PHYSICAL REVIEW D VOLUME 37, NUMBER 1 1 JANUARY 1988

# $CP$ violation in $K_L \to \pi^+\pi^-\gamma$

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(Received 29 June 1987)

We study the process $K_L \to \pi^+\pi^-\gamma$, looking for new information on $CP$ violation. We find that $CP$ violation in this process is dominated by the inner-bremsstrahlung contribution which gives no information different from what is known in $K_L \to \pi\pi$. Any $CP$ violation in the direct emission appears to be too small to be observed. We also find that the direct-emission term is dominated by long-distance contributions to the electromagnetic form factor, and can be reasonably understood within a pole model.

## I. INTRODUCTION

$CP$ violation is a very difficult phenomenon to observe.[1] Within the standard model it is predicted that any $CP$-violating process is proportional to a combination of Kobayashi-Maskawa angles, $s_1^2 s_2 s_3 \sin\delta$, which is a very small number. This small number gets enhanced in some rare decays and it is therefore important to study those decays. To be able to distinguish between different models for $CP$ violation it is necessary to measure some signal other than $\epsilon$, where $\epsilon$ is the basic parameter that measures $CP$ violation coming from $K^0$-$\bar{K}^0$ mixing, and all the models of $CP$ violation are adjusted to reproduce its value. What we want to see is $CP$ violation that has an origin different from mixing. This will give us some insight into the problem of identifying the origin of the phenomenon. In this paper we study the rare decay $K_L \to \pi^+\pi^-\gamma$ in order to see if it is useful in the study of $CP$ violation.

It is conventional to divide the decay $K_L \to \pi^+\pi^-\gamma$ into an inner-bremsstrahlung (IB) and a direct-emission (DE) contribution. The IB is a $CP$-violating process that has already been observed, but yields no information different from that in $K_L \to \pi\pi$ decays. The DE term is predominantly $CP$ conserving, but it can have a $CP$-violating piece too. This term is interesting in itself and this process provides a good opportunity to study it because of the $CP$ suppression of the usually dominant IB term. A related process has been recently studied[2] by McGuigan and Sanda.

In this paper we first recalculate the IB term using the lowest-order chiral Lagrangians. We then proceed to examine the DE portion of the decay. In Sec. III we consider the $CP$-conserving contributions to this term, and in Sec. IV we study the possibility of a $CP$-violating piece in the DE.

## II. INNER BREMSSTRAHLUNG

We start by labeling the momenta of the particles involved in the reaction as follows:

$$K_L(k) \to \pi^+(p^+)\pi^-(p^-)\gamma(q,\epsilon) ,$$

where $\epsilon$ is the photon polarization. It is then convenient to classify the possible forms for the decay amplitude into three groups:

$$\frac{p^+\cdot\epsilon}{p^+\cdot q} - \frac{p^-\cdot\epsilon}{p^-\cdot q} , \tag{1a}$$

$$\epsilon_{\alpha\beta\gamma\delta} p_\alpha^+ p_\beta^- q_\gamma \epsilon_\delta , \tag{1b}$$

$$(p^+\cdot\epsilon)(p^-\cdot q) - (p^-\cdot\epsilon)(p^+\cdot q) . \tag{1c}$$

The forms (and others equivalent after kinematic substitutions) are the only possible ones up to third order in momenta that are gauge invariant. The total invariant amplitude for the process must then be a superposition of these terms multiplied by some scalar functions. Forms (1a) and (1c) correspond to electric transitions, while form (1b) corresponds to magnetic transitions. Although form (1c) is just form (1a) multiplied by a scalar it is convenient to treat them separately.

One then divides the amplitude into two parts: IB, which will be proportional to form (1a) and which comes from a photon radiated off one of the charged pions, and DE, which is proportional to form (1b) or (1c). Experimentally the IB contribution to the decay rate is extracted from the data by fitting the lower end of $d\sigma/dq$ to an amplitude falling as $1/q$ (Ref. 3).

