Chiral-symmetry-breaking corrections in two-photon decays of pseudoscalar mesons

Riazuddin
Department of Physics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

Fayyazuddin
Department of Physics, Umal Qurah University, Makkah Al-Mukkarama, Saudi Arabia

(Received 10 February 1987)

Chiral-symmetry-breaking corrections of order m_P^2/m_V^2 to the main effect which arises from the axial-vector anomaly in π^0, η^0, and $\eta'^0\to 2\gamma$ decays are estimated by using gauge invariance and vector-meson dominance, which holds at about the 25% level. The inclusion of these corrections requires a large η-η' mixing angle $\approx -(25\pm4)^\circ$ and substantial violation of nonet symmetry for F_8 and F_0, the axial-vector decay constants for η_8 and η_0.

I. INTRODUCTION

If up-, down-, and strange-quark masses are set equal to zero, then the chiral SU(3)$\times$SU(3) symmetry is a property of the QCD Lagrangian. The spontaneous breaking of this chiral symmetry (resulting in the appearance of eight massless pseudoscalar Nambu-Goldstone bosons) or equivalently PCAC (partial conservation of axial-vector current) plus the axial-vector anomaly fixes[1] the two-photon decays of pseudoscalar mesons in terms of the matrix elements

$$\langle 0 \mid A_{k\lambda} \mid P_k(q)\rangle = iF_P q_\lambda \tag{1}$$

($k=3,8,0$ with the corresponding $P_k=\pi^0,\eta_8,\eta_0$, and we write for simplicity $F_{\eta_8}=F_8, F_{\eta_0}=F_0$). Unlike F_π (which is known experimentally, $F_\pi=93$ MeV), F_8 and F_0 are not simply related to other well-known physical processes. With certain assumptions to be stated below for F_8, the recent measurements of η and η' radiative widths[2] have been interpreted[3,4] as an indication of large mixing angle ($-\theta\approx 20^\circ$) between η_8 and η_0. This result, of course, depends on the value of F_8/F_π used. If one uses $F_8/F_\pi=1$, one obtains

$$-\theta=(17.5\pm4.5)^\circ\ ,\quad F_0/F_\pi=1.08\pm0.11\ . \tag{2}$$

Recently the value $F_8/F_\pi=1.25$, which has been obtained by the chiral one-loop renormalization[5] of F_8 and F_π, has been used. On the other hand, $\pi^0\to2\gamma$ and $\eta_8\to2\gamma$ amplitudes are not renormalized[3] at the chiral one-loop level and have thus the standard expressions of current algebra; the use of these with $F_8/F_\pi=1.25$ and the experimental radiative widths of π^0, η, and η' gives the solution[3]

$$-\theta=(23\pm3)^\circ\ , \tag{3a}$$

$$F_0/F_\pi=1.04\pm0.04\ . \tag{3b}$$

As is well known, the current-algebra calculation for the radiative widths crucially involves the extrapolation of the matrix elements of the divergence of the axial-vector current,

$$\left\langle \gamma(k_1)\gamma(k_2)\left|\frac{\partial A_{k\lambda}}{\partial x_\lambda}\right|0\right\rangle$$

from $q^2=(k_1+k_2)^2=0$ to $-q^2=+m_P^2$ ($m_P=m_\pi$, $m_\eta,m_{\eta'}$). Because m_π is small compared to a light-hadron mass (e.g., $m_\rho\approx770$ MeV), one may hope that the neglect of this extrapolation is justified, but for η and η' mesons the extrapolation is potentially dangerous. The purpose of this paper is to study this question and to estimate the corrections due to explicit breaking of chiral symmetry when the pseudoscalar Goldstone bosons acquire their masses (or equivalently quark masses are not set equal to zero). We show that the gauge invariance and vector-meson dominance enable us to estimate (q^2/m_V^2) corrections to the main effect, which arises from the axial-vector anomaly term. As will be discussed, these corrections are small (1.2%) for π^0 decay but could be substantially large for η and η' decays ($\sim17\%$ for η decay and 40% for decay in the amplitudes). However, these estimates may be off by about 25% since our use of vector-meson dominance may be off by this amount. When these corrections are included, we obtain

