Charm and the rise of the $p\bar{p}$ total cross section

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We give a detailed description of the $p\bar{p}$ forward amplitude through CERN SPS collider energies, using the flavored Pomeron model as an effective parametrization of nonperturbative QCD. We show that the rise in the total cross section between CERN ISR and SPS collider energies is consistent with the onset of charmed-particle production up to the level of a few millibarns, along with other processes, and in agreement with available data. In contrast with our estimates of charm production, perturbative QCD charm-production calculations are well below the data. We give estimates of the $\bar{p}$ and $K^{\pm}$ multiplicities at SPS collider energies. We also present a simplified version of the flavoring model in order to facilitate comparisons between it and other parametrizations.

Recent measurements[1] of the $p\bar{p}$ total cross section at the CERN SPS collider have prompted much discussion regarding the rate at which this cross section increases. Much of the discussion has centered around asymptotic expressions for σ_{tot}, such as the Froissart bound or the critical Pomeron. We have recently argued,[2] however, that even at these seemingly high energies the $p\bar{p}$ amplitude is still sensitive to the perturbative effects of massive flavor (and baryon) production, and that it is inappropriate to make comparisons of existing data with asymptotic formulas. Block and Cahn[3] have recently shown that it is unlikely that the cross section is rising as $\ln^2 s$ in the current energy regime. This supports the argument that current energies are not asymptotic. In this paper we present our interpretation of the new data and describe the level of charmed-particle production which is needed for consistency with the observed rise in σ_{tot} between (CERN) ISR and SPS collider energies. We also describe a simplified model which contains the essence of the flavoring threshold physics and which lends itself as a convenient parametrization to which future data can be compared.

The flavoring model views the bare Pomeron as built up (through unitarity) by multiperipheral production processes. Here the bare Pomeron, including flavoring corrections, refers to the input to the Reggeon field theory. Production of high-mass states (baryon-antibaryon or charm-anticharm, for instance) occurs significantly only at energies above thresholds which are delayed by multiperipheral kinematics. It is only at energies above all these thresholds that one can expect to see the full Pomeron amplitude. The flavoring model is not an asymptotic model, but is rather a threshold-perturbative model of QCD nonperturbative effects which exhibits very directly the effects of finite scales. The rising total cross section is correlated directly to the cross sections for producing the various flavors, so that our predictions are strongly constrained by experiment. Indeed, because of s-channel unitarity, we regard consistency with the various particle multiplicities as a requirement that the correct theory of diffraction *must* satisfy. The explicit inclusion of charm represents an important consistency check of our approach. In other models of diffraction, the constraints of correct particle composition are typically not dealt with. An important result of our analysis is that in our current parametrization, a moderate amount of charm production is needed at SPS collider energies (3–6 mb). Our model is consistent with existing data for charm production at $\sqrt{s} \leq 63$ GeV. Perturbative QCD predictions for this cross section fall far below experiment.[4] This underscores the importance of soft (nonperturbative QCD) processes which are the underlying basis of our model.

THE FLAVORED POMERON AMPLITUDE

We use a j-plane representation for our amplitude. The full amplitude includes the absorptive effects of inelastic diffraction, as well as a parametrization of associated production of strange particles (the latter distinguishes K^+ from K^- multiplicities). Charm associated production is discussed below. We write the (even signatured) Pomeron amplitude in this approximation as

$$T(s,0) = -\int_{c-i\infty}^{c+i\infty} \frac{dj}{2\pi i} (se^{-i\pi/2}/s_0)^j \frac{A_j}{\sin(\pi j/2)} , \tag{1}$$

with $A_j = N_j / D_j$,

$$N_j = \beta e^{-b_0 j} \left[1 + \sum_k g_k \frac{e^{-b_k j}}{(j-j_k)^2} \right] , \tag{2}$$

and

$$D_j = j - \hat{\alpha} - \sum_n g_n \frac{e^{-b_n j}}{j} . \tag{3}$$

In these expressions, the g_k ($k = D$, diffraction with

$g_D < 0$, and also associated production) and the g_n ($n = s\bar{s}, B\bar{B}, c\bar{c}$) are external and internal couplings, respectively. (B represents "baryon" here, not "bottom." Bottom production is negligible.) The b_n are threshold parameters, whose function in a one-dimensional model such as ours is illustrated by the following simple example. The imaginary part of the *unflavored* bare Pomeron amplitude at $t=0$ is given by

$$\mathrm{Im}\hat{T} = \beta \int \frac{dj}{2\pi i} \frac{e^{(y-b_0)j}}{j-\hat{\alpha}} = \beta e^{(y-b_0)\hat{\alpha}} \theta(y-b_0) . \quad (4)$$