The calculation of this term was done long ago,[4] and here we present the result for completeness. The diagrams contributing to the process are those of Fig. 1. To describe the $\Delta S = 1$ $CP$-violating interactions, we use the phenomenological Lagrangian

$$L_w = g\,\mathrm{Tr}(\lambda_7 D_\mu M\, D^\mu M^\dagger) , \tag{2}$$

where $M = \exp(i\phi_a\lambda^a/F_\pi)$, $F_\pi$ is the usual pion decay constant 93 MeV and $D_\mu M = \partial_\mu M + ieA_\mu[Q,M]$ with $Q$ given by

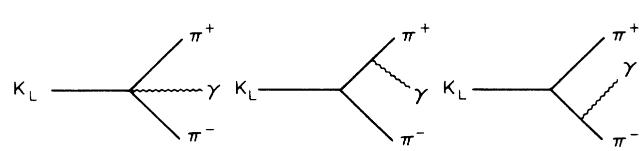


FIG. 1. Diagrams contributing to $K_L \to \pi^+\pi^-\gamma$.

 

$$Q=\begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} , \tag{3}$$

and $\phi_a$ are the usual pseudoscalar mesons.

Normalizing the overall amplitude to $A(K_S \to \pi^+\pi^-) \equiv A_0$ one obtains

$$M_{\rm IB} = -e\eta_{+-}A_0 \left[ \frac{p^+\cdot\epsilon}{p^+\cdot q} - \frac{p^-\cdot\epsilon}{p^-\cdot q} \right] , \tag{4}$$

where

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \mid H_w \mid K_L \rangle}{\langle \pi^+\pi^- \mid H_w \mid K_S \rangle} .$$

And after a numerical integration over phase space, in which we consider photons with energies larger than 20 MeV, one gets the branching ratio

$$\frac{\Gamma_{\rm IB}(K_L \to \pi^+\pi^-\gamma)}{\Gamma_{K_L}} = 1.4\times 10^{-5} , \tag{5}$$

which compares very well with the experimental number $(1.52\pm0.16)\times 10^{-5}$ (Ref. 3).

Note that if the *CP*-conserving DE term is proportional to form (1b) there is no interference with IB and we can separate the branching ratio into its IB and DE contributions. This can be seen by noting that after summing over the photon polarization there are not enough independent momenta to contract with the $\epsilon$ tensor.

It is also interesting to point out that the first diagram in Fig. 1 cancels contributions from the other diagrams that are generated by the momentum-dependent part of the $K\pi\pi$ vertex, to produce the gauge-invariant form Eq. (4).

## III. *CP*-CONSERVING DIRECT EMISSION

The lowest-order DE process is an $M1$ transition,[5] and within the chiral-Lagrangian framework it is generated by the effective Wess-Zumino term. The amplitude can be understood in terms of pole diagrams similar to those included in the treatment of $K_L \to \gamma\gamma$ (Ref. 6). The relevant nonvanishing diagrams are shown in Fig. 2.

The $\phi\phi\phi\gamma$ vertex is described by the effective Lagrangian[7]

$$L_{\phi\phi\phi\gamma} = \frac{ie}{\pi^2 F_\pi^3}\epsilon_{\mu\nu\alpha\beta}A_\mu {\rm Tr}(Q\,\partial_\nu\phi\,\partial_\alpha\phi\,\partial_\beta\phi) . \tag{6}$$

With this we can calculate the amplitude for the $\pi^0$-pole diagram $A_\pi$:

$$A_\pi = A(K_L \to \pi^0)\frac{1}{M_K^2 - M_\pi^2}\frac{e}{4\pi^2F_\pi^3}\epsilon_{\alpha\beta\gamma\delta}p_\alpha^+ p_\beta^- q_\gamma \epsilon_\delta . \tag{7}$$

To include the contributions of the $\eta$ and $\eta'$ poles we will use the usual mixing prescription:

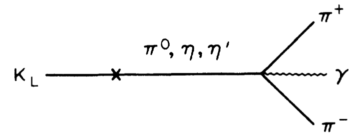


FIG. 2. Pole contributions to the direct emission.

