# Are the $e^+e^-$ and $lp$ multiplicity distributions Poissonian?

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We show that all available multiplicity distributions for $\nu p$, $\bar{\nu}p$, $\mu^+p$, and $e^+e^-$ processes, both for central rapidity intervals and for the whole phase space, are well described by a two-parameter Poisson-type formula. This formula is obtained when small clusters (resonances) are randomly produced.

## I. INTRODUCTION

Standard hadronization models used to describe the final hadronic state in $e^+e^-$ and deep-inelastic $lp$ scattering, such as the uncorrelated cluster models[1] and the recursive cascade models,[2,3] are based on the idea that rapidity correlations have short range. In the first type of models, clusters are produced randomly, and the corresponding multiplicity distributions are of Poisson type (i.e., have $D^2 \propto \langle n \rangle$), provided that the cluster properties are independent of energy. In the recursive cascade models, the multiplicity distributions are also approximately Poisson-type (see below). On the contrary, the QCD branching models[4] lead to $D \propto \langle n \rangle$ and to approximate Koba-Nielsen-Olesen (KNO) scaling.[5] The available experimental data, both in $e^+e^-$ and in deep-inelastic $lp$ scattering, exhibit a dispersion $D$ of the multiplicity distribution which is approximately proportional to the average multiplicity $\langle n \rangle$ (as it should be for KNO scaling) and not to $\langle n \rangle^{1/2}$ (as expected in a short-range-order picture).

One can argue that, at large $Q$, there are modifications to the short-range-order hadronization schemes, due to gluon radiation, which can be described by perturbative QCD. However, since the behavior of $D$ discussed above is already valid at small $Q$, one can hardly put the blame on perturbative QCD corrections. Moreover, in $e^+e^-$ at large $Q$, there are now $e^+e^-$ data[6] available in which the two-jet events have been isolated. This event sample has the same features as the whole inclusive sample. Furthermore, soft processes also exhibit similar features at not too high energies.

On the other hand, if one considers the shapes of the multiplicity distributions, one realizes that they are close to Poisson[7] or even exactly Poissonian[6] when the whole rapidity interval is considered. However, in central rapidity intervals, they are found[6] to be much broader than Poisson.

This puzzling situation has been discussed quite often in the literature. Some authors have argued that the observed approximate KNO scaling is fortuitous, and that the data, both for hadronic processes at not too high energy[8] and for $e^+e^-$ (Ref. 9), are consistent with the short-range-order picture—plus corrections due to hard-gluon radiation in $e^+e^-$ at high $Q$ (Ref. 9). In the following we produce further evidence in this direction, by examining recent data in $e^+e^-$ (Refs. 6 and 7) and $lp$ scattering.[10,11] We show in particular that for all these data, including the data of Ref. 6 for central rapidity intervals, the shapes of the multiplicity distributions can be described by a Poisson-type distribution.

## II. POISSON-TYPE DISTRIBUTIONS

We call Poisson-type distributions, the ones obtained from a Poisson distribution of clusters having decay properties independent of energy. In this case one has

$$D^2 \equiv \langle n^2 \rangle - \langle n \rangle^2 = K_{\rm eff} \langle n \rangle \ ,$$

where $K_{\rm eff} = \langle K^2 \rangle / \langle K \rangle$, and $\langle K \rangle$ is the average charged multiplicity per cluster. In the cluster model one usually considers[12] two rather extreme situations.

(a) The cluster decay distribution is just a delta function (this unphysical case is taken as representative of narrow cluster decay).

(b) The cluster decay distribution is itself Poissonian (this case being take as representative of broad cluster decay).

In the first case, the multiplicity distributions of produced particles can be obtained in a straightforward way. One writes first a Poisson distribution for clusters, with $\langle n_c \rangle = \langle n \rangle / \langle K \rangle = (\langle n \rangle / D)^2$. When written in a KNO form (i.e., $\langle n \rangle P_n$ versus $z = n / \langle n \rangle$), the distribution of clusters is identical to the distribution of final particles. Hence,

$$\psi(z) = \langle n \rangle P_n = \langle n_c \rangle P_{n_c} = \langle n_c \rangle e^{-\langle n_c \rangle} \frac{\langle n_c \rangle^{n_c}}{n_c!} \ , \qquad (1)$$

where $z = n / \langle n \rangle = n_c / \langle n_c \rangle = n_c (D / \langle n \rangle)^2$. Equation (1) gives the multiplicity distribution in the KNO form in terms of a single parameter $\langle n_c \rangle = (\langle n \rangle / D)^2$. To obtain $P_n$ requires, of course, the knowledge of a second parameter ($\langle n \rangle$).