With $y = \ln(s/s_0)$, we see that the threshold for this term is $s_{\mathrm{th}} = s_0 e^{b_0}$. When D_j is expanded in powers of g_n, the $(g_n)^N$ term corresponds to N pairs ($K\bar{K}$, etc.) produced.

The parameter $\hat{\alpha}$ in Eq. (3) is the unflavored Pomeron pole intercept. If only u and d quarks existed, this would be the intercept of the bare Pomeron one would use as input in Reggeon field theory. Detailed phenomenology[5] has determined a value of 0.85 for $\hat{\alpha}$.

The factor of $1/j$ in Eq. (3), and the factors $1/(j-j_k)^2$ in Eq. (2), are inserted to increase the strength of the singularity in j, and thus smooth the resulting amplitude as a function of s near thresholds, thus avoiding any jumps or hard kinks in the cross sections. Such factors arise naturally in strong-coupling multiperipheral models. They assure an amplitude which rises just above $s = s_{\mathrm{th}}$ as $\ln^2(s/s_{\mathrm{th}})$.

The above model has been described in great detail in our earlier papers.[2] The main new feature which we have added in order to fit the SPS collider data is a term for charm—i.e., the sum in Eq. (3) includes not only ($s\bar{s}$) and ($B\bar{B}$), but also ($c\bar{c}$) production. A fit to the total cross section has been presented elsewhere;[2(b)] we give here an updated fit and a more detailed discussion of the nature of this fit. A fit to the elastic differential cross section, which is consistent with the present phenomenology, has been presented in Ref. 6.

PHENOMENOLOGY OF THE FLAVORED POMERON

Our fitting procedure involves a simultaneous least-squares fit to the vacuum contribution ($pp + p\bar{p}$) to the total cross section and to the multiplicity data $\langle n_{K^\pm} \rangle, \langle n_{\bar{p}} \rangle$. We use 11 representative energies from $\sqrt{s} = 4.5$ GeV to $\sqrt{s} = 900$ GeV. The nature of the fit at ISR energies and below is not changed from that which we presented in Ref. 2. The SPS collider data are consistent with the pattern established by these lower-energy data.

We present in Fig. 1 the multiplicity data and our fits to these data. The model is in qualitative agreement with both the ISR and SPS collider data. (Only SPS collider data at pseudorapidity $|\eta| < 3$ have been measured, as shown in Fig. 1. We do not agree with the extrapolation estimate to $|\eta| < 5$ proposed in Ref. 7.) It is the correlation between these multiplicity data and the total cross section which distinguishes the flavoring model from others in current use. Note that there are reliable data only up through ISR energies, so that the $K^\pm$ and $\bar{p}$ components of the model are constrained almost entirely by these data.

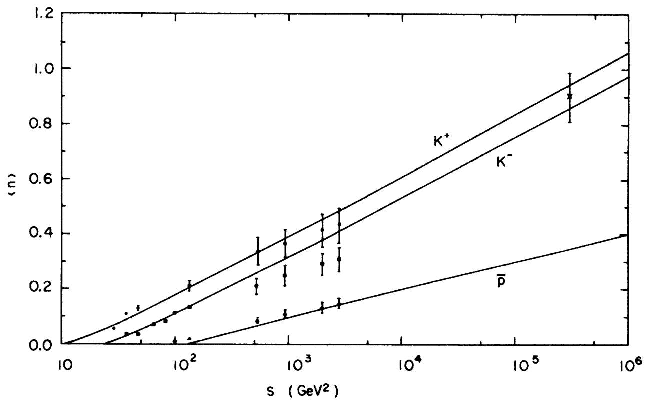

FIG. 1. Multiplicities of $K^\pm$ and $\bar{p}$ in pp scattering. The point marked x is the average of the K^+ and K^- multiplicities for $|\eta| \le 3$ at the SPS collider.