$$\begin{aligned} |\eta\rangle &= \cos\theta\,|\eta^8\rangle - \sin\theta\,|\eta^0\rangle , \\ |\eta'\rangle &= \sin\theta\,|\eta^8\rangle + \cos\theta\,|\eta^0\rangle , \end{aligned} \tag{8}$$

where $|\eta^8\rangle$ and $|\eta^0\rangle$ represent the SU(3) octet and singlet, respectively. Because it is outside the octet there is some uncertainty in the way to include the $|\eta^0\rangle$ into our calculation. We assume that the ratios of its transition amplitudes to those of the $\pi^0$ are given by nonet-symmetry predictions. We can then calculate the total amplitude following the prescription of Ref. 6 to include SU(3) breaking. We obtain

$$\begin{aligned} \frac{\langle \eta^8 \mid L \mid K_L \rangle}{\langle \pi^0 \mid L \mid K_L \rangle} &= \sqrt{1/3}(1+\xi) , \\ \frac{\langle \eta^0 \mid L \mid K_L \rangle}{\langle \pi^0 \mid L \mid K_L \rangle} &= -2\sqrt{2/3}\rho , \\ \frac{\langle \pi^+\pi^-\gamma \mid L \mid \eta^8 \rangle}{\langle \pi^+\pi^-\gamma \mid L \mid \pi^0 \rangle} &= \sqrt{1/3}\left[\frac{F_\pi}{F_8}\right]^3 , \\ \frac{\langle \pi^+\pi^-\gamma \mid L \mid \eta^0 \rangle}{\langle \pi^+\pi^-\gamma \mid L \mid \pi^0 \rangle} &= \sqrt{2/3}\left[\frac{F_\pi}{F_0}\right]^3 , \end{aligned} \tag{9}$$

where $\xi$ measures SU(3) breaking and $\rho=1$ gives the simple quark-model prediction.[6]

With these results and $A_\pi$ given by Eq. (7),

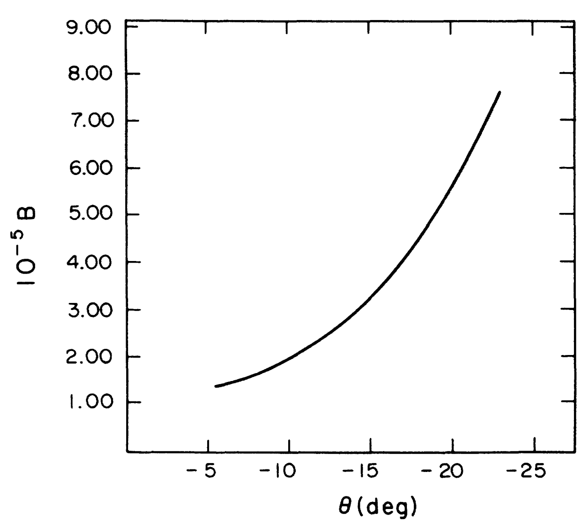


FIG. 3. Branching ratio for the direct emission as a function of the mixing angle.

$$A_{\mathrm{DE}}(K_L \to \pi^+\pi^-\gamma) = A_\pi \left\{ 1 + \frac{M_K^2 - M_\pi^2}{M_K^2 - M_\eta^2} [\sqrt{1/3}(1+\xi)\cos\theta + 2\sqrt{2/3}\sin\theta] \left[ \sqrt{1/3} \left[ \frac{F_\pi}{F_8} \right]^3 \cos\theta - \sqrt{2/3} \left[ \frac{F_\pi}{F_0} \right]^3 \sin\theta \right] \right.$$
$$\left. + \frac{M_K^2 - M_\pi^2}{M_K^2 - M_{\eta'}^2} [\sqrt{1/3}(1+\xi)\sin\theta - 2\sqrt{2/3}\cos\theta] \left[ \sqrt{1/3} \left[ \frac{F_\pi}{F_8} \right]^3 \sin\theta + \sqrt{2/3} \left[ \frac{F_\pi}{F_0} \right]^3 \cos\theta \right] \right\} . \tag{10}$$

