Cascade simulation of ultrarelativistic collisions

G. Bertsch and M. Gong
Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

L. McLerran
Physics Department, MS106, Fermilab, P.O. Box 500, Batavia, Illinois 60510

V. Ruuskanen and E. Sarkkinen
Physics Department, University of Jyvaeskylae, Seminaarinkatu 15, SF-40100, Jyvaeskylae, Finland
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The dynamics of hot matter produced in ultrarelativistic heavy-ion collisions is studied with a cascade simulation. We model the putative quark-gluon plasma with independent globs of high-density matter. The hadronic phase is treated by explicit tracking of pion coordinates. We find that the pions make 0–2 collisions with globs and 1–3 collisions with other pions, under conditions expected for heavy ions at collider energies. The entropy increases by about 20% during the phase transition. The transverse momentum in the final state is almost entirely due to the momentum with which pions are emitted from the globs, except at extremely high densities where the hydrodynamic expansion of the pure plasma phase is significant.

I. INTRODUCTION

An important objective in studying ultrarelativistic collisions of heavy ions is to find out which observables are sensitive to a phase transition of hadronic matter into a quark-gluon plasma. One possibility of a useful observable is the transverse momentum of the particles in the final state, measured as a function of their density in rapidity.[1,2] Qualitatively, the effect of a phase transition is to lower the pressure of the matter for a given entropy density. If the entropy is conserved, then it is related to the number of particles in the final state. Thus there will be less transverse momentum for a given entropy in the final state if a phase change takes place.

A detailed hydrodynamic study of the evolution of the high-density matter and the relation between transverse momentum and multiplicity has been made by von Gersdorff *et al.*[3] and Kataja *et al.*[4] These authors find a definite signature of the phase transition, but as they point out the assumption of hydrodynamics may be unjustified in the rapidly expanding hadronic phase. In this work we examine the expansion in a more microscopic model of the hadronic phase, treating the evolution by a cascade simulation of the pion motion in the final state.

We will follow the previous work in modeling the plasma phase and the initial conditions of the expansion. For the hadronic phase, we explicitly follow the motion of individual pions. Their interactions with each other and with the plasma material will be described in detail below. We first review the description of the plasma phase and the initial expansion.

II. UNDERLYING ASSUMPTIONS

The initial condition for the simulation is the boost-invariant cylinder of high energy density, as proposed by Bjorken.[5] This is formed at time $\tau=0$ by two nuclei passing through each other, leaving a wake of the excited matter. We need only specify for the initial condition the radius of the cylinder and some measure of the excitation of the medium, either the energy density or the entropy density. It is convenient to discuss the bulk properties of the two phases in terms of the entropy density. In simple models, the expansion and phase change is isentropic, which allows one to use information about the final-state entropy to specify the earlier conditions in the high-density matter. It is generally assumed that the pions are highly relativistic in the final state, in which case the entropy is related to the particle density in rapidity by

$$\frac{dN}{dy}=\frac{\zeta(3)}{4\zeta(4)}\frac{dS}{dy}=\frac{\pi R^2\tau\sigma}{3.6} . \tag{2.1}$$

Here R is the radius of the cylinder and σ is the entropy density.

The equation of state of the two phases is derived from entropy of a thermal distribution of massless particles. The formulas for the entropy density in the two phases are given by

$$\sigma_h=c_h T^3=\frac{12\pi^2}{90}T^3 , \tag{2.2a}$$

$$\sigma_{qg}=c_{qg} T^3=\frac{148\pi^2}{90}T^3 , \tag{2.2b}$$

where T is the temperature. The coefficients are obtained taking three kinds of pion for the hadronic phase, and two quark flavors and eight gluon colors for the plasma phase. The pressure of the pion gas at temperature of interest is close to that for massless particles, which is given by

$$P_h=\frac{\sigma_h T}{4} . \tag{2.3}$$

The pressure of the plasma phase has a similar contribution from the quark and gluon constituents, together with a bag pressure term B:

$$P_{qg}=\frac{\sigma_{qg}T}{4}-B \ . \tag{2.4}$$

We want to specify the model by the transition temperature between the two phases, T_c. This occurs when the two pressures are equal, which allows us to write (1.4) in terms of T_c instead of B:

$$P_{qg}=\frac{c_{qg}}{4}(T^4-T_c^4)+\frac{c_h}{4}T_c^4 \ . \tag{2.5}$$

We shall also need the energy density in the quark-gluon phase. This is given by

$$E_{qg}=\frac{3\sigma_{qg}}{4}T+B=\frac{c_{qg}}{4}(3T^4+T_c^4)-\frac{c_h}{4}T_c^4 \ . \tag{2.6}$$

If we assume that entropy is conserved and neglect transverse expansion, the time τ_q at which the hadronic phase begins to form may be simply calculated. Equation (2.2b) is evaluated at the transition temperature, giving the result

$$\tau_q=\frac{3.6dN/dy}{\pi R^2 c_{qg}T_c^3} \ . \tag{2.7}$$

A similar formula may be obtained for the time τ_h at which the plasma phase disappears. The only difference is that c_{qg} is replaced by c_h. At intermediate times, the fraction f of the volume in the hadronic phase is

$$f=\frac{\tau-\tau_q}{\tau_h-\tau_q} \ , \tag{2.8}$$

assuming that entropy is conserved and the expansion is purely longitudinal.