Although Eq. (1) is strictly true only for infinitely narrow clusters (case a), it can be shown to be also approximately true (to a high degree of accuracy) for clusters decaying according to the Poisson law (case b). This can, of

 

TABLE I. The values of the normalized moments $C_3$, $C_4$, and $C_5$, obtained with the two-jet Lund Monte Carlo program with standard parameters (see Ref. 3), for a $u$-$uu$ string fragmentation (corresponding to $\nu p \to \mu^+ X$), at two different energies and for two different rapidity span selections, are compared with the corresponding values obtained from Eq. (4) (and the corresponding equations for $C_4$ and $C_5$), using the value of $C_2$ obtained in the Monte Carlo simulation. The errors in the Monte Carlo figures are about 1%.

| | $W=4$ GeV | | | | $W=12$ GeV | | | |
|---|---|---|---|---|---|---|---|---|
| | All $y$ | | $\lvert y \rvert <0.5$ | | All $y$ | | $\lvert y \rvert <0.5$ | |
| | MC | Eq. (1) | MC | Eq. (1) | MC | Eq. (1) | MC | Eq. (1) |
| $C_3$ | 1.29 | 1.29 | 3.29 | 3.28 | 1.29 | 1.29 | 3.72 | 3.71 |
| $C_4$ | 1.62 | 1.62 | 7.87 | 7.78 | 1.62 | 1.63 | 9.44 | 9.46 |
| $C_5$ | 2.17 | 2.17 | 21.4 | 21.0 | 2.16 | 2.18 | 26.9 | 27.6 |

course, be checked by comparing the corresponding distributions. However, it is more transparent to consider the moments of these distributions, which are given by very simple expressions, in the two cases (a) and (b). The expression of $\langle n \rangle = \langle K \rangle \langle n_c \rangle$ is, of course, the same in both cases. For the second moment one has

$$C_2 \equiv \frac{\langle n^2 \rangle}{\langle n \rangle^2} = 1 + \frac{K_{\text{eff}}}{\langle n \rangle} = 1 + \frac{\langle K \rangle + \eta}{\langle n \rangle} , \tag{2}$$

where $\eta=0$ for case (a) and $\eta=1$ for case (b). Likewise one finds

$$C_3 \equiv \frac{\langle n^3 \rangle}{\langle n \rangle^3} = 1 + \frac{3(\langle K \rangle + \eta)}{\langle n \rangle} + \frac{\langle K \rangle^2 + \eta^2 + 3\langle K \rangle \eta}{\langle n \rangle^2} . \tag{3}$$

For case (a) ($\eta=0$) Eqs. (2) and (3) imply

$$C_3 = C_2^2 + C_2 - 1 , \tag{4}$$

with similar relations between $C_i (i>3)$ and $C_2$. These relations obviously determine the shape of the multiplicity distribution, Eq. (1), once the second moment $C_2$ has been fixed. We see from (3) that relation (4) can also be obtained in case (b) ($\eta=1$) in the limit $\langle K \rangle / \langle n \rangle \to 0$. The same is true for the relations involving higher moments. The condition $\langle K \rangle \ll \langle (n) \rangle$ is satisfied for small clusters ($\langle K \rangle$ of order 1) and high average multiplicities, but it is no longer valid when considering rapidity intervals of small length, where $\langle n \rangle$ becomes very small.[6] However, in this case one finds from Eq. (2) (with $\eta=1$) and the experimental values of $D$ and $\langle n \rangle$ (Ref. 6), that $\langle K \rangle \ll \eta$. In this limit, Eq. (4) also holds.[13] Numerically we have computed $C_3$ in both cases ($\eta=0$ and $\eta=1$) for all available rapidity intervals (from $\lvert y \rvert <0.1$ to the full one).[6] In each case, the value of $\langle K \rangle$ is obtained from Eq. (2) and the experimental values of $D$ and $\langle n \rangle$. It turns out that the values of $C_3$ obtained in the two cases, differ from each other by at most 3% [the difference in the values of $D_3 = \langle (n - \langle n \rangle)^3 \rangle^{1/3}$ being at most 5%]. Thus Eq. (1) provides a good approximation to the multiplicity distribution obtained in a model in which small clusters, having decay distributions not broader than Poisson, are randomly produced.