Using the most reasonable values for $\xi$, $F_8$, and $F_0$ (case IV of Ref. 6 and with $\rho = 1$) we find that the result depends critically on the mixing angle and that there is, therefore, a large uncertainty. For the favored value of mixing angle, $\theta = -20°$, Eq. (10) gives (after a numerical integration over phase space with photon energy larger than 20 MeV) a branching ratio

$$\frac{\Gamma_{\mathrm{DE}}(K_L \to \pi^+\pi^-\gamma)}{\Gamma_{K_L}} = 5.7 \times 10^{-5} \tag{11}$$

which is about twice as large as the experimental result of[3] $(2.89 \pm 0.28) \times 10^{-5}$. Keeping all the other parameters fixed, we plot in Fig. 3 the branching ratio [Eq. (11)] as a function of the mixing angle $\theta$. We notice that one could accommodate the experimental result with values for the different parameters involved that are not unreasonable, in particular with a mixing angle of $\theta = -14°$. The conclusion is then that the DE could be explained as being mostly a *CP*-conserving process that is very similar to $K_L \to \gamma\gamma$, but there is sufficient uncertainty to incorporate other effects.

Up to now we have parametrized three sources of uncertainty: SU(3) corrections are included in the parameter $\xi$ and in the use of physical values for $F_\pi$, $F_8$, and $F_0$; the unknown $\rho$ measures the uncertainty in the inclusion of the SU(3)-singlet state; and, of course, there is the mixing angle $\theta$. We must also consider that some of the coupling constants can have an important momentum dependence.

The experimental results of Ref. 3 seem to indicate that this is in fact true. They obtain a best fit to their data with an amplitude such as that of Eq. (7) modified by a $\rho$ propagator. This suggests that one can try to extract the electromagnetic form factor involved in the DE by using the phenomenologically successful idea of vector-meson dominance for the photon couplings.

A consistent way to incorporate this idea is to use the effective chiral Lagrangian expanded to include vector mesons.[8] We expect the lowest-order DE term to originate in the anomaly as before, and the relevant Lagrangian for this is given in Ref. 9. The nonvanishing diagrams contributing to the DE are then those of Fig. 4.

The diagrams of Fig. 4(a) generate an amplitude of the form (7), while the diagrams of Fig. 4(b) will have a form factor proportional to the $\rho$ propagator in them. The vertices are given by[8]

$$L_{V\phi\phi\phi} = \frac{-ig}{4\sqrt{2}\pi^2 F_\pi^3} \left[ 1 - \frac{3}{2} \left[ \frac{g^2 F_\pi^2}{M_\rho^2} \right] + \frac{3}{8} \left[ \frac{g^2 F_\pi^2}{M_\rho^2} \right]^2 \right]$$
$$\times \epsilon^{\mu\nu\alpha\beta} \mathrm{Tr}(V_\mu \partial_\nu \phi\, \partial_\alpha \phi\, \partial_\beta \phi) ,$$

$$L_{V\phi\phi} = \frac{-3g^2}{16\pi^2\sqrt{2}F_\pi} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \rho^\nu$$
$$\times [\partial^\alpha \omega^\beta \sqrt{2}\pi^0 + \partial^\alpha \rho^\beta (\sqrt{1/6}\eta^8 + \sqrt{1/3}\eta^0)] , \tag{12}$$
$$L_{\pi^+\pi^-\rho} = \frac{ig}{\sqrt{2}} \rho_\mu (\pi^+ \partial^\mu \pi^- - \pi^- \partial^\mu \pi^+) ,$$
$$L_{\mathrm{EM}} = \frac{\sqrt{2}e}{g} A_\mu \left[ M_\rho^2 \rho^{0\mu} + \tfrac{1}{3} M_\omega^2 \omega^\mu - \frac{\sqrt{2}}{3} M_\phi^2 \phi^\mu \right] ,$$

where the coupling $g$ is identified with $\sqrt{2}F_{\rho\pi\pi}$ and $V_\mu$ is the vector-meson nonet.[8] This means that the phenomenological coupling $g$ is being obtained by fitting $\rho \to 2\pi$ and that to include the SU(3) singlet we are assuming nonet symmetry.