$$-\theta=(25\pm4)^\circ\ , \tag{4a}$$

$$F_0/F_\pi=0.64\pm0.07\ , \tag{4b}$$

substantially different from (2) and (3), particularly for F_0 which indicates large nonet-symmetry breaking for F_0 and F_8. The result for the η-η' mixing angle is obtained from the η_8 sum rule only and is thus independent of the pure-gluon component which might be present in η and η' in addition to $\bar{q}q$ states η_8 and η_0. The large mixing angle may be consistent with the one used in the linear mass formula for pseudoscalar mesons. The consistency of a large mixing angle with the quadratic mass formula has been discussed in Ref. 4. Below we give the details of our calculation.

II. CHIRAL-SYMMETRY-BREAKING CORRECTIONS

The S-matrix element for $P\to 2\gamma$ is defined by

$$\langle\gamma(k_1)\gamma(k_2)\,|\,P(q)\rangle = i(2\pi)^4\delta^4(q-k_1-k_2)\times\langle\gamma(k_1)\gamma(k_2)\,|\,J_P(0)\,|\,0\rangle\ , \tag{5a}$$

with

$$\langle\gamma(k_1)\gamma(k_2)\,|\,J_P(0)\,|\,0\rangle=\epsilon^*_\mu(k_1)\epsilon^*_\nu(k_2)(-\Gamma^k_{\mu\nu})\ , \tag{5b}$$

where

$$\Gamma^k_{\mu\nu}=-i\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}A^k_P\ . \tag{5c}$$

Now we have the Ward identity

$$-iq_\lambda M^k_{\mu\nu\lambda}=M^k_{\mu\nu}\ , \tag{6a}$$

where separating out the pseudoscalar-meson P pole and possible pole P' due to the radial excitation of P, we have

$$\begin{aligned}M^k_{\mu\nu\lambda}&=\int \alpha^4x\,d^4y\,e^{-ik_1x}e^{-ik_2y}\langle 0\,|\,T(V^{\rm em}_\mu(x)V^{\rm em}_\nu(y)A_{k\lambda}(0))\,|\,0\rangle\\ &=i\Gamma^k_{\mu\nu\lambda}-\sum_P\frac{F_{kP}iq_\lambda}{q^2+m_P^2}\Gamma^{k,P}_{\mu\nu}-\sum_{P'}\frac{F_{kP'}iq_\lambda}{q^2+m_{P'}^2}\Gamma'^{k,P'}_{\mu\nu}\ ,\end{aligned} \tag{6b}$$

where

$$\langle 0\,|\,A_{k\lambda}\,|\,P(q)\rangle=iF_{kP}q_\lambda\ , \tag{6c}$$

and for $k=3$, $P=\pi$ only with $F_{3\pi}=F_\pi$ while for $k=8,0$, $P=\eta$ and η', and similarly for radial excitations P'. In Eq. (6a),

$$M^k_{\mu\nu}=\int\alpha^4x\,d^4y\,e^{-ik_1x}e^{-ik_2y}\times\langle 0\,|\,T(V^{\rm em}_\mu(x)V^{\rm em}_\nu(y)\partial_\lambda A_{k\lambda}(0))\,|\,0\rangle\ . \tag{7}$$

Using now PCAC with the axial-vector anomaly

$$\frac{\partial}{\partial x_\lambda}A_{k\lambda}(x)=F_Pm_P^2P_k(x)-\frac{i}{16\pi^2}S^k_P\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}(x)F_{\alpha\beta}(x)\ , \tag{8}$$

with $k=3,8,0$, i.e., separating out the pseudoscalar-meson P pole and other possible pole P' due to its radial excitation and keeping the anomaly term, we obtain

$$M^k_{\mu\nu}=\sum_P\frac{F_{kP}m_P^2}{q^2+m_P^2}\Gamma^{k,P}_{\mu\nu}+\sum_{P'}\frac{F_{kp'}m_{P'}^2}{q^2+m_{P'}^2}\Gamma'^{k,P'}_{\mu\nu}+\frac{i}{2\pi^2}S^k_P\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\ . \tag{9}$$

