Model for light-cone quark confinement dynamics

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In the framework of the light-cone formalism it is shown that the recent relativistic model, based on nonstatic spin-wave functions that are approximate eigenstates of $\mathbf{J}^2$, can be derived from a scalar separable confinement interaction, both for the π and ρ mesons and the N and Δ baryons.

Since the 1960s the nonrelativistic quark model (NQM) has been successful in describing hadrons as bound states of constituent quarks with an effective mass $m_q=m_u=m_d\simeq m_N/3$ moving independently in a scalar, e.g., harmonic-oscillator, confinement potential.[1] In the 1970s the model was improved by including a color hyperfine (chf) interaction motivated by the short-range one-gluon exchange of perturbative QCD generating appropriate hadron mass splittings.[2]

For several years now relativistic constituent-quark models (RCQM) formulated in the light-cone Fock approach are being developed, both for mesons[3] and baryons,[4] whose momentum distributions are relativistic generalizations of the usual harmonic-oscillator model. Recently the approach has been refined to meet the requirement that such light-cone wave functions be, at least approximately, eigenstates of $\mathbf{J}^2$ (called the Melosh case hereafter[5]).

In this work we use quark confinement dynamics in terms of a scalar separable interaction[6] in the equation of motion for the meson and baryon to determine their momentum wave functions and the composition of their spin-wave functions. We find that the dynamics allows for wave functions of the Melosh type.

We start with the light-meson case and assume that the π and ρ states are dominated by the valence-quark–antiquark configuration. A consistent framework for a description of intrinsically relativistic light-quark motion is provided by the light-cone formalism.[7] With this valence-quark dominance assumption any meson state with momentum

$$P^\mu=(P^+,P^-,\mathbf{P}_\perp)=(P^0+P^3,(m_H^2+\mathbf{P}_\perp^2)/P^+,\mathbf{P}_\perp)$$

is described by the light-cone wave function $\psi(x,\mathbf{k}_\perp,\lambda)$, the probability amplitude for finding its constituents with helicity λ_i and momentum $p_i^+=x_iP^+$, $\mathbf{p}_{\perp i}=x_i\cdot\mathbf{P}_\perp+\mathbf{k}_{\perp i}$, so that $\sum x_i=1$ and $\sum \mathbf{k}_{\perp i}=0$. It is invariant under all kinematical Lorentz transformations that contain the Lorentz boost along the three-direction.

In general, the ground states of the π and ρ mesons, ψ, may be expanded,

$$\psi^H(x,\mathbf{k}_\perp,\lambda)=\sum_{n=1}^{2}\phi_n^H(x,\mathbf{k}_\perp)I_n^H(x,\mathbf{k}_\perp,\lambda)\ , \tag{1}$$

in terms of the Lorentz-invariant spinor amplitudes

$$I_1^\pi=m_\pi\bar{u}_1\gamma_5 v_2,\quad I_2^\pi=\bar{u}_1\hat{P}\gamma_5 v_2\ , \tag{2a}$$

$$I_1^\rho=m_\rho\bar{u}_1\hat{\epsilon}v_2,\quad I_2^\rho=\bar{u}_1\hat{P}\hat{\epsilon}v_2\ , \tag{2b}$$

where we denote $\hat{P}=P_\mu\gamma^\mu$, etc., while u_λ, v_λ are the light-cone spinors of quarks and antiquarks, and ϵ_μ is the polarization vector of the ρ meson.[8] The momentum distributions ϕ_n in (1) can be obtained from the equation of motion[9]

$$(m_H^2-M^2)\psi(x,\mathbf{k}_\perp)=\int[dx'd^2k'_\perp]V\psi(x',\mathbf{k}'_\perp)\ , \tag{3}$$

where m_H^2 is the eigenvalue and

$$M^2=\sum_{i=1}^{2}(\mathbf{k}_{\perp i}^2+m_q^2)/x_i \tag{4}$$

is the invariant $q\bar{q}$ mass squared.

In spite of the common belief that quantum chromodynamics is the correct theory of strong interactions we are far from knowing the structure of the quark-quark forces. The usual assumption has been to consider a Lorentz-*scalar* confining potential together with a short-range Fermi-Breit interaction playing the crucial role of mass splittings.[2] While a vector interaction preserves chiral invariance, it is known not to confine quarks properly.[10] Thus, we will consider here mainly the scalar $q\bar{q}$ interaction

$$U_S=-(\bar{u}_1u'_1)v_s(\bar{v}'_2v_2) \tag{5}$$

and comment on the vector $q\bar{q}$ interaction

$$U_V=(\bar{u}_1\gamma_\mu u'_1)v_v(\bar{v}'_2\gamma^\mu v_2) \tag{6}$$

only briefly.