The charm component of the model is constrained not by direct measurement of the charm multiplicity, but rather by the total cross section itself. The $K^\pm$ and $\bar{p}$ contributions are tightly constrained by the total cross section and multiplicity data through ISR energies. In our model, the only mechanism for explaining the additional rise in σ_{tot} from ISR to SPS collider energies is through charm production. Experimental estimates of the charm cross section at $\sqrt{s} \le 63$ GeV are typically on the order of a few hundred microbarns. We find that our model requires a charm cross section of 0.8–1.0 mb at $\sqrt{s} = 63$ GeV. This value is above the upper bound of 0.6 mb suggested in Ref. 8, but because this energy is not far above the effective charm threshold, we do not consider the discrepancy significant. The predicted charm cross section at $\sqrt{s} = 546$ GeV ranges from 5 to 6 mb, depending on the cross section at $\sqrt{s} = 63$ GeV. The fit described here has $\sigma_c = 1$ mb at $\sqrt{s} = 63$ GeV. We have included in the theoretical $K^\pm$ multiplicities the contribution from charm decays. These contributions are necessarily included in the data. There is no appreciable difference between the new results for multiplicities and those presented earlier[9] when we had no charm component in the model, since the charm cross section is so much smaller than the strange-particle cross section at these energies.

Our fitting routine was designed to give a reasonable overall fit, not to minimize χ^2. The choice of 11 representative energies streamlines the process. We also adjusted our χ^2 to give equal weight to the total cross section and multiplicity data. Nevertheless, the actual χ^2 value for the total-cross-section data alone, including *all* data for $\sqrt{s} > 15$ GeV, is 9.8 for 11 degrees of freedom. For $\sqrt{s} > 5$ GeV, we find $\chi^2 = 65$ for 24 degrees of freedom, with 41 units of χ^2 coming from a single data point. Thus, our fit is of an excellent statistical quality.

Table I shows the various contributions to the total cross section at $\sqrt{s} = 62.3$ GeV (ISR) and $\sqrt{s} = 546$ GeV (SPS collider). It is interesting to note the processes which account for the substantial (17 mb) rise in σ_{tot} over this interval. Baryon production accounts for the bulk of the increase, but inelastic diffractive absorption largely

TABLE I. Contributions to the total cross section at ISR ($\sqrt{s}=62.3$ GeV) and at SPS collider ($\sqrt{s}=546$ GeV) energies. The first entry σ_0 is the contribution from central production (nondiffractive) of strange and nonstrange mesons. The second entry σ_B is due to central baryon production. The third entry σ_C is due to central production of charm. The fourth column σ_D is the inelastic diffractive cross section, and the fifth σ_Λ is associated production of strange particles. All cross sections are in mb.

c.m. energy (GeV)	σ_0	σ_B	σ_C	σ_D	σ_Λ	Total
62.3	40.7	10.2	1.0	−13.3	5.8	44.4
546	44.5	28.1	5.9	−25.9	8.3	60.9

cancels it out. Appreciable contributions come from charm and from $K\bar{K}$ production (with no accompanying heavier flavors). The total pion-plus-$K\bar{K}$ production cross section is rising as $s^{\bar{\alpha}-1}$, with $\bar{\alpha}\simeq 1.02$. Finally, the associated production term also adds an appreciable contribution to the increase. We see that the rise cannot easily be attributed to any single factor.