Substituting in the Ward identity (6a) and using (6b), we obtain the sum rule

$$-\sum_P F_{kP}\Gamma^{k,P}_{\mu\nu}-\sum_{P'}F_{kP'}\Gamma'^{k,P'}_{\mu\nu}=-q_\lambda\Gamma^k_{\mu\nu\lambda}+\frac{i}{2\pi^2}S^k_P\epsilon_{\mu\nu_\alpha\beta}k_{1\alpha}k_{2\beta}\ . \tag{10a}$$

For $k=8$ or 0 when $P=\eta$ and η', writing η and η' in terms of η_8 and η_0 as

$$\begin{aligned}\eta&=\cos\theta\,\eta_8-\sin\theta\,\eta_0\ ,\\ \eta'&=\sin\theta\,\eta_8+\cos\theta\,\eta_0\ ,\end{aligned} \tag{10b}$$

we get, from Eq. (6c) [cf. Eq. (1)],

$$\begin{aligned}&F_{8\eta}=\cos\theta\,F_8,\quad F_{8\eta'}=\sin\theta\,F_8\ ,\\ &F_{0\eta}=-\sin\theta\,F_0,\quad F_{0\eta'}=\cos\theta\,F_0\ .\end{aligned} \tag{10c}$$

Then writing

$$\Gamma^{3,\pi}_{\mu\nu}\equiv\Gamma^3_{\mu\nu}$$

and

$$\begin{aligned}&\cos\theta\,\Gamma^{8,\eta}_{\mu\nu}+\sin\theta\,\Gamma^{8,\eta'}_{\mu\nu}=\Gamma^8_{\mu\nu}\ ,\\ &-\sin\theta\,\Gamma^{0,\eta}_{\mu\nu}+\cos\theta\,\Gamma^{0,\eta'}_{\mu\nu}=\Gamma^0_{\mu\nu}\ ,\end{aligned}$$

we can rewrite the sum rule (10a) as

$$-F_P\left[\Gamma^k_{\mu\nu}+\frac{F_{P'}}{F_P}\Gamma'^k_{\mu\nu}\right]=-q_\lambda\Gamma^k_{\mu\nu\lambda}+\frac{i}{2\pi^2}S^k_P\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\ , \tag{10d}$$

where F_P is now defined as in Eq. (1). Chiral symmetry requires that $F_{P'}$ vanishes in the chiral limit, so that

$$F_{P'}=r_{P'}\frac{m_P^2}{m_{P'}^2}F_P\ , \tag{11}$$

where $r_{P'}$ has been estimated[6] to be $2\sqrt{2}$.

Now to estimate $\Gamma'^k_{\mu\nu}$ relative to $\Gamma^k_{\mu\nu}$, i.e., $A_{P'\to 2\gamma}/A_{P\to 2\gamma}$, we use the quark annihilation model[7] which gives

$$\begin{aligned}\Gamma_{\gamma\gamma}(P)&=\frac{4\pi\alpha^2}{3}\left[\sum_i\frac{Q_i^2}{\sqrt{3}}\frac{m_P}{m_i^2+m_P^2/4}\right]^2|\,\psi_i^P(0)\,|^2\\ &=4\pi\alpha^2\,|\,A_{P\to 2\gamma}\,|^2\frac{m_P^3}{16}\ ,\end{aligned} \tag{12a}$$

giving

$$|A_{P\to 2\gamma}|=\frac{4}{\sqrt{3}}\sum_i \frac{Q_i^2}{\sqrt{3}}\frac{1}{(m_i^2+m_P^2/4)m_P^{1/2}}|\psi_i^P(0)| \; . \tag{12b}$$

This gives, for example, for the pion case,

$$\left|\frac{A_{\pi'\to 2\gamma}}{A_{\pi\to 2\gamma}}\right|=\left[\frac{m_\pi}{m_{\pi'}}\right]^{1/2}\frac{1+(m_\pi/2m_{\rm NS})^2}{1+(m_{\pi'}/2m_{\rm NS})^2}\left|\frac{\psi^{\pi'}(0)}{\psi^{\pi}(0)}\right| , \tag{13a}$$

where $m_{\rm NS}$ is the average of nonstrange-quark masses ($\simeq 300$ MeV). Potential models, which give good fit to masses of heavy-quarkonium states, would give[8]

$$\left|\frac{\psi^{\pi'}(0)}{\psi^{\pi}(0)}\right|=\left|\frac{\psi_2(0)}{\psi_1(0)}\right|\simeq\sqrt{3/7} \tag{13b}$$