Following Ref. 6 we make a separable ansatz

$$v=U^{(0)}f(M^2)f(M'^2)\ . \tag{7}$$

As a consequence, Eq. (3) admits solutions ψ with momentum distributions

$$\phi_n^H(M^2)=c_n^Hf(M^2)/(m_H^2-M^2)\ . \tag{8}$$

For the scalar potential, the coefficients c_n^H obey the linear equations

$$c_1^H = U_S^{(0)}(R_H^{(2)}c_1^H + R_H^{(0)}c_2^H) ,$$
$$c_2^H = U_S^{(0)}(R_H^{(0)}c_1^H + R_H^{(2)}c_2^H) , \qquad (9)$$

with

$$\left.\begin{matrix} R_H^{(2)} \\ R_H^{(0)} \end{matrix}\right\} = \int \frac{dx\, d^2k_\perp f^2(M^2)}{16\pi^3 x(1-x)(m_H^2 - M^2)} \times \begin{cases} x(1-x)m_H^2 + m_q^2 \\ (m_q m_H) \end{cases} . \qquad (10)$$

Equation (9) follows from (3) when the terms $U_S I'_m$ that occur in (3) are expanded in terms of the independent invariants I_n. For (10) to hold, the relative momentum q' of the p'_i in the spin sums

$$\sum_\lambda u'\bar{u}' = \hat{p}' + m_q, \quad \sum_\lambda v'\bar{v}' = \hat{p}' - m_q$$

is neglected, i.e., $p'_i \to x'_i P_H$. A typical term $U_S I'_1$ for the pion involves

$$\sum \bar{u}_1 u'_1 \bar{v}_2 v'_2 I'^{\pi}_1 = m_\pi \bar{u}_1(\hat{p}'_1 + m_q)\gamma_5(\hat{p}'_2 - m_q)v_2$$
$$= -m_\pi \bar{u}_1(x'_1\hat{P} + m_q)(m_q + x'_2\hat{P})\gamma_5 v_2$$
$$= -(x'_1 x'_2 m_\pi^2 + m_q^2)I_1^\pi - m_q m_\pi I_2^\pi .$$

Since Eqs. (9) for the pion and ρ meson are identical for the scalar interaction (5), as expected, there is no mass splitting, i.e., $m_\pi = m_\rho$. The resulting solution $x_\rho = x_\pi \equiv x$ is given by

$$x = \frac{c_1}{c_2} = \frac{U_S^{(0)}R^{(0)}}{1 - U_S^{(0)}R^{(2)}} = \frac{1 - U_S^{(0)}R^{(2)}}{U_S^{(0)}R^{(0)}} = 1 \qquad (11)$$

in conjunction with the determinantal constraint $U_S^{(0)} = (R^{(0)} + R^{(2)})^{-1} < 0$. [Note that $c_1 = -c_2$ leads to $U_S^{(0)} = (R^{(2)} - R^{(1)}) > 0$ and is thereby excluded.] Now, we use (11), (8), and (1) to obtain

$$\psi^H(x,k_\perp,\lambda) = \phi(x,k_\perp)\chi^H(x,k_\perp,\lambda) ,$$
$$\phi(x,k_\perp) = \frac{f(M^2)}{m_H^2 - M^2} , \qquad (12)$$
$$\chi^H(x,k_\perp,\lambda) = I_1^H + I_2^H .$$

This nonstatic-relativistic spin-wave function χ^H is identical to the Melosh solution studied elsewhere by one of us.[4] Thus the separable ansatz (7) provides a dynamical rationale for such wave-function models. However, the Melosh states are degenerate because the two invariants in (1) imply a doubling of eigenstates.

Had we now chosen the vector confining interaction (6) instead of (5), the resulting linear equations for the π and ρ coefficients

$$c_1^\pi = 4U_V^{(0)}(R_\pi^{(2)}c_1^\pi + R_\pi^{(0)}c_2^\pi) ,$$
$$c_2^\pi = -2U_V^{(0)}(R_\pi^{(0)}c_1^\pi + R_\pi^{(2)}c_2^\pi) ,$$
$$c_1^\rho = 2U_V^{(0)}(R_\rho^{(2)}c_1^\rho + R_\rho^{(0)}c_2^\rho), \quad c_2^\rho = 0$$

would lead to unphysical constraints on the strength $U_V^{(0)}$. A separable form of the vector interaction as a relativistic model of confinement seems to be ruled out being in contradiction with the Melosh solution, which is known to give reasonable descriptions of the low- and high-energy hadron properties.

The nucleon and Δ cases can be analyzed in a similar way. It is easy to show that the corresponding equations of motion for N and Δ are identical and lead to the Melosh solutions of Ref. 11 when the separable scalar confining interaction has three-body character,

$$U_S = U_S^{(0)} f(M^2) f(M'^2)(\bar{u}_1\bar{u}'_1)(\bar{u}_2 u'_2)(\bar{u}_3 u'_3) .$$

We conclude that a class of separable relativistic confinement interactions leads to wave-function solutions, which are a product of a potential-dependent momentum distribution and a Lorentz-invariant spinor amplitude. The latter is the nonstatic spin-wave function used in recent phenomenological analyses of low- and high-Q^2 hadron structure. Such models provide a basis for more refined studies, which would include a short-range vector interaction for mass splittings.

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