A well-known complication, not mentioned so far, is due to energy-momentum-conservation constraints which introduce negative correlations and thus are responsible for a narrowing of the multiplicity distributions.

In order to examine its consequences regarding the validity of Eq. (1), we have performed a simulation using the two-jet Lund Monte Carlo[3] model. We have found that Eq. (4), and the corresponding equations for the higher moments, are satisfied to a good degree of accuracy, not only in a central rapidity interval (where the effect of the energy-momentum-conservation constraints is strongly suppressed), but also in the full interval (see Table I).

## III. COMPARISON OF THE POISSON-TYPE DISTRIBUTION WITH DATA AT VARIOUS ENERGIES AND IN VARIOUS RAPIDITY INTERVALS

We are going to consider the available data on $\nu p$, $\bar{\nu} p$ (Ref. 10), $\mu p$ (Refs. 11 and 14), and $e^+e^-$ (Refs. 6 and 7) at various energies. For each process, we take, at each energy, the value of $\langle n \rangle / D$, determined from experiment, and use the Poisson-type multiplicity distribution in Eq. (1).

The comparison of the obtained multiplicity distribution in the complete rapidity interval with the experimental data is given in Figs. 1–3. The used value of $\langle n \rangle / D$

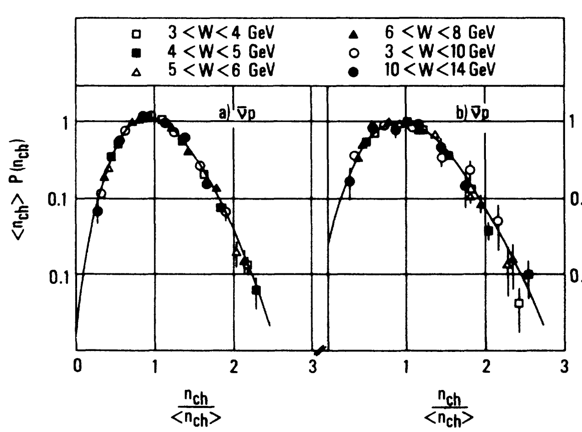


FIG. 1. Multiplicity distributions for (a) $\nu p$ and (b) $\bar{\nu} p$ scattering for various intervals of $W$. Data from Grässler *et al.* (Ref. 10). The curve is obtained from Eq. (1) with $(\langle n \rangle / D)^2 = 8.4$ for $\nu p$ and $(\langle n \rangle / D)^2 = 5.5$ for $\bar{\nu} p$.

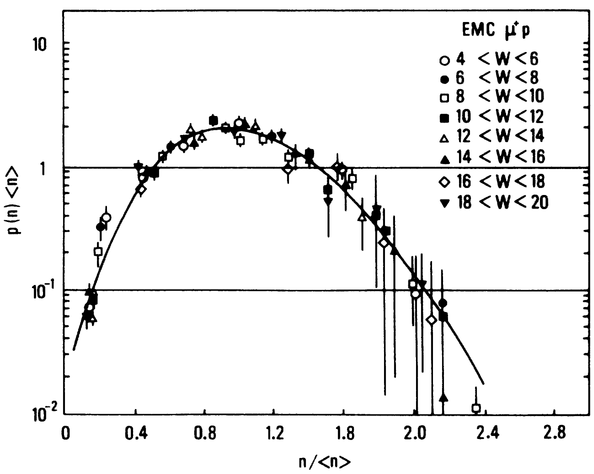


FIG. 2. Multiplicity distribution for $\mu^+p$ scattering for different $W$ regions. Data from Ref. 11. The curve is obtained from Eq. (1) with $(\langle n \rangle /D)^2 = 6.5$.