In previous work, the evolution of the hadronic final state required specifying a freeze-out density, below which interactions cease. In our model, both the middle mixed phase and the final hadronic phase are treated microscopically. We do not need to specify a freeze-out density or assume entropy conservation. Instead, we model the π-π collisions explicitly in propagating the system through the freeze-out point.

We also postulate a microscopic mechanism by which the phase transition takes place. This process may generate entropy, depending on how rapidly the expansion takes place. An upper limit to the entropy generation in our model is obtained if all of the internal energy of the plasma phase is converted to internal energy in the hadronic phase. The increase is then given by

$$\frac{dS/dy\mid_{\text{final}}}{dS/dy\mid_{\text{initial}}}\le\frac{\sigma_h}{\sigma_{qg}}\frac{E_{qg}}{E_h}=\frac{1-c_h/4c_{qg}}{\frac{3}{4}}\approx 1.3 \ . \tag{2.9}$$

III. PHYSICAL ASPECTS OF THE MODEL

Our starting point in time is τ_q,the point at which the plasma phase begins to convert to hadronic matter. There are two quite different scenarios that can be imagined here, depending on whether the plasma phase supercools or breaks up immediately. We only consider the latter situation in this paper. We assume that the plasma becomes unstable with respect to the breakup into droplets as the system expands. The droplets of quark-gluon matter will be called globs. At τ_q we describe the system by a collection of globs, having a thermal velocity distribution at temperature T_c. The globs evaporate by emitting pions, shrinking in size and eventually disappearing. They can also absorb pions that are incident upon them. We neglect the interaction of the globs with each other, allowing them to freely move throughout the medium. This introduces an error in the momentum transport as does the initial condition of a thermal momentum distribution at τ_q. We estimate the error below, and find that it is small except at extremely high multiplicities.

The globs are treated as spheres of plasma with a definite radius r_g and surface thickness d. The energy of the glob is obtained from the plasma energy density in the volume:

$$E=E_{qg}\frac{4\pi}{3}r_g^3 \ . \tag{3.1}$$

There are two considerations in choosing an initial radius of the globs. First, a glob cannot be so small that it only contains a few pions, or else it would be a known physical state. Second, we assume that the globs are much smaller than the transverse dimension of the cylinder, to allow the globs to participate in a transverse hydrodynamic expansion. Of course, the physical reality could well be that plasma does not break up into globs but remains connected, evaporating pions from an external surface. That would surely have more noticeable observational consequences than the conservative assumption we make here. We have somewhat arbitrarily chosen an initial plasma radius of 1 fm and a surface thickness of 1 fm.

As the globs emit pions, the plasma radius shrinks to preserve the energy density of the plasma phase. The total number of pions associated with a glob depends on details of the dynamics. If the expansion is slow, so that a given glob absorbs and emits many pions, the conversion will be adiabatic, and Eq. (2.2a) can be used to find the number of pions created from each glob. On the other hand, if the expansion takes place with little reabsorption of pions, energy conservation determines the number of created pions. Each pion is emitted with an average energy of 2.7 T_c, so the number of pions per glob would be the ratio of (3.1) to this energy. We assume in this work a transition temperature $T_c=200$ MeV. Then the globs of radius 1 fm have an energy of 13 GeV from Eq. (3.1). These transform into roughly 18 pions under adiabatic conditions and 24 pions under sudden expansion conditions. The model will give a prediction for the entropy production during the phase change. In contrast, the hydrodynamic equations by themselves do not give a unique prediction under the conditions found here.

We determine the rate at which the globs emit pions by detailed balance. We assume that the globs absorb all incident pions that come within a distance r_g+d of the glob. The rate at which they are emitted must equal the absorption rate when the globs are in a thermal bath of pions at temperature T_c. This condition requires that the emission rate be

$$W = \pi(r_g + d)^2 c_h T_c^3 / 3.6 \ . \tag{3.2}$$

Besides the possibility of being captured by globs, the pions interact by elastic scattering with each other. We model the elastic scattering with the phases shifts δ_l^I for isospin I and the $l = 0,1$ partial waves. These are obtained indirectly from various empirical data involving the π-π system.[6] Our parametrization of the $I = 0$ s-wave channel is based on the low-energy phase shifts deduced from the π-π correlation in the K_{e4} decay,[7] and the higher-energy phase shifts inferred from the $\pi N \to \pi\pi N$ reaction.[8] Both data sets are fit by a simple resonance model,