We show in Fig. 2 the ratio ρ of the real to imaginary parts of the pp elastic amplitude at $t=0$. We have made the same assumptions as in our earlier work, and this figure can be considered an update to Ref. 2(a). We parametrize the very-low-energy ($s<11$ GeV^2) cross section by a simple fit to the data, and include an ω-exchange contribution to the amplitude. The real part of the Pomeron amplitude is determined from the crossing-even dispersion relation. We stress that we do not get a simple Regge phase, $\rho_R=-\cot\pi\alpha/2$, since our amplitude has not just a leading real pole, but also a sequence of complex poles. However, at high energy, we expect the leading pole α_P (see below) to dominate, and ρ should approach

$$\rho\rightarrow -\cot\frac{\pi\alpha_P}{2}=0.15 \ .$$

This is consistent with experimental measurements, although we note that ρ is still well below 0.15, according to the model, even at SPS collider energies.

We have investigated the role of associated charm production, which is not included in the above results. The only experimental information on associated charm production comes from a 400-GeV/c Fermilab experiment,[10] which finds the data to be consistent with, but not to require, 40% of the charm cross section (≈ 0.1 mb) coming from associated production. In the context of charm flavoring effects on σ_{tot}, 400 GeV/c is rather low, and we therefore did not try to fit this small cross section. Generally, our model predicts that the ratio of associated charm production to central $c\bar{c}$ production decreases at high energies, similar to the case for kaons.

There are not sufficient data to pin down the parameters of a charm-associated-production term in our model. However, we offer the following as evidence that a reasonable amount of associated charm production is consistent with our model. We expect the associated charm-production threshold to be below the central charm-production threshold, because the total mass ($\Lambda_c\bar{D}\bar{p}$, for instance) is less than ($D\bar{D}p\bar{p}$). If we arbitrarily set $b_{\Lambda_c}=b_c-1.5$, and demand charm-associated-production cross sections at ISR on the order of $0.1-0.2$ mb, we can achieve a satisfactory description of the total cross section which differs somewhat from our earlier fit. The two fits are shown in Fig. 3 and the parameters are listed in Table II. The second fit, which includes associated charm production, has somewhat more baryon production and predicts a charm cross section of only 4.2 mb at $\sqrt{s}=546$ GeV. This reduces to 3 mb if $\sigma_c=0.8$ mb at $\sqrt{s}=63$ GeV. In light of existing data, this is probably the most realistic fit. The multiplicities are changed in the second fit by only a few percent relative to Fig. 1.

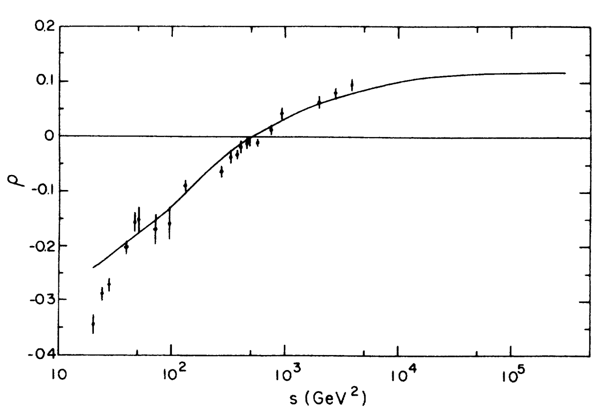

FIG. 2. The ratio ρ of the real to the imaginary part of the pp elastic amplitude at $t=0$.

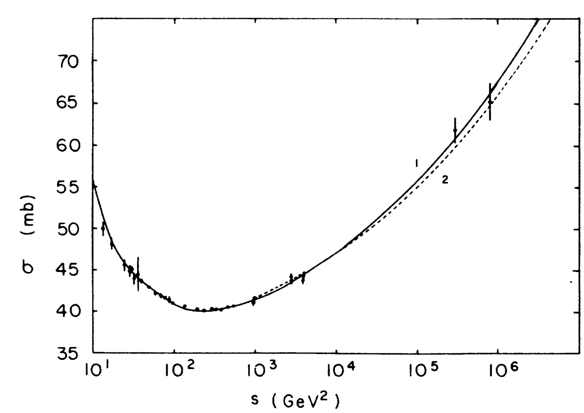

FIG. 3. The vacuum contribution to σ_{tot} for the full model without (fit 1) and with (fit 2) associated production of charm. The predicted cross sections at $\sqrt{s}=2$ TeV are 76.6 mb and 74.3 mb, respectively.