Setting $m_{\pi'}\simeq 1300$ MeV, we see that

$$\left|\frac{A_{\pi'\to 2\gamma}}{A_{\pi\to 2\gamma}}\right|\simeq\frac{1}{20} \; . \tag{13c}$$

This together with (11) is too small. In any case, together with (11), we see that

$$\frac{F_{P'}}{F_P}\left|\frac{A_{P'\to 2\gamma}}{A_{P\to 2\gamma}}\right|=O(m_P/m_{P'})^{5/2} \tag{14}$$

at least while we are interested in terms of $O(m_P^2/m_{P'}^2)$. In view of the above and also due to lack of data on radial excitations for η_8 and η_0 we shall neglect the contributions from the radial excitations in the sum rule to obtain

$$-F_P\Gamma^k_{\mu\nu}=-q_\lambda\Gamma^k_{\mu\nu\lambda}+\frac{i}{2\pi^2}S_P^k\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta} \; . \tag{15}$$

We now estimate the term $q_\lambda\Gamma^k_{\mu\nu\lambda}$ which on dimensional grounds will be $O(q^2/m_V^2)$ where m_V is a typical vector-boson or axial-vector-boson mass. What we need is the matrix element

$$i\Gamma^k_{\mu\nu\lambda}=-\langle\gamma(k_1)\gamma(k_2)|\tilde{A}_k(0)|0\rangle ,$$

where a tilde denotes that $\tilde{A}_{k\lambda}$ does not contain any pseudoscalar-meson pole. This we calculate in the vector-meson-dominance model for which we first calculate $\langle V_i(k_1)V_j(k_2)|A_{k\lambda}|0\rangle$ to be dominated by the axial-vector-meson A_k pole. The gauge-invariant axial-vector-meson coupling with two vector mesons (here i,j are indices corresponding to vector mesons ρ, ω, and ϕ) is defined as

$$-i\langle V_i(k_1)V_j(k_2)|A_k(q)\rangle=\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)\epsilon_\lambda(q)\times\{[q^2-(k_1^2+k_2^2)]\tfrac{1}{2}\epsilon_{\nu\mu\alpha\lambda}(k_1-k_2)_\alpha+(\epsilon_{\nu\lambda\alpha\beta}k_{2\mu}-\epsilon_{\mu\lambda\alpha\beta}k_{1\nu})k_{1\alpha}k_{2\beta}\}b^{ijk} \; . \tag{16}$$

Then defining the matrix elements of the axial-vector current $A_{k\lambda}$ in a gauge-invariant way as

$$\langle V_i(k_1)V_j(k_2)|A_{k\lambda}|0\rangle=\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)\times[f_A^{ijk}(q^2)\epsilon_{\nu\mu\alpha\lambda}(k_1-k_2)_\alpha+h^{ijk}(q^2)\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}q_\lambda+g_1^{ijk}(q^2)(\epsilon_{\nu\lambda\alpha\beta}k_{2\mu}-\epsilon_{\mu\lambda\alpha\beta}k_{1\nu})k_{1\alpha}k_{2\beta}+g_2^{ijk}(q^2)(\epsilon_{\nu\lambda\alpha\beta}k_{1\mu}-\epsilon_{\mu\lambda\alpha\beta}k_{2\nu})k_{1\alpha}k_{2\beta}] , \tag{17}$$

we see that pseudoscalar-meson pole contributions to the above matrix elements are

$$\sum_P\frac{F_{kP}q_\lambda}{q^2+m_P^2}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\gamma^{(k)}_{V_iV_jP} , \tag{18a}$$

where $k=3,8,0$ and for $k=3$, $P=\pi$ only while for $k=8$ or 0, $P=\eta$ and η' [here γ_{VVP} is the coupling strength for $V\to V'P$ and F_{kP} are defined in Eq. (6c)]. Thus,

