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Motion of the Earth and the detection of weakly interacting massive particles

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If the galactic halo is composed of weakly interacting massive particles (WIMP's), then cryogenic experiments may be capable of detecting the recoil of nuclei struck by the WIMP's. Earth's motion relative to the galactic halo produces a seasonal modulation in the expected event rate. The direction of nuclear recoil has a strong angular dependence that also can be used to confirm the detection of WIMP's. I calculate the angular dependence and the amplitude of the seasonal modulation for an isothermal halo model.

One of the outstanding problems in astrophysics is understanding what comprises the galactic halo (see Ref. 1 for review). Several extensions of the standard model provide particle physics with "missing-mass" candidates.[2] Many of these candidates (photinos, Higgsinos, massive Dirac neutrinos, cosmions) are lumped together as weakly interacting massive particles (WIMP's), since they are expected to have masses of several GeV and have weak (or nearly weak) interactions with baryons. Goodman and Witten[3] and Wasserman[4] realized that cryogenic detectors could potentially detect the nuclear recoil due to WIMP scattering. In the last few years, several groups have proposed experiments that utilize the low heat capacity of cryogenic materials to detect WIMP's (Ref. 5). These proposed experiments measure either the number of events above a fixed threshold or the energy of individual events; some of the proposed experiments will also measure the direction of nuclear recoil. The major experimental problem for all of these detector schemes is the need to discriminate between signal and noise.

Drukier, Freese, and Spergel[6] realized that Earth's motion relative to the galactic halo provides a distinctive signal for WIMP detection. Since WIMP's are nondissipative, the galactic halo is not rapidly rotating, but rather is supported by the random velocity of the particles. The Sun moves on a nearly circular orbit around the galactic center with velocity $v_{\odot} \approx 220 \pm 20$ km/s. This motion relative to the rest frame of the WIMP's results in a strong angular dependence in the event rate. Earth moves around the Sun with a velocity of 30 km/s in an orbit inclined 60° relative to the disc of the galaxy. Earth's motion produces a seasonal variation in the laboratory's velocity relative to the halo; this motion provides a distinctive temporal modulate of the observed event rate. In this paper, I will calculate the amplitude of the modulation and show how this can be used to provide a unique signature of galactic WIMP's in cryogenic detectors.

Consider a WIMP of mass m_x moving with velocity

$$\mathbf{v} = \cos\alpha\hat{\mathbf{x}} + \sin\alpha \sin\beta\hat{\mathbf{y}} + \sin\alpha \cos\beta\hat{\mathbf{z}}$$

in the laboratory frame. This WIMP scatters off of a nucleus of mass m_n which recoils with energy

TABLE I. This table shows the dependence of the amplitude of the seasonal modulation and the directional asymmetry on the detector threshold. The first column lists the minimum velocity of particle detectable by the experiment. The second column lists the fraction of the incident flux detected as a function of threshold velocity. The third column lists the ratio of events in the forward direction to events in the backward direction above the threshold. The ratio of the event rate above the threshold in July to the event rate above the threshold in January is shown in column 4.

$v_{\rm th}/v_{\rm halo}$	Fraction of incident flux detected	Forward/back	July/January
0.00	1.00	4.00	1.04
0.20	0.97	4.17	1.04
0.40	0.90	4.66	1.05
0.60	0.78	5.44	1.07
0.80	0.65	6.56	1.08
1.00	0.50	8.10	1.11
1.20	0.37	10.18	1.13
1.40	0.25	12.98	1.16
1.60	0.16	16.73	1.20
1.80	0.10	21.77	1.24
2.00	0.06	28.54	1.28

$$E_n = \frac{m_n m_x^2}{(m_n+m_x)^2} v^2(1-\mu) , \qquad (1)$$

where $\mu = \cos\theta$ and θ is the WIMP scattering angle in the center-of-mass frame. The velocity $\mathbf{u}$ of the nuclear recoil makes an angle γ relative to the x axis:

$$\cos\gamma = \frac{\hat{\mathbf{x}}\cdot\mathbf{u}}{|\mathbf{u}|} = \left[\frac{1-\mu}{2}\right]^{1/2}\cos\alpha - \left[\frac{1+\mu}{2}\right]^{1/2}\cos\xi\,\sin\alpha , \qquad (2)$$

where $\cos\xi = \mathbf{u}\cdot(\hat{\mathbf{x}}\times\mathbf{v})/|\mathbf{u}\cdot(\hat{\mathbf{x}}\times\mathbf{v})|$.