Again we use Eq. (8) to describe the mixing between $\eta^8$ and $\eta^0$ to obtain

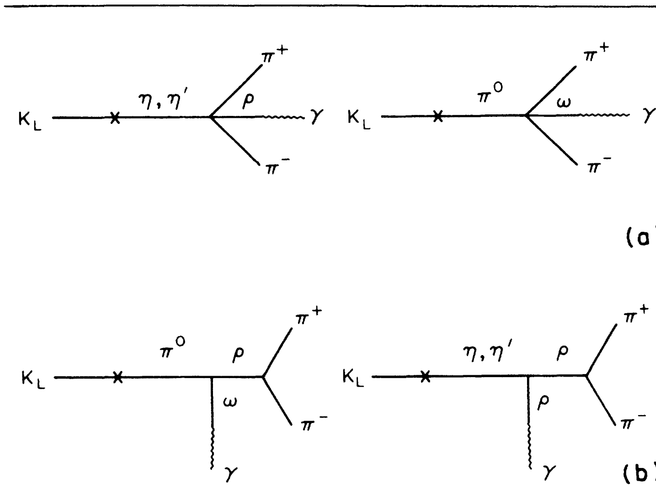


FIG. 4. Contributions to the vector-meson model of direct emission: (a) contact terms and (b) pole ($\rho$-propagator) terms.

$$A_1=-i\frac{\langle\pi^0|L|K_L\rangle}{M_K^2-M_\pi^2}\frac{0.2e}{4\pi^2F_\pi^3}\epsilon^{\mu\nu\alpha\beta}p_\mu^+p_\nu^-q_\alpha\epsilon_\beta$$
$$\times\left\{1+\frac{M_K^2-M_\pi^2}{M_K^2-M_\eta^2}[\sqrt{1/3}(1+\xi)\cos\theta+2\sqrt{2/3}\sin\theta]\left[\frac{1.2}{\sqrt{3}}\left[\frac{F_\pi}{F_8}\right]^3\cos\theta-1.3\sqrt{2/3}\left[\frac{F_\pi}{F_0}\right]^3\sin\theta\right]\right.$$
$$\left.+\frac{M_K^2-M_\pi^2}{M_K^2-M_{\eta'}^2}[\sqrt{1/3}(1+\xi)\sin\theta-2\sqrt{2/3}\cos\theta]\left[\frac{1.2}{\sqrt{3}}\left[\frac{F_\pi}{F_8}\right]^3\sin\theta+1.3\sqrt{2/3}\left[\frac{F_\pi}{F_0}\right]^3\cos\theta\right]\right\},\tag{13}$$
$$A_2=i\frac{\langle\pi^0|L|K_L\rangle}{M_K^2-M_\pi^2}\frac{3e}{\pi F_\pi}\left[\frac{1}{(M_\rho^2-M_K^2)+2M_Kq^0}\right]\epsilon^{\mu\nu\alpha\beta}p_\mu^+p_\nu^-q_\alpha\epsilon_\beta$$
$$\times\left\{1+\frac{M_K^2-M_\pi^2}{M_K^2-M_\eta^2}[\sqrt{1/3}(1+\xi)\cos\theta+2\sqrt{2/3}\sin\theta]\left[\sqrt{3}\left[\frac{F_\pi}{F_8}\right]\cos\theta-\sqrt{6}\left[\frac{F_\pi}{F_0}\right]\sin\theta\right]\right.$$
$$\left.+\frac{M_K^2-M_\pi^2}{M_K^2-M_{\eta'}^2}[\sqrt{1/3}(1+\xi)\sin\theta-2\sqrt{2/3}\cos\theta]\left[\sqrt{3}\left[\frac{F_\pi}{F_8}\right]\sin\theta+\sqrt{6}\left[\frac{F_\pi}{F_0}\right]\cos\theta\right]\right\},$$

where $A_1$ comes from the diagrams of Fig. 4(a) and $A_2$ from the ones of Fig. 4(b); and $\rho$ has been set equal to 1.

The two amplitudes $A_1$ and $A_2$ interfere in a different way depending on the mixing angle, but $A_2$ is, in general, larger than $A_1$. Within the context of the model we take this to mean that the momentum dependence of the vertex is indeed important. We expect the form extracted as a $\rho$ propagator to be the major effect and, thus, the calculated branching ratio should be less sensitive to uncertainties than, Eq. (11).

We calculate the branching ratio by numerically performing the integration over phase space to obtain the results shown in Fig. 5. It is interesting to see that we can now fit the experimental result with a mixing angle of $-20.5°$, which is very close to the presently favored value of $\theta$ (Ref. 11). We find this encouraging in view of the strong dependence of the branching ratio on the mixing angle. We cannot be too optimistic though, because of the sensitivity to sources of uncertainty already mentioned.