[the only parameter in Eq. (1)], is given in the figure caption. Experimentally, the value of $\langle n \rangle /D$ is practically independent of the energy. Hence Eq. (1) leads to approximate KNO scaling.[15] Note, however, that the derivation of Eq. (1) in the context of cluster models requires that $\langle n \rangle \propto D^2$, so that $\langle K \rangle \equiv D^2/\langle n \rangle$ is independent of $s$. This point is discussed in detail in Secs. IV A–IV C and V.

Experimental results on the $e^+e^-$ multiplicity distributions at 29 GeV from the High Resolution Spectrometer (HRS) Collaboration (SLAC storage ring PEP) have been presented in Ref. 6. In this work the multiplicity distributions are given in various rapidity intervals $|y| < y_0$ centered at $y^* = 0$. Moreover, results are presented separately for a two-jet event sample (selected using sphericity and aplanatarity cuts), and for the whole inclusive sample. Results on the $\mu p$ multiplicity distributions in various rapidity intervals from the European Muon Collaboration (EMC) are also available.[14]

In order to test the Poisson-type distribution in Eq. (1) in central rapidity intervals we proceed as above. For each rapidity interval we use Eq. (1) with the value of $\langle n \rangle /D$ measured in that interval (see Tables I and II in Ref. 6 and Table II in Ref. 14). The comparison with the experimental distributions is given in Figs. 4 and 5. The multiplicity distributions obtained in Ref. 6 from the best fit to the data using a negative binomial is also shown. We see that the Poisson-type distribution also fits the data well. The data of Ref. 6 for the whole inclusive sample are also well described (not shown).

Note that in the whole rapidity interval, $\langle n \rangle \gg \langle K \rangle$ and therefore the shape of the distribution in Eq. (1) is close to a Poisson distribution of particles with average $\langle n \rangle$. On the contrary, in small rapidity intervals, $\langle n \rangle$

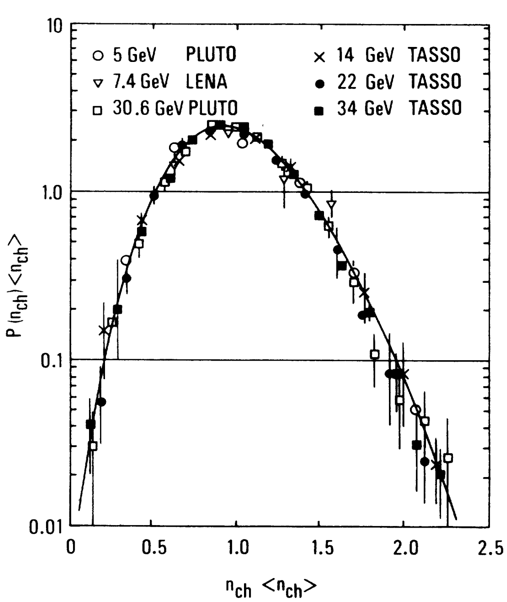


FIG. 3. Multiplicity distribution for $e^+e^-$ annihilation for different values of $W$. Data from a compilation by the TASSO Collaboration (Ref. 7). The curve is obtained from Eq. (1) with $(\langle n \rangle /D)^2 = 7.85$.