TABLE II. Parameters in the two fits described in the text. Fit 1 assumes no associated production of charm, whereas Fit 2 assumes a small such contribution.

Parameters	Not varied[a]	Parameter	Fit 1	Fit 2
β	289 mb GeV^2	g_K	0.49	0.49
b_0	1.8	g_B	1.75	2.06
$\hat{\alpha}$	0.85	g_C	5.40	1.63
g_Λ	0.40	b_K	1.02	1.02
g_D	0.22	b_B	2.77	2.88
b_Λ	0.19	b_C	5.04	4.41
b_d	1.24	g_{Λ_c}		8.15
$j_K=\alpha^{\text{cut}}_{K^* x K^*}$	-0.6			
$j_D=\alpha^{\text{cut}}_{\hat{P}x\hat{P}}$	0.7			

[a]These parameters were determined by lower-energy phenomenology or (in the case of the cut trajectory intercepts) by standard Regge theory.

We show in Fig. 3 the extrapolations of the two fits to Fermilab Tevatron collider energies. At $\sqrt{s}=2$ TeV, fit 1 (central $c\bar{c}$ production only) predicts $\sigma=76.6$ mb, to be compared with 74.3 mb if we include associated charm production. Associated production itself accounts for only 0.5 mb at $\sqrt{s}=2$ TeV, as this component is only slowly increasing with energy.

The high-energy form of the forward flavored amplitude in our approximation at SPS collider energies is roughly

$$T \sim s^{\alpha_P} ,$$

where $\alpha_P=1.09$ is the intercept of the flavored bare Pomeron pole with the parameters of our fits. It is significant that the convergence of the flavoring contributions noted in our early paper [Ref. 2(a)] continues to hold. We find that strange-quark production renormalizes the Pomeron intercept from the unflavored value of $\hat{\alpha}=0.85$, to $\bar{\alpha}=1.02$; baryon production renormalizes this to 1.08; charmed quarks add only about 0.01 to $\alpha_P=1.09$. One would expect that bottom and top (and heavier flavors) will add only very small amounts to the Pomeron intercept. Thus, the flavoring renormalization converges rapidly, and our value for α_P is probably close to the intercept of the bare Pomeron of the Reggeon field theory.

A SIMPLIFIED FLAVORING MODEL

One feature of our model is that, because it attempts to accurately describe the nonperturbative QCD dynamics of the pp interaction, it is rather complicated. It is not the type of model that lends itself to simple comparison to new data. We wish to offer here a simpler version of the flavoring model, one which retains the basic features and does provide a simple parametrization of the data. We invite our colleagues to use it in this manner and compare it to other models.

Noting that by Fermilab energies, $K\bar{K}$ systems can be considered low-mass systems, we choose to sum the $K\bar{K}$ effects and consider only the flavoring due to $B\bar{B}$ and $c\bar{c}$ production. This restricts the validity of our simplified model to Fermilab energies and above. Next, because the diffractive and baryon cross sections happen to have similar energy dependence, we combine them into a single term in D_j. Then we drop the associated production terms, since they are relatively small. We then have a simple j-plane model, with

$$N_j=\beta e^{-b_0 j}$$

and D_j given by Eq. (3) with only two terms in the sum. The intercept $\hat{\alpha}$ is now replaced by the value which includes flavoring due to strange quarks, namely, $\bar{\alpha}=1.02$. Finally, recognizing that incorporation of a single $B\bar{B}$ or $c\bar{c}$ pair is adequate to describe the data, we expand Eq. (3) to first order in g_n, to find near the $B\bar{B}$, $c\bar{c}$ thresholds s_1, s_c that

$$\operatorname{Im}T=\beta e^{\bar{\alpha}y}+\frac{\beta g_1}{\bar{\alpha}}[e^{\bar{\alpha}y_1}(y_1-1/\bar{\alpha})+1/\bar{\alpha}]\theta(y_1)$$
$$+\frac{\beta g_c}{\bar{\alpha}}[e^{\bar{\alpha}y_c}(y_c-1/\bar{\alpha})+1/\bar{\alpha}]\theta(y_c) . \tag{5}$$

In this expression,

$$y=\ln(s/s_0) , \tag{6a}$$

$$y_1=\ln(s/s_1) , \tag{6b}$$

$$y_c=\ln(s/s_c) . \tag{6c}$$

We set $b_0=\ln(s_0)=0$, since a rescaling of the other parameters can accommodate any change in b_0.