If we plot, as in Fig. 6, the ratio of $d\Gamma/dq$ obtained in this way to $d\Gamma/dq$ obtained from the simple analysis

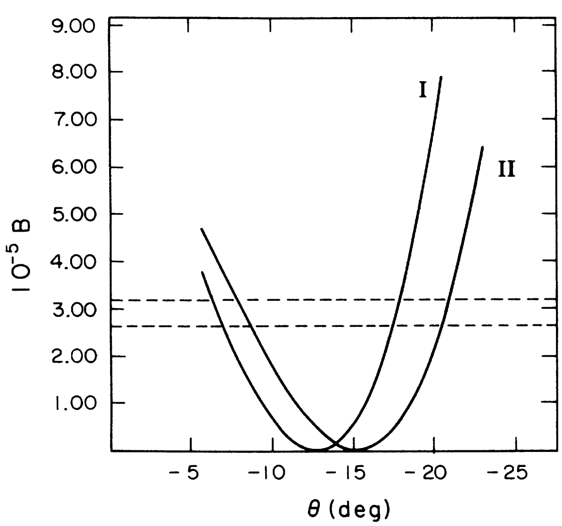


FIG. 5. DE branching ratio as a function of the mixing angle in the vector-meson model. Curve I represents the limit of no SU(3) breaking, and curve II uses the values that Ref. 6 found most reasonable. The two straight, dotted lines represent the experimental results of Ref. 4.

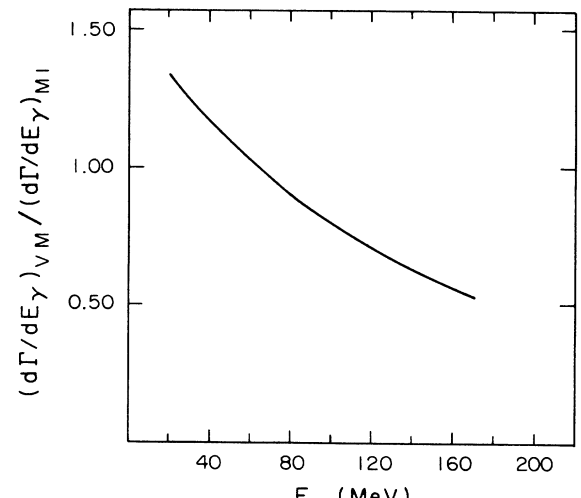


FIG. 6. Ratio of the vector-meson-model prediction for $d\Gamma/dE_\gamma$ to the simple model with no form factors. Both are evaluated at the angle where they give a best fit to the experimental number.

leading to Eq. (11) we roughly see that the two differ by a form factor falling as the square of the $\rho$ propagator. This is what experimentalists found in Ref. 3. We also find that the two never differ by more than a factor of 2 in the allowed region.

## IV. *CP*-VIOLATING DIRECT EMISSION

We now turn our attention to the question of whether or not there can be substantial *CP* violation beyond the IB term. So far we have seen that the decay is well described by a *CP*-violating $E1$ IB term and a *CP*-conserving $M1$ DE term that do not interfere. Let us explore the different ways of seeing any additional *CP* violation.

To generate a charge asymmetry we would need a *CP*-violating $M2$ term or a *CP*-conserving $E2$ term. As has been noted by many authors[9] these higher-order multipoles are expected to be small. This is confirmed by the experimental results[3] which find no evidence for an $E2$ term. Therefore, we do not expect any detectable charge asymmetry.

It has been pointed out, however,[10] that an $E1$ contribution to the DE would interfere with IB, and the experimental results[3] are consistent with the presence of such a term at the 13% level.

From the phenomenological point of view that we are using, chiral Lagrangians with more than two derivatives can produce a term like this. However, we expect a term with four derivatives to be suppressed with respect to the lowest-order Lagrangian by a factor of $(M_K^2/\Lambda^2)$, where $\Lambda$ is the chiral-symmetry scale parameter $\sim 1$ GeV. One also expects this term to give a contribution to a *CP*-violating signal enhanced with respect to $\epsilon'$ by a factor of about 22 (Ref. 1) because of the removal of the $\Delta I = \frac{3}{2}$ suppression. In a handwaving sense, it is therefore conceivable to have a contribution at the percent level.