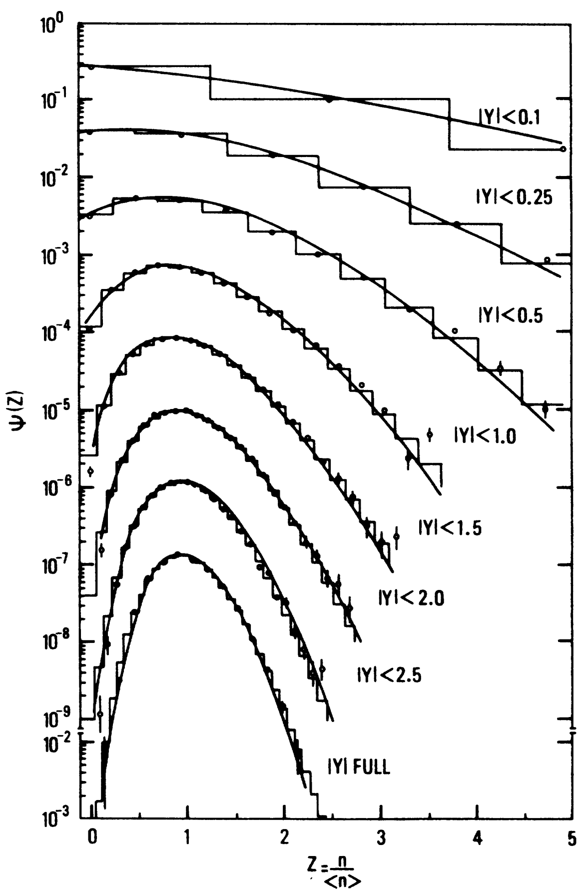


FIG. 4. Multiplicity distributions for two-jet events of $e^+e^-$ annihilation at 29 GeV for various rapidity span selections. Data from Ref. 6. The curves are obtained from Eq. (1), with the values of $\langle n \rangle /D$ given in Ref. 6. The histograms show the best fit to the negative binomial, obtained in Ref. 6.

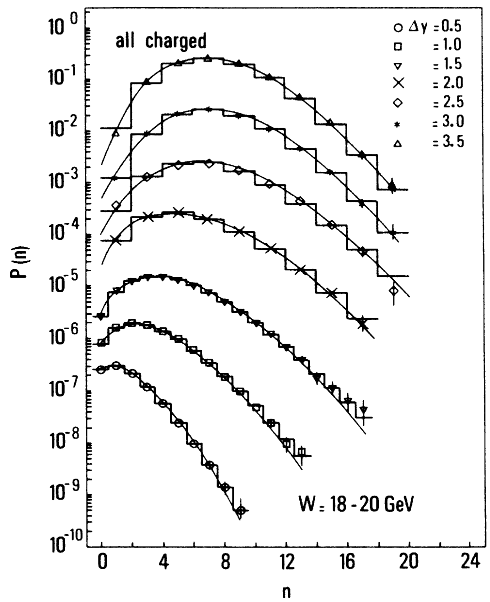


FIG. 5. Multiplicity distributions for inclusive events in $\mu p$ scattering at $W=18-20$ GeV for various rapidity-span selections. Data from Ref. 14. The curves are obtained from Eq. (1) with the values of $\langle n \rangle / D$ given in Ref. 14. The histograms show the best fit to the negative binomial, obtained in Ref. 14.

gets closer to $\langle K \rangle$ and the Poisson distribution of clusters, Eq. (1), is then much broader than the corresponding Poisson distribution of particles.

## IV. DISCUSSION AND REMARKS

*(a) Energy dependence of $K_{\rm eff}$.* The values of $K_{\rm eff}=D^2/\langle n \rangle$ obtained using the available data, increase with increasing energy. They range from $K_{\rm eff}\sim 0.4$ to $K_{\rm eff}\sim 1.4$. This had to be expected in view of the well-known linear relation between $D$ and $\langle n \rangle$ (Wroblewski relation). The energy dependence of $K_{\rm eff}$ can be qualitatively understood as a consequence of the energy-momentum-conservation constraints. As discussed above, these constraints produce a narrowing of the multiplicity distribution. Since this effect is larger at low energies, the value of $K_{\rm eff}$ increases with $s$. With a simulation using the Lund Monte Carlo program for $e^+e^-$, one finds[9] a linear relation $D=a\langle n \rangle + b$, for a very large range of values of $\langle n \rangle$. The energy-momentum-conservation constraints play an important role in building up this linear dependence.

The increase of $K_{\rm eff}$ with $s$ mimics, in the case of narrow clusters, a KNO-scaling situation in which the cluster multiplicity $\langle K \rangle \approx K_{\rm eff}$ increases with $s$, while the average number of clusters, $\langle n_c \rangle \approx \langle n \rangle / K_{\rm eff}$, remains approximately constant. However, in the short-range-order models, this can only occur in a limited range of energy. A simulation with the two-jet Lund Monte Carlo program shows that $K_{\rm eff}$ is practically constant above top DESY PETRA energies.