The WIMP distribution function $f(v,\alpha,\beta)$ determines the event rate per nucleon in the detector:

$$dR = f(v,\alpha,\beta) v^3 dv\, d\cos\alpha\, d\beta \frac{d\sigma}{d\mu} d\mu \frac{d\xi}{2\pi} , \qquad (3)$$

where $0<v<\infty$, $-1<\cos\alpha<1$, $0<\beta<2\pi$, $-1<\mu<1$, and $0<\xi<2\pi$. I will assume that the nucleon-WIMP scattering is isotropic: $d\sigma/d\mu = \sigma_0/2$, in this paper.

The rate of events in which the nucleus recoils with energy E into a cone of opening angle γ can be found from Eqs. (1)–(3):

$$\frac{dR}{dE\, d\cos\gamma} = \frac{\sigma_0}{4\pi}\int_{v_{\min}}^{\infty} v^3 dv \int_{-1}^{1} d\cos\alpha \int_0^{2\pi} d\beta\, f(v,\alpha,\beta) \left|\frac{\partial(E,\cos\gamma)}{\partial(\mu,\xi)}\right|^{-1} , \qquad (4)$$

where $v_{\min}^2 = (m_x+m_n)^2 E/2m_x^2 m_n$ is the minimum WIMP velocity that can produce a nuclear recoil of energy E. A change of variables

$$\cos\theta = [\cos\alpha - \cos\gamma\sqrt{(1-\mu)/2}]/[\sin\gamma\sqrt{(1+\mu)/2}] ,$$

simplifies this expression:

$$\frac{dR}{dE\, d\cos\gamma} = \frac{(m_x+m_n)^2}{2\pi m_n m_x^2}\sigma_0 \int_{v_{\min}}^{\infty} v\, dv \int_0^{\pi} d\theta \int_0^{2\pi} d\beta\, f[v,\alpha(\gamma,\theta,\mu),\beta] . \qquad (5)$$

I will assume that WIMP's in the halo have an isotropic velocity distribution function

$$f(v) = (\rho_0/m_x \pi^{3/2} v_{\text{halo}}^3)\exp(-v^2/v_{\text{halo}}^2) ,$$

where ρ_0 is the local WIMP density. A more general distribution function will be considered in a subsequent paper.[7] If the halo density distribution is $\propto r^{-2}$, then $v_{\text{halo}} = \sqrt{3/2}\,v_\odot$. Viewed in the laboratory frame,

$$f(\mathbf{v}+\mathbf{v}_\odot+\mathbf{v}_E) = \frac{\rho_0}{\pi^{3/2} m_x v_{\text{halo}}^3} \times \exp[-(\mathbf{v}+\mathbf{v}_E+\mathbf{v}_\odot)^2/v_{\text{halo}}^2] , \qquad (6)$$

v_E is the component of Earth's motion parallel to $v_\odot$.

Combining Eqs. (5) and (6),

$$\frac{dR}{dE\, d\cos\gamma} = \frac{\rho_0\sigma_0}{\sqrt{\pi}} \frac{(m_x+m_n)^2}{2m_x^3 m_n v_{\text{halo}}} \times \exp\left[\frac{-[(v_E+v_\odot)\cos\gamma - v_{\min}]^2}{v_{\text{halo}}^2}\right] . \qquad (7)$$

Equation (7) shows that the nuclear recoil direction has a very strong angular dependence. The number of events in the forward direction will significantly exceed the number of events in the backward direction for any energy threshold E_{th}. This strong effect suggests that even weak angular resolution would be a powerful tool that could discriminate between the dark-matter signal and the background.

Earth's motion around the Sun produces a seasonal modulation in the total number of events expected above the energy threshold of the detector.[6] During July, there is a component of Earth motion around the Sun parallel to the Sun's motion around the galactic center. This adds 15 km/s to the motion of the laboratory relative to the nonrotating galactic halo. During January, this component is antiparallel and reduces Earth's velocity relative to the halo. Integrating (7) over all angles yields the ratio of the July event rate to the January event rate: Table I lists the amplitude of the seasonal modulation effect and the asymmetry in recoil direction as a function of threshold velocity v_{th}. The second column shows the fraction of incident flux detected at the threshold value. The seasonal modulation effect is $O(v_E/v_{\text{halo}}) \approx 0.1$, while the angular dependence effect is $O(v_\odot/v_{\text{halo}}) \approx 1$.

In a real detector, the predicted rates would have to be convoluted with the quantum detector efficiency of the experiment and the measurement uncertainties. This will reduce the amplitude of the effects. However, these effects should be observable in the laboratory and can aid in the search for the "missing mass."

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