Unfortunately this is not the case. If we compare the $E1$ and $M1$ contributions calculated in Secs. II and III we can see that they are of the same order. Moreover, there is no region of phase space for which the $M1$ amplitude is much larger than the $E1$ amplitude. This appears at first surprising, because the $E1$ is a *CP*-violating amplitude. However, its contribution is so important because of the enhancement produced by the $1/q$ dependence of an IB amplitude.

A DE $E1$, however, is not enhanced by the $1/q$ dependence. And it is still *CP* violating. Even if the DE *CP*-violating amplitude were as large as $\epsilon$, its effect on the branching ratio would be smaller by a factor of about $10^{-3}$ than the corresponding IB.

We can confirm this by finding one of the possible Lagrangians with four derivatives that contributes to $K_L \to \pi^+\pi^-\gamma$:

$$L_{\rm HO} = \frac{g_1}{\Lambda^2}\mathrm{Tr}[\lambda_7(M\,D_\mu D_\nu M^\dagger)D^\mu M\,D^\nu M] + \mathrm{H.c.}\ , \quad (14)$$

where HO stands for higher order. The Langrangian contributes both to a three-meson vertex and to a three-meson–one-photon vertex. We can calculate the tree diagrams of Fig. 1 to obtain

$$A_{\rm HO} = \frac{2g_1 e}{\Lambda^2 F_\pi^3} M_K^2(M_K^2 - M_\pi^2)\left[\frac{p^+\cdot\epsilon}{p^+\cdot q} - \frac{p^-\cdot\epsilon}{p^-\cdot q}\right] + \frac{8g_1 e}{\Lambda^2 F_\pi^3}(p^+\cdot\epsilon\, p^-\cdot q - p^+\cdot\epsilon\, p^-\cdot q)\ . \quad (15)$$

Its contribution to the IB amplitude forces us to redefine $g$ in terms of $A_0$:

$$\eta_{+-}A_0 = \frac{2(M_K^2 - M_\pi^2)}{F_\pi^3} g\left[1 - \frac{g_1 M_K^2}{g\Lambda^2}\right]\ , \quad (16)$$

which if we take $g \approx g_1$ increases the value of $g$ by about 30%. In the presence of this term, the branching ratio obtains an additional contribution

$$B_{\rm int} \simeq 8.2\times 10^{-8}\ , \quad (17)$$

which is far too small to be observed.

Finally let us consider the possibility of observing an interference between $K_L \to \pi^+\pi^-\gamma$ and $K_S \to \pi^+\pi^-\gamma$ (Ref. 9). It is natural to define

$$\frac{A(K_L \to \pi^+\pi^-\gamma)_{CP}}{A(K_S \to \pi^+\pi^-\gamma)} = \epsilon + \epsilon'_{\pi\pi\gamma} = \eta_{+-\gamma}\ . \quad (18)$$

However, $A(K_L \to \pi^+\pi^-\gamma)_{CP}$ is the sum of IB and, to next order, a possible $E1$ DE term. The IB contributes an uninteresting term $\eta_{+-}$ to $\eta_{+-\gamma}$. The DE depends on the region of phase space we are looking at. We could try to maximize the effect by looking at the region of the Dalitz plot where the photon has its highest energy. At that point, using $\Lambda \approx 2M_K$,

$$\frac{A(\mathrm{DE}, E1)}{A(K_S \to \pi^+\pi^-\gamma)} \approx 0.02\eta_{+-}\left[\frac{g_1}{g}\right]\ . \quad (19)$$

We can again see, with $g_1/g \approx 1$, that the largest contribution we can expect from a DE *CP* violation to $\epsilon'_{\pi\pi\gamma}$ is $0.02\epsilon$, which is not far from the most optimistic estimates, for $\epsilon'$ (Ref. 1).

In view of this result we must conclude then that this decay is not likely to give any new information on *CP* violation. On the other hand, we feel that it suggests evidence for the presence of an electromagnetic form factor in the DE process, which can be well explained within the vector-meson-dominance assumption.

## ACKNOWLEDGMENTS

We want to thank J. F. Donoghue and B. R. Holstein for useful conversations and comments, and for suggesting this problem.

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