*(b) $e^+e^-$ multiplicity distribution at CERN LEP energies.* With a value of $K_{\rm eff}\equiv D/\langle n \rangle^2$, constant from top PETRA energies to LEP energies, the value of the unique parameter $\langle n \rangle / K_{\rm eff} = (\langle n \rangle / D)^2$ in Eq. (1) will decrease substantially between these two energy ranges. As a consequence, the $e^+e^-$ multiplicity distribution will violate KNO scaling, getting narrower with increasing $s$. (See, however, the restrictions mentioned in the last paragraph of Sec. V.)

*(c) Dependence of $K_{\rm eff}$ on the size of the rapidity interval.* From the data of Ref. 6, one obtains the following behavior of $K_{\rm eff}$: for the whole rapidity interval, the value of $K_{\rm eff}$ is about one (both for the two-jet and the inclusive event samples), increases when the length of the rapidity interval decreases, reaches a broad maximum of about 1.4 in the two-jet sample (1.7 in the inclusive one), and then decreases toward one for very small rapidity intervals. As mentioned above, the value of $K_{\rm eff}$ increases monotonically with $s$. Therefore, the fact that its value is close to one at 29 GeV is fortuitous. However, its variation with the size of the interval is again quite obvious from energy-momentum-conservation constraints. The induced negative correlations are smaller in central rapidity intervals, and $K_{\rm eff}$ increases. For very small rapidity intervals, the clustering effect vanishes and $K_{\rm eff}$ tends to one. Again such a behavior is obtained in the two-jet Lund Monte Carlo program.

The same type of dependence of $K_{\rm eff}$ on the size of the rapidity interval has been observed in $\mu p$ scattering.[14] The value of $K_{\rm eff}$ at the maximum turns out to be approximately independent of energy. This result, which strongly supports our cluster-model interpretation, is discussed in more detail in Sec. V.

*(d) Consequences for cluster size.* A Monte Carlo simulation in which clusters are identified with a realistic mixture of directly produced particles and known resonances leads a charged average cluster multiplicity $\langle K \rangle \sim 1.4$ (Ref. 16). It is therefore interesting that all measured values of $K_{\rm eff}$, in the whole rapidity interval, are smaller than 1.4 and that they reach a maximum of about 1.4 in central rapidity intervals. This favors the view that clusters are small and narrow and can be identified with known resonances. A similar conclusion was reached in Ref. 17 from an analysis of hadronic data.

*(e) Hadronic processes.* In multiple-scattering models for hadron-hadron collisions such as the dual parton model[17] one has a sum of contributions containing a different number of strings. Multistring configurations give substantial contributions at high energies. Therefore, one might expect that the equations in Sec. II are irrelevant in this case. However, although this is true at a quantitative level, these equations are still qualitatively consistent with a multiple-scattering picture. Indeed Eqs. (2) and (3) with $\eta=1$ can be interpreted as the result of a superposition, with Poisson weights, of identical strings, having average multiplicity $\langle K \rangle$, and fragmenting according to a Poisson law ($\eta=1$), or Poisson type if $\eta\neq 1$ (for instance, $\eta=1.4$). Quantitatively, the multiplicity distribution for hadronic processes obtained from Eq. (1), is expected to be narrower than the experimental one, especially at high energies. Indeed, the distribution of the weights of the various multichain contributions (ob-

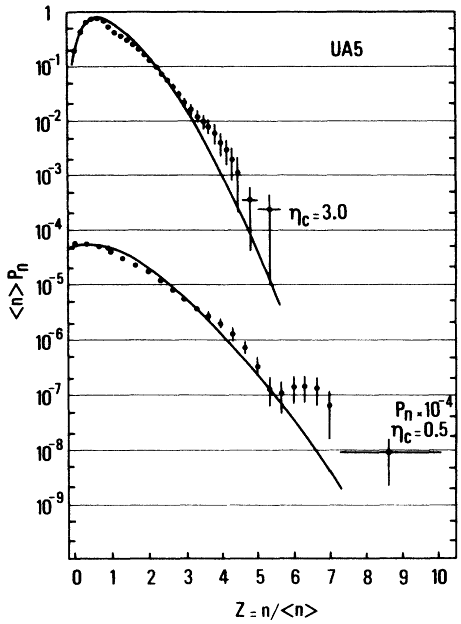


FIG. 6. Multiplicity distributions for $\bar{p}p$ scattering at $\sqrt{s} = 540$ GeV, for two pseudo-rapidity-span selections $|\eta| < 0.5$ and $|\eta| < 1.5$. Data from Ref. 18. The curves are obtained from Eq. (1) with the values of $\langle n \rangle / D$ given in Ref. 18.

