PHYSICAL REVIEW D VOLUME 37, NUMBER 5 1 MARCH 1988

Electromagnetic width of the σ meson from the σ model

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(Received 7 October 1987)

We calculate the decay width of the σ (or ϵ) scalar meson into two photons, in the framework of the linear σ model. We are motivated in part by a recent experimental determination from photon-photon annihilation into pion pairs, which suggested a surprisingly large value. We argue that the model takes into account a plausible hybrid structure of the scalar mesons, as having fermion-antifermion as well as boson pair constituents. We show that the corresponding two contributions tend to cancel, leading, on the contrary, to a rather small width.

Photon-photon experiments have provided a large number of results on the electromagnetic widths of mesons in recent years. Some, like those concerning scalar mesons,[1–3] were not known before. A surprising result for the scalar-isoscalar resonance ϵ is presented in Ref. 3: the width is found to lie between 4 and 16 keV, which is an order of magnitude larger than for other mesons in the same mass range. In this Brief Report, we propose to use the linear σ model[4] to estimate this width.

From the theoretical point of view, the scalar mesons are not easy to understand,[5] their electromagnetic properties are no exception. For the ϵ, say, one can find numbers in the literature ranging from 0.1 (Ref. 6) to 25 (Ref. 7). Our calculation seems to favor rather small values, $\gamma \leq 1$ keV, thereby disagreeing with Ref. 3. This would be more in line, on the other hand, with what is found experimentally for other scalar mesons.[1,2]

Before we proceed, the question to ask is to what extent one can identify the meson occurring in the σ model with the resonance listed by the Particle Data Group, $\epsilon(1300)$ (nowadays called f_0). From the Lagrangian (2), one cannot at first sight reconciliate both the mass and the width of this particle. Indeed, one finds, for the total width,

$$\Gamma_{\sigma\to\pi\pi}=\frac{3}{32\pi f_\pi^2}\frac{(m_\sigma^2-m_\pi^2)^2}{m_\sigma^2}(m_\sigma^2-4m_\pi^2)^{1/2} . \qquad (1)$$

Clearly, Γ increases very rapidly as a function of m_σ; setting $\Gamma=600$ MeV (at the upper bound of the experimental determination), we get $m_\sigma \simeq 611$ MeV, which is down by a factor of 2 compared to m_ϵ. [This can be somewhat improved in the framework of more general effective Lagrangians. If, for instance, the spin-1 mesons ρ and A_1 are introduced, the pion couplings get renormalized and the σ mass (for a given width) is pushed up by about 200 MeV.] Yet, with such a small mass, the σ meson has proved very useful in several instances in the past, as a means to mimic the $\pi\pi$ attraction in the channel $J=0$, $I=0$. It plays a role, for instance, in models of the nucleon-nucleon interaction,[8] in the current-algebra "puzzle" $\eta'\to\eta\pi\pi$ (Ref. 9), and so on. In this sense, one may believe that the σ is also involved in the reaction $\gamma\gamma\to\pi\pi$, which, at threshold, cannot be understood from the Born approximation alone.[10] (For an alternative way to treat the final-state interaction there, see Ref. 11.) This is in fact the point of view adopted in the work of Mennessier on this reaction,[12] where $m_\sigma \simeq 650$ MeV. This work, used in conjunction with the data of the DM2 group, has led to the large width quoted above from Ref. 3. The main virtue of the σ model rests with its ability to describe $\pi\pi$ scattering correctly, at low energies using the tree-level amplitudes,[13] but also, to a reasonably good extent, at medium energies.[14] Concerning its fermionic content, one may recall that the σ-model evaluation of $\pi^0\to\gamma\gamma$, from a one-nucleon loop, is very accurate.[15] This is of course because it is nonperturbative[16] and coincides with the QCD result, which would be exact at vanishing pion mass. No such theorem holds for the σ. One may nevertheless be curious to know what the result is in this case.