$$h_P^{ijk}(q^2)=\sum_P\frac{F_{kP}\gamma^{(k)}_{V_iV_jP}}{q^2+m_P^2} \tag{18b}$$

and one has the Goldberger-Treiman relation

$$2f_A^{ijk}(0)=\sum_P F_{kP}\gamma^{(k)}_{V_iV_jP} \; . \tag{18c}$$

Now if we use Eqs. (10c) and write

$$\cos\theta\,\gamma^{(8)}_{V_iV_j\eta}+\sin\theta\,\gamma^{(8)}_{V_iV_j\eta'}=\gamma^{(8)}_{V_iV_j\eta_8} ,$$
$$-\sin\theta\,\gamma^{(0)}_{V_iV_j\eta}+\cos\theta\,\gamma^{(0)}_{V_iV_j\eta'}=\gamma^{(0)}_{V_iV_j\eta_0} , \tag{18d}$$

we can rewrite Eq. (18c) as

$$2f_A^{ijk}(0)=F_P\gamma^{(k)}_{V_iV_jP} , \tag{18e}$$

where F_P is now defined as in Eq. (1).

The axial-vector-meson pole contribution to Eq. (17) is

$$f_A^{ijk}(q^2)=\sum_A\frac{F_{kA}b^{ijk}[q^2-(k_1^2+k_2^2)]}{2(q^2+m_A^2)} , \tag{19a}$$

$$h_A^{ijk}(q^2)=-\sum_A\frac{F_{kA}b^{ijk}[q^2-(k_1^2+k_2^2)]}{m_A^2(q^2+m_A^2)} , \tag{19b}$$

$$g_1^{ijk}(q^2)=\sum_A \frac{b^{ijk}F_{kA}}{q^2+m_A^2} , \tag{19c}$$

$$g_2^{ijk}(q^2)=0 , \tag{19d}$$

where F_A and F_V are defined by

$$\langle 0 | A_{k\lambda}(0) | A(q)\rangle = F_{kA}\epsilon_\lambda(q) , \tag{20a}$$

$$\langle 0 | V_{i\mu} | V_i(k_1)\rangle = F_{V_i}\epsilon_\mu(k_1) . \tag{20b}$$

For the vector mesons on their mass shell ($k_1^2=-m_{V_i}^2$, $k_2^2=-m_{V_j}^2$), Eqs. (18e) and (19a) give, for $q^2=0$,

$$\sum_A 2F_{kA}b^{ijk}\frac{m_{V_i}^2+m_{V_j}^2}{2m_A^2}=F_P\gamma_{V_iV_jP}^{(k)} . \tag{21}$$

Thus, finally using Eqs. (17) and (19), vector-meson dominance for $\langle\gamma(k_1)\gamma(k_2)|\tilde{A}_{k\lambda}|0\rangle$ gives ($k_1^2=0=k_2^2$)

$$\begin{aligned} -i\Gamma_{\mu\nu\lambda}^k &= \langle\gamma(k_1)\gamma(k_2)|\tilde{A}_{k\lambda}|0\rangle \\ &= \sum_{i,j}\sum_A \frac{F_{kA}}{q^2+m_A^2}\frac{F_{V_i}F_{V_j}}{m_{V_i}^2 m_{V_j}^2}\left[\frac{q^2}{2}b^{ijk}\epsilon_{\nu\mu\alpha\lambda}(k_1-k_2)_\alpha-\frac{2}{m_A^2}\frac{q^2}{2}b^{ijk}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}q_\lambda\right. \\ &\qquad + b^{ijk}(\epsilon_{\nu\lambda\alpha\beta}k_{2\mu}-\epsilon_{\mu\lambda\alpha\beta}k_{1\nu})k_{1\alpha}k_{2\beta} \\ &\qquad \left. + \left[-\frac{q^2}{2}b^{ijk}\right]\left[\frac{1}{m_{V_i}^2}\epsilon_{\nu\lambda\alpha\beta}k_{1\mu}-\frac{1}{m_{V_j}^2}\epsilon_{\mu\lambda\alpha\beta}k_{2\nu}\right]k_{1\alpha}k_{2\beta}\right] , \end{aligned} \tag{22}$$

which is explicitly gauge invariant. Substituting Eq. (22) in the sum rule (15) and using Eqs. (5c) and (21), we obtain (with $q^2=-m_P^2$)

$$A_P^k=\frac{1}{F_P}\left[\frac{1}{2\pi^2}S_P^k-\sum_{i,j}\frac{m_P^2}{m_{V_i}^2+m_{V_j}^2}\frac{F_{V_i}F_{V_j}}{m_{V_i}^2m_{V_j}^2}F_P\gamma_{V_iV_jP}^{(k)}\right] . \tag{23}$$