tained from unitarity[18]) is broader than Poisson weights. Moreover, there are fluctuations in the position of the chain ends which are not considered in Eqs. (1)–(4). The comparison with $\bar{p}p$ data[19] shows that Eq. (1) gives indeed a too narrow distribution (see Fig. 6).

The fact that here the underlying physical picture is different from that for "one-string" processes, can be confirmed by looking at the values of $K_{\text{eff}}$ obtained from the $pp$ data at $\sqrt{s} = 540$ GeV (Ref. 19). One obtains, in the various rapidity intervals, values of $K_{\text{eff}}$ ranging from 1.7 to 9.3. This shows that here "clusters" are no longer small and supports a multistring picture for high-energy hadronic processes.

## V. CONCLUSION AND PHYSICAL INTERPRETATION

The particle production in one-string processes is expected to correspond to independent emission of clusters. Therefore the multiplicity distributions for two-jet events in $e^+e^-$ and $lp$ scattering are expected to be Poissonian. This expectation seems to be at variance with experiment. The data in the whole rapidity interval show a ratio $\langle n \rangle / D$ approximately constant corresponding to KNO scaling rather than to a Poisson law. As a consequence the average charged multiplicity per cluster, $\langle K \rangle \equiv D^2 / \langle n \rangle$, has an energy dependence. This energy dependence is different in different rapidity intervals.

In this paper we have shown that a Poisson multiplicity distribution in clusters [Eq. (1)] with the parameters $\langle n \rangle$ and $D$ taken from experiment, reproduces very well the multiplicity distributions at all energies and for all rapidity intervals. This interesting result, obtained here for the first time, is, in itself, a numerical regularity with no physical interpretation. Indeed the interpretation of Eq. (1) based on independent emission of clusters, requires a value of $\langle K \rangle$ independent of both $s$ and the size $\Delta y$ of the rapidity interval and this is at variance with the data. However, we have argued in Sec. IV that the observed dependence of $\langle K \rangle$ in $s$ and $\Delta y$ is a consequence of energy-momentum (plus some effect resulting from gluon radiation), and, therefore, the above interpretation of Eq. (1) can be maintained. This claim is strongly supported by the recent data on $e^+e^-$ (Ref. 6) and $\mu p$ (Ref. 14) in limited rapidity intervals. Indeed, these data provide us with the value of $\langle K \rangle = D^2 / \langle n \rangle$ in a central rapidity interval, where the effects of energy conservation are strongly suppressed. The value of $\langle K \rangle$ obtained from $\mu p$ reactions in the interval $|y| < 1$ and in the energy range $8 \leq W \leq 18$ GeV, is $\langle K \rangle \sim 1.3 \div 1.4$ (Ref. 14), while its value in $e^+e^-$ at $Q = 29$ GeV and $|y| < 1$ for the two-jet sample is $\langle K \rangle \sim 1.4$ (Ref. 6).

As discussed in Sec. IV, a value of $\langle K \rangle \sim 1.4$ is precisely the one found in a Monte Carlo simulation where the clusters are identified with a realistic mixture of directly produced particles and known resonances.[16] The same value of $\langle K \rangle$ is needed in the context of the dual parton model to reproduce multiplicity distributions, long-range and short-range correlations in $\bar{p}p$ collisions[17]).

Of course one can argue that the above picture for $e^+e^-$ and $lp$ scattering may be blurred at high energies by the increasing contribution of gluon radiation. However, it is important to show, as we have done in this paper, that all the one-string processes can be described, at present energies, by a simple picture in which small clusters are randomly produced, and, furthermore, that both in one-string processes (for two-jet events) and in hadronic reactions, clusters can be identified with known resonances.

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