The σ model involves a pair of nucleons, a triplet of pions, and a scalar meson; it relies on the Lagrangian (in the spontaneously broken phase)

$$\begin{aligned}\mathcal{L}=\;&\bar\psi[i\partial\!\!\!/-m_N+g(\sigma+i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5)]\psi\\&+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}+\tfrac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma-\tfrac{1}{2}m_\sigma^2\sigma^2-\tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2\\&-\lambda v\sigma(\sigma^2+\boldsymbol{\pi}^2)-\frac{\lambda}{4}(\sigma^2-\boldsymbol{\pi}^2)^2 ,\end{aligned} \qquad (2)$$

where $v=-f_\pi\simeq -93.3$ MeV, $\lambda=(m_\sigma^2-m_\pi^2)/2f_\pi^2$, and $g=m_N/f_\pi$. Being neutral, the σ does not couple to the photon via minimal coupling. To lowest order, the diagrams for $\sigma\to\gamma\gamma$ involve a proton loop and a charged-pion loop. The amplitude thereby separates into two contributions:

$$T_N^{\mu\nu}=ge^2\left[\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{tr}[(\not p+\not k+m_N)\gamma^\mu(\not p+\not k-\not k_1+m_B)\gamma^\nu(\not k+m_N)]}{[(p+k)^2-m_N^2][(p+k-k_1)^2-m_N^2](k^2-m_N^2)}+(\mathrm{crossed})\right],$$

$$T_\pi^{\mu\nu}=2\lambda ve^2\left[\int\frac{d^4k}{(2\pi)^4}\frac{(2p+2k-k_1)^\mu(p+2k-k_1)^\nu}{[(p+k)^2-m_\pi^2][(p+k-k_1)^2-m_\pi^2](k^2-m_\pi^2)}+(\mathrm{crossed})\right.$$
$$\left.-2\int\frac{d^4k}{(2\pi)^4}\frac{g^{\mu\nu}}{[(p+k)^2-m_\pi^2](k^2-m_\pi^2)}\right]. \tag{3}$$

The calculation is most easily performed using the rules of d-dimensional integration.[17] One then realizes that the apparent divergences cancel exactly. (This may have been expected since the model is renormalizable and there is no term in the Lagrangian of the same form as would be the counterterm. More surprisingly, we found that the same phenomenon occurs also in the case of derivative coupling.) This leaves the following finite result:

$$iT^{\mu\nu}=\left[g^{\mu\nu}-\frac{k_1^\nu k_2^\mu}{k_1\cdot k_2}\right](t_N+t_\pi), \tag{4a}$$

$$t_N=\frac{2gm_N\alpha}{\pi}\left[1-\frac{1}{2}\left[1-\frac{4m_N^2}{m_\sigma^2}\right]\int_0^1\frac{dx}{x}\ln\left[1-\frac{m_\sigma^2}{m_N^2}x(1-x)\right]\right], \tag{4b}$$

$$t_\pi=\frac{-\lambda f_\pi\alpha}{\pi}\left[1+\frac{2m_\pi^2}{m_\sigma^2}\left[\int_0^1\frac{dx}{x}\ln\left|1-\frac{m_\sigma^2}{m_\pi^2}x(1-x)\right|+i\pi\ln\frac{1+\sqrt{1-4m_\pi^2/m_\sigma^2}}{1-\sqrt{1-4m_\pi^2/m_\sigma^2}}\right]\right]. \tag{4c}$$

The correct formula for t_N can be found in the 1949 paper by Steinberger[18] and t_π also has appeared in the literature a long time ago[19] (the main novelty in the present paper is to put the two contributions together). Note that the use of the Lagrangian (2) has determined the magnitude of the couplings and has also fixed the relative phase of the amplitudes uniquely. t_π has been used to estimate the σ electromagnetic width by Eliezer.[7] However, a misprint has propagated in the expression for the integrand in the last line of (4): the ratio $z=m_\sigma^2/m_\pi^2$ is replaced in both these references by its reverse. As a consequence, the following estimate for the integral (call it I) is used in Ref. 7:

$$I\simeq-\frac{1}{z}\int_0^1 dx\,(1-x), \tag{5}$$

which is not correct. Instead, a small calculation shows that the following expansion holds:

$$I=\tfrac{1}{2}(\ln z)^2+C+\frac{2}{z}+\cdots, \tag{6}$$

where $C\simeq-5$. Because of this, and some other discrepancies in numerical factors, we do not corroborate the results of Ref. 7. Numerically, some care is needed in the evaluation of I because of the two logarithmic singularities of the integrand. As an example, taking $m_\sigma=650$ MeV we find $t_N=3.06$ and $t_\pi=-3.77-i4.03$ (MeV). It is interesting to note that the nucleon contribution is quite smaller than expected. This is because the two terms in the parentheses multiplying the coupling constant in t_N are of opposite sign, such that their sum is close to $m_\sigma^2/4m_N^2$ instead of being ~ 1. As a result, there is a significant amount of cancellation between the nucleon and the pion loop amplitudes. The electromagnetic width, finally, is given by

$$\gamma_{\sigma\to\gamma\gamma}=\frac{1}{16\pi m_\sigma}|t_N+t_\pi|^2, \tag{7}$$

and some typical results are

$$\begin{aligned} m_\sigma&=600, & \gamma&=0.56,\\ m_\sigma&=650, & \gamma&=0.62,\\ m_\sigma&=700, & \gamma&=0.68,\\ m_\sigma&=750, & \gamma&=0.74, \end{aligned} \tag{8}$$

where masses are in MeV and widths in keV. Clearly, we find rather small numbers. Invoking unitarity, we can argue that the imaginary part of $T^{\mu\nu}$ in (4) should be reliably close to the exact one. Indeed, it is fair to assume that the 2π intermediate state dominates the unitarity relation:

$$2\,\mathrm{Im}\langle\gamma\gamma|\mathcal{T}|\sigma\rangle\simeq\langle\gamma\gamma|\mathcal{T}|\pi\pi\rangle\langle\sigma|\mathcal{T}|\pi\pi\rangle^*$$

On the right-hand side, only the magnitudes are relevant. The first term then would correspond in our model to the Born approximation for $\gamma\gamma\to\pi\pi$, which is correct up to, perhaps, a factor of $\sqrt{2}$. The second term is simply related to the σ width and is also roughly correct. The numbers given above can therefore be considered as reasonable lower bounds, independent of the model assumed. We note here that the results of Ref. 20 on scalar gluonium disagree with this argument. By comparison, a recent quark model estimate by Barnes, Dooley, and Isgur[11] yields $\gamma=2.2$ keV (the mass being 750), while a former calculation (also in the quark model) by Budnev and Kaloshin[21] resulted in $\gamma=8$ keV.

Perhaps one would prefer to see a quark instead of a nucleon loop occurring in the photonic annihilation. This can easily be implemented in a similar framework, starting from a "quark-sigma model."[22] In the formula for t_N [Eq. (4b)] one would only have to change m_N into a constituent-quark mass m_q and multiply the amplitude by a color factor $\frac{5}{9}N_c$. This modification enhances the fermionic contribution and the resulting widths are roughly a factor of 2 larger than those presented above. For example, if $m_\sigma=750$, now $\gamma=1.67$. This approach is close in spirit to the Törnqvist model of the scalar mesons,[5] where they are considered as mixtures of $q\bar{q}$ and $2q2\bar{q}$ states in the form of meson pairs. It is not possible, however, to follow exactly this model, because of the presence of phenomenological form factors which would jeopardize gauge invariance here. If the width is indeed small, then the σ model cannot explain the enhancement in the $\gamma\gamma\rightarrow\pi^+\pi^-$ cross section. The consistency of these data with current algebra has been questioned.[23] In fact, a recent calculation using chiral perturbation theory to one loop, also leads to a rather small improvement over the Born approximation result.[24]

B.M. is grateful to T. N. Truong for suggesting the unitarity argument following (8). Division de Physique Théorique is Laboratoire Associé au CNRS.

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