We replace $m_{V_i}^2+m_{V_j}^2$ by the average vector-meson mass $\bar{m}_V^2=\frac{1}{2}(m_{V_i}^2+m_{V_j}^2)$ and can then eliminate $F_P\gamma_{VVP}^{(k)}$, by assuming vector dominance of $P\to\gamma\gamma$ matrix elements, which succeeds at $\lesssim 25\%$. The vector-dominance model gives

$$A_P^k=\sum_{i,j}\frac{F_{V_i}F_{V_j}}{m_{V_i}^2m_{V_j}^2}\gamma_{V_iV_jP}^{(k)} , \tag{24}$$

so that we can write the sum rule (23) as

$$A_P^k=\frac{1}{F_P}\frac{1}{2\pi^2}S_P^k\left[1+\frac{m_P^2}{2\bar{m}_V^2}\right]^{-1} . \tag{25}$$

Note that in this sum rule $k=3$, 8, and 0 and correspondingly $P=\pi^0$, η_8, and η_0.

III. NUMERICAL RESULTS

We have

$$S_\pi=\tfrac{1}{2},\quad S_{\eta_8}=\frac{1}{2\sqrt{3}},\quad S_{\eta_0}=\sqrt{2/3} \tag{26}$$

and for vector bosons which are coupled to γ's,

$$\bar{m}_V^2=\frac{m_\rho^2+m_{\omega_8}^2}{2}\approx 1.23m_\rho^2 , \tag{27}$$

using $m_\rho\simeq 770$ MeV, $m_{\omega_8}\simeq 930$ MeV. Define

$$A(P\to 2\gamma)=\frac{1}{4\pi^2}\tilde{A}(P\to 2\gamma) , \tag{28}$$

so that

$$\Gamma(P\to 2\gamma)=\frac{m_P^3\alpha^2}{64\pi^3}|\tilde{A}(P\to 2\gamma)|^2 , \tag{29}$$

where $P=\pi^0$, η, or η'. Now from Eqs. (25) and (26) we have

$$\tilde{A}_{\pi^0}=4\pi^2A_{\pi^0}=\frac{1}{F_\pi}(1-0.012) , \tag{30a}$$

$$\tilde{A}_{\eta_8}=4\pi^2A_{\eta_8}=\frac{1}{F_8}(1-0.18) , \tag{30b}$$

$$\tilde{A}_{\eta_0}=4\pi^2A_{\eta_0}=\frac{1}{F_0}(1-0.38) , \tag{30c}$$

where we have used $m_\pi=135$ MeV, $m_{\eta_8}=560$ MeV, and $m_{\eta_0}=948$ MeV. The value of m_{η_8} is that which one gets from Gell-Mann–Okubo mass formula. Once the value of m_{η_8} is fixed as above one gets m_{η_0} from the relation

$$m_{\eta_0}^2+m_{\eta_8}^2=m_\eta^2+m_{\eta'}^2$$

and the experimental values of m_η and $m_{\eta'}$. It may be noted that the sum rule (25) is not sensitive to small variations in m_{η_8}. The Gell-Mann–Okubo formula is at least valid to the same accuracy as vector-meson dominance.

Equation (30a) gives

$$\Gamma_{\pi^0}=7.45\text{ eV} \tag{31}$$

to be compared with 7.64 eV without the chiral-symmetry-breaking correction and with its experimental value[9]

$$\Gamma_{\pi^0}^{\text{expt}}=7.57\pm0.32\text{ eV}\;. \tag{32}$$

Thus we see that with an increased accuracy of the experimental measurement of Γ_{π^0}, the chiral-symmetry-breaking correction calculated here is testable.