We have used the parametrization of Eq. (5) to fit the total cross section. There are five free parameters: β, g_1, s_1, g_c, and s_c. $\bar{\alpha}$ was fixed at 1.02. (We use the notation g_1 rather than g_B because this term represents both the baryon-flavoring contribution and the diffractive absorptive part; one should not be misled into comparing this term to the baryon cross section.) The result of our fit is given in Fig. 4. We have only used data above $s=350$ GeV^2. This fit has a χ^2 of 9.9 for 9 degrees of freedom, and is certainly acceptable from this standpoint. The model was constrained to give a charm cross section of 1 mb at $s=4000$ GeV^2. We predict a charm cross section of about 8.4 mb at $\sqrt{s}=546$ GeV, which is the highest, and probably the least reliable, of the several fits described in this paper. The total cross section at $\sqrt{s}=2$

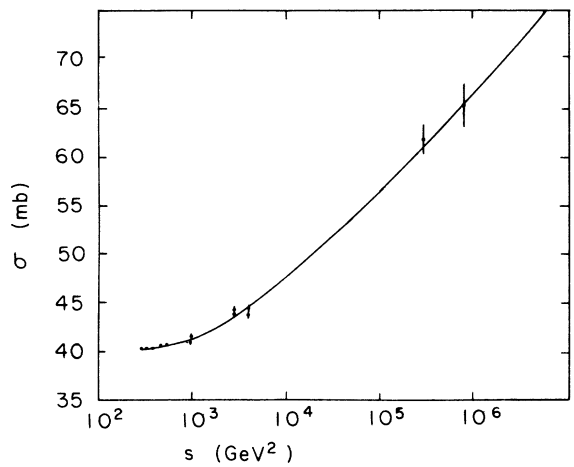

FIG. 4. The vacuum contribution to σ_{tot} for $s \geq 350$ GeV^2, compared to the simplified flavoring model. The predicted cross section at $\sqrt{s}=2$ TeV is 73 mb.

TeV is 73 mb in this model.

The parametrization of Eq. (5) should be adequate through Tevatron energies, but probably not SSC energies. This is just the first iteration of Eq. (1), and corresponds to just a single $B\bar{B}$ or $c\bar{c}$ pair being produced. Our full model indicates that this is not a bad approximation at $\sqrt{s}=900$ GeV; it certainly will not continue to be valid as the energy increases significantly. Bottom-quark production is still on the order of microbarns at SPS collider energies (Ref. 11) and is not yet important in our model. Moreover, higher-order j-plane cut effects will start to play an increasingly important role, as will flavoring renormalization in the low-order vertex and eikonal j-plane cuts.

A simple formula for the ratio $\rho = \text{Re}T(0)/\text{Im}T(0)$ cannot be written down, for the reasons discussed in connection with the full model [see also Ref. 2(a)]. This situation is compounded here, as the simplified model is by construction valid only in a more restricted energy range than is the full model. Because the dispersion relation is sensitive to the imaginary part of T over a wide range of values of s, our simplification of the imaginary part has not resulted in a concurrent simplification of the real part. We note that this conclusion also applies if one chooses to use a Sommerfeld-Watson transform to find the real part, instead of a dispersion relation. The latter approach involves evaluating the real part of Eq. (1). In this case, the $\cot(\pi j/2)$ poles require one to evaluate the integrand at positive integral values of j if $s < s_{\text{th}}$ and at negative values if $s > s_{\text{th}}$. A complete knowledge of the j-plane structure of A_j is therefore required, and our simplified model does not contain this.

