high energy $\langle n \rangle_h$ can become much greater than $\langle n \rangle_s$ and λ significantly less than 1; hence, when $K \gg 1$, B would approach 1.

In the Appendix we give very crude estimates of the various terms in (5.45). With a wide margin of error, we obtain $B \approx 0.4$. While its remarkable agreement with the observed value[7] 0.41 ± 0.01 at $\sqrt{s}=540$ GeV should not be taken too seriously, the result cannot be totally fortuitous. At the very least, the geometrical branching model cannot be ruled out on phenomenological grounds.

VI. CONCLUSION

We have formulated a model for multiparticle production that has the following properties.

(a) It treats hadrons as extended objects so that impact parameter is an essential variable in the description of their collisions.

(b) At each impact parameter the mechanism of particle production is of the Furry-type branching, which agrees in leading order with the results in perturbative theory. Since no small parameters are involved, the branching process is assumed to be still relevant at low p_T even though perturbative theory is not. The multiplicity distribution at each impact parameter is broad, and cannot be identified with the narrow distributions of hard processes, such as in e^+e^- annihilation.

(c) Until the collision energy is high enough so that the production of jets becomes important, the model possesses various scaling properties: (α) constant average transverse momentum, (β) geometrical scaling, and (γ) KNO scaling.

(d) Since the model incorporates the eikonal description of high-energy collisions, it is able to reproduce all the main features of elastic scattering, such as the slope of the diffractive peak and the first dip in $d\sigma/dt$, as well as the total cross section.

(e) The model can give a KNO curve on multiplicity distribution that agrees with the ISR data to a high degree of accuracy.

(f) It relates the efficiency of particle production to the opacity of hadrons at each impact parameter.

(g) It yields the correct forward-backward multiplicity correlation measured at ISR.

(h) All the scaling properties are violated at higher energies because of important contributions from jet production. The incorporation of hard processes in the model has been formulated in a way consistent with unitarity.

(i) Geometrical scaling defines the soft component; the p_T cutoff for the jet cross section is not chosen arbitrarily.

(j) We anticipate that the model can quantitatively account for the (α) increase of the total cross section, (β) breaking of geometrical scaling, (γ) breaking of KNO scaling, (δ) increase in F-B multiplicity correlation, and (ε) increase in average p_T.

In counting the number of free parameters in the model, one should, as always bear in mind what data the model is to describe. For example, the eikonal function $\Omega(R)$ is an essential part of our model, but it is not free because it is constrained by σ_{el}/σ_{tot} and the first dip of $d\sigma/dt$. If one adopts the Chou-Yang and Durand-Lipes model for diffractive scattering, it can even be calculated. Thus, taking, in addition $\sigma_{in}(s)$, $\langle n \rangle$, and C_2 as given by minimum-bias experiments, parton distribution function of the nucleon, and jet fragmentation function by hard collision experiment, we have essentially only two parameters in the model.

(1) γ: This parameter relates production efficiency to hadron opacity. It is over-constrained by multiplicity moments and the F-B correlation parameter.

(2) v_0: This is the minimum virtuality associated with a jet. It is adjusted to fit the total cross section at $\sqrt{s}=540$ GeV.

While much remains to be checked, the geometrical branching model has thus far successfully met a variety of phenomenological tests.

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APPENDIX

The formalism described in this paper allows an unambiguous computation of the correlation parameter B. There is one adjustable parameter v_0, which is the virtuality cutoff. It is to be adjusted to fit $\sigma_{\rm in}$ above jet threshold, a task that involves extensive numerical work.[36] We give here a crude estimate of B. The degree of reliability of this estimate should not be taken to reflect the soundness of the model, on the basis of which (5.45) is derived.

We first consider the jet correlation parameter γ_{12}^j defined in (5.24). We approximate $f(s,v)$, defined in (4.25), by

$$f_0 v^{-1}\exp(-\alpha\sqrt{v})\theta(v-v_0) ,$$

the normalization factor being determined by (4.29). From Ref. 47 we expect $\tilde{j}_1(v)$ defined in (5.36) to be proportional to $v^{1/4}$. The proportionality constant cancels in the calculation of $\overline{j_1 j_2}/\overline{j_1}^2$. Taking $\alpha\sqrt{v_0}$ to be 0.5 yields $\gamma_{12}^j\approx 0.1$. A 100% modification of this number would affect the final value of B by less than 8%.

We can use the UA1 data[5] to get a rough estimate of $\langle n_1\rangle_h$ and $\langle n_1\rangle_s$. If we approximate $\langle l_1\rangle_h$ by $\langle n_1\rangle_s$, we get $\rho_l\approx 1$ and $\rho_j\approx 0.5$. From e^+e^- annihilation data[47], we have $\gamma_2^j\approx 0.1$, while for γ_2^l we approximate it by the ISR value: $\gamma_2^l\approx C_2-1=0.2$. Taking μ_h to be roughly $\mu\approx 1.06$, we obtain

$$B_h\approx\frac{0.06+4\times 0.1\times(0.5)^2}{0.2+2\times 0.1\times(0.5)^2}\approx 0.64 .$$

For ρ we get from the UA1 data roughly $\rho\approx 2$. For r we use the value given in (3.17). Thus

$$K\approx\frac{1}{0.06+(1.2-1.06)/0.36}\approx 2.2 ,$$

$$L\approx\frac{0.2+2\times 0.1\times(0.5)^2}{0.06+(1.2-1.06)/0.36}\approx 0.55 .$$

At $\sqrt{s}=540$ GeV we infer from Ref. 5 that $\lambda\approx 0.85$. Since B_s should be identified with the value 0.17 below the jet threshold on the grounds of extended scaling of the soft component, we have finally, from (5.45),

$$B\approx\frac{0.17+0.15\times 2.2+0.18\times 0.55\times 0.64}{1+0.15\times 2.2+0.18\times 0.55}\approx 0.39 .$$

Despite the crudeness of the approximations made in estimating the values of various terms, we believe that the margin of error for the final value of B should not exceed the 50% level.

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