The experimental result[2] $\Gamma(\eta\to2\gamma)=0.53\pm0.08$ keV and the average value (as quoted in Ref. 2) of $\Gamma(\eta'\to2\gamma)=4.42\pm0.34$ keV together with (29b) and $F_\pi=93$ MeV, give

$$\begin{aligned}\tilde{A}_\eta&=(1.01\pm0.08)F_\pi^{-1}\;,\\ \tilde{A}_{\eta'}&=(1.27\pm0.05)F_\pi^{-1}\;,\end{aligned} \tag{33}$$

where

$$\tilde{A}_\eta=\cos\theta\,\tilde{A}_{\eta_8}-\sin\theta\,\tilde{A}_{\eta_0}$$

and

$$\tilde{A}_{\eta'}=\sin\theta\,\tilde{A}_{\eta_8}+\cos\theta\,\tilde{A}_{\eta'}\;.$$

Using the experimental values (33), the η_8 sum rule (30b) gives the value of θ quoted in Eq. (4a). Using this value of θ in the η_0 sum rule (30c) then gives the result (4b) for F_0/F_π.

Finally if η and η' have pure gluonium component G_0 so that Eqs. (10b) are replaced by[10]

$$\begin{aligned}\eta&=\cos\theta_1\eta_8-\sin\theta_1(\cos\theta_2\eta_0-\sin\theta_2G_0)\;,\\ \eta'&=\sin\theta_1\eta_8+\cos\theta_1(\cos\theta_2\eta_0-\sin\theta_2G_0)\;,\\ G&=\sin\theta_2\eta_0+\cos\theta_2G_0\;,\end{aligned} \tag{34}$$

where G_0 has no coupling to photons and we have neglected the very small η_8 component in G. Then

$$\begin{aligned}\eta_8&=\cos\theta_1\eta+\sin\theta_1\eta'\;,\\ \eta_0&=\frac{1}{\cos\theta_2}(-\sin\theta_1\eta+\cos\theta_1\eta'+\sin\theta_2G_0)\;,\end{aligned} \tag{35}$$

so that the η_8 sum rule (30b) gives then

$$-\theta_1=(25\pm4)^\circ \tag{36a}$$

and the η_0 sum rule (30c) gives (as G_0 has no coupling to photons)

$$\frac{F_0/\cos\theta_2}{F_\pi}=0.64\pm0.07$$

or

$$\frac{F_0}{F_\pi}=(0.64\pm0.07)\cos\theta_2\;, \tag{36b}$$

so that (4b) is then upper limit for F_0/F_π in this case.

ACKNOWLEDGMENT

The authors would like to acknowledge the support of the King Abdulaziz City of Science and Technology under Grant No. AR7-162.

[1] S. L. Adler, Phys. Rev. **117**, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969).

[2] JADE Collaboration, W. Bartel *et al.*, Phys. Lett. **160B**, 421 (1985); A. Weinstein *et al.*, Phys. Rev. D **28**, 2896 (1984).

[3] J. F. Donoghue, B. R. Holstein, and Y. C. R. Lin, Phys. Rev. Lett. **55**, 2766 (1985).

[4] G. Grunberg, Phys. Lett. **168B**, 141 (1986).

[5] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[6] N. V. Krasnikow and A. A. Pivovarov, Phys. Lett. **112B**, 397 (1982); S. G. Gorishny, A. L. Katev, and S. A. Larin, *ibid.* **135B**, 457 (1984).

[7] N. Isgur, Phys. Rev. D **13**, 129 (1976); **23**, 817(E) (1981); M. Budnev and A. E. Kaloshin, Phys. Lett. **86B**, 351 (1979); J. E. Paschalis and G. J. Gounaris, Nucl. Phys. **B222**, 473 (1983). For a review, see J. H. Field, Report No. DESY 85-110, 1985 (unpublished).

[8] C. Quigg and J. L. Rosner, Phys. Rep. **56C**, 167 (1979).

[9] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[10] See, for example, M. S. Chanowitz, Phys. Rev. Lett. **44**, 59 (1980).