We do not feel that it is a drawback to our model that we cannot write down a simple expression for ρ. If the amplitude were a simple sum of 2 or 3 Regge exchanges, a simple expression for ρ would ensue. However, our thesis has been from the outset that the dynamics are more complicated than simple Regge exchange, and in our full model neither the real nor the imaginary part of the amplitude is simple (except in limiting cases such as $s \to s_{\text{th}}$ or $s \to \infty$). Any model which accurately fits the total cross section and has sufficient theoretical foundation to satisfy the dispersion relation, will fit the real part as well. It is, therefore, most important to understand the total cross section. Our proposal of a simplified flavoring amplitude, Eq. (5), is designed to render the flavoring model more manageable, so that it can be subjected to more thorough scrutiny than has been the case.

SUMMARY AND CONCLUSIONS

We summarize by listing the main results of this analysis and the experimental tests which can be used to evaluate the flavoring model. The crucial ingredient in our approach is the correlation between particle multiplicities and σ_{tot}. We have presented an estimate of a few (3–6) mb for the charm cross section at $\sqrt{s}=546$ GeV, based on our two fits to the full model. Including a modest amount of associated charm production led to smaller predicted total charm cross sections, and to a less rapidly rising total pp cross section. At Tevatron energies, this led to a predicted pp cross section of about 74 mb. We have emphasized that charm production is predominantly a nonperturbative QCD process and expect our predicted cross section to be more reliable than the value one obtains from perturbative QCD with an arbitrary multiplicative "K factor." This and the $K^{\pm},\bar{p}$ multiplicities are predictions which can be tested. Our total cross-section predictions can only be extrapolated to a few TeV with any degree of confidence. We can say that further thresholds (bottom and top) will further renormalize the Pomeron, though probably not much. The cross section ought to rise roughly as $s^{\alpha_P - 1}$, with $\alpha_P \approx 1.1$, until higher-order terms in the Reggeon field theory become important.

Inasmuch as this is largely a model based on a self-consistent treatment of finite-energy effects in the context of perturbative Reggeon field theory, we are not able to make detailed predictions at significantly higher energies. Still, it is remarkable that the rather simple parametrization we use is successful over such a wide energy range, fitting in smoothly with low-energy parametrizations involving the unflavored bare Pomeron.[5] We feel this is an indication of the importance of the correlation between heavy-particle production and the shape of the total cross section. The further simplified model presented in the last section should provide a useful way to compare the flavoring approach to other interpretations of the high-energy total cross section.

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[1]G. J. Alner *et al.*, Z. Phys. C **32**, 153 (1986); M. Bozzo *et al.*, Phys. Lett. **147B**, 392 (1984).

[2](a) J. W. Dash, S. T. Jones, and E. K. Manesis, Phys. Rev. D **18**, 303 (1978); (b) J. W. Dash and S. T. Jones, Phys. Lett. **157B**, 229 (1985), and references therein.

[3]M. M. Block and R. N. Cahn, in *Strong Interactions and Gauge Theories,* proceedings of the 21st Rencontre de Moriond, Les Arcs, France, 1986, edited by J. Tran Thanh Van (Editions Frontiéres, Gif-sur-Yvette, 1986), Vol. 2, p. 679.

[4]See, for example, M. Witherell, in *Proceedings of The Salt Lake City Meeting,* Meeting of the Division of Particles and Fields of the APS, Salt Lake City, Utah, 1987, edited by C. DeTar and J. Ball (World Scientific, Singapore, 1987), p. 135.

[5]N. F. Bali and J. W. Dash, Phys. Lett. **51B**, 99 (1974); Phys. Rev. D **10**, 2102 (1974).

[6]J. W. Dash and S. T. Jones, Phys. Rev. D **33**, 1512 (1986).

[7]K. Alpgard *et al.*, Phys. Lett. **121B**, 209 (1983).

[8]See, for example, A. Kernan and G. Van Dalen, Phys. Rep. **106**, 297 (1985).

[9]J. W. Dash and S. T. Jones, Z. Phys. C **22**, 49 (1984).

[10]M. E. Duffy *et al.*, Phys. Rev. Lett. **57**, 1522 (1986).

[11]G. Arnison *et al.*, Phys. Lett. **155B**, 442 (1985).