$$\delta_0^0 = \arctan\left[\frac{\Gamma_\sigma/2}{E_\sigma - E}\right] \tag{3.3}$$

with $\Gamma_\sigma = 2.06q$ and $E_\sigma = 5.8 m_\pi$. Here

$$q = \tfrac{1}{2}\sqrt{s - (2m_\pi)^2}$$

is the momentum of a pion in the c.m. frame. We also use a ρ resonance formula for the $I = 1$ p-wave channel. The resonance parameters are $E_\rho = 770$ MeV and $\Gamma_\rho = 150$ MeV. We use a functional form for $\Gamma_\rho(q)$ that has correct threshold behavior and that falls off at large q:

$$\Gamma_\rho(q) = 0.095q\left[\frac{q/m_\pi}{1 + (q/E_\rho)^2}\right]^2 . \tag{3.4}$$

The $I = 2$ s-wave phases are weakly repulsive and not very well known. We use a scattering length approximation in this channel, which is sufficient to fit the phase shift of Ref. 9. The parametrization is

$$\delta_0^2 = -0.12q/m_\pi \ . \tag{3.5}$$

The cross sections in states of good isospin are related to the phase shifts by

$$\sigma_l^I = \frac{8\pi}{q^2}(2l+1)\sin^2\delta_l^I \ . \tag{3.6}$$

We have neglected coherence between different partial waves and assumed isotropic scattering in the model. The physical quantities studied in this work depend only on the isospin-averaged cross section, which is given by

$$\sigma_{\rm av} = \tfrac{1}{9}\sigma^0 + \tfrac{1}{3}\sigma^1 + \tfrac{5}{9}\sigma^2 \ . \tag{3.7}$$

An important quantity for the discussion of the dynamics of the pion gas in the final state is the thermally averaged cross section. We define it as the average over two-particle phase space of the cross section (3.7) weighted by the relative velocity of the pions times their Bose-Einstein statistical factors. Our average cross section is shown in Fig. 1 as a function of temperature. It is about 20 mb at temperatures around the assumed phase transition temperature. It drops sharply at low temperature, which will cause the pions to freeze-out relatively early in the expansion. At higher temperatures our model becomes unreliable because of the neglect of other resonances and higher partial waves.

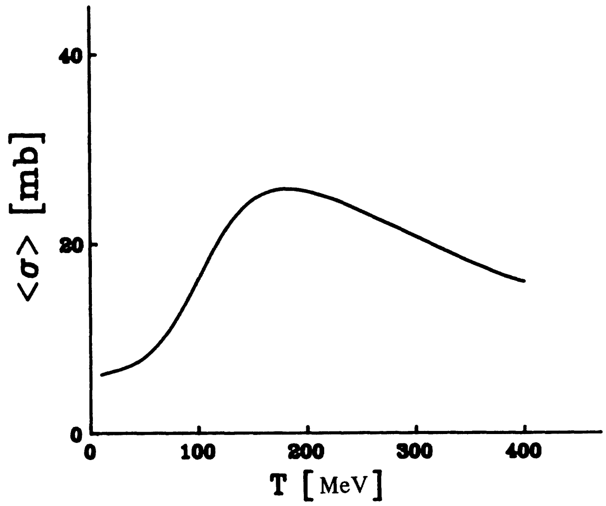

FIG. 1. Thermally averaged π-π scattering cross sections, from Eq. (3.3)–(3.7).

IV. NUMERICAL ASPECTS OF THE MODEL

The computer program to simulate the above processes requires procedures for initializing the distribution of globs, and then tracking the emission, absorption, and scattering of pions. We view the system in a fixed frame, with space-time events labeled by coordinates (r,t). For the boost-invariant initial condition, the local rapidity at a given point is

$$y = \text{arctanh}(t/z) \ . \tag{4.1}$$

The proper time at this rapidity is given by

$$\tau = t/\cosh y \ . \tag{4.2}$$

The first task of the program is to initialize the globs at proper time τ_q. The program updates the state of the system at fixed time intervals Δt, starting from $t = \tau_q + \Delta t$. At each time step, up to some maximum, the spatial position z_i is determined at which $\tau(i) = \tau_q$. Then globs are added to the intervals $\pm[z_{i-l}, z_i]$, chosen randomly from a uniform rapidity distribution. This process is repeated at each time step up to some maximum rapidity. Particles of higher rapidity are neglected.

The glob initialization also assigns a momentum, which is taken to be thermally distributed in the local rest frame. Then in the fixed frame the momentum four-vector is the Lorentz transform of the thermal four-vector by the rapidity (4.1).

The emission rate for pions is calculated from Eq. (3.2). If the probability that a pion is emitted in a given time step exceeds $\frac{1}{2}$, the program creates a pion on the outer surface of the glob, at a random position on the surface. The momentum of the pion is determined from a thermal distribution, with the orientation distributed randomly. If the number of pions remaining in the glob falls below five, all the pions are released at the glob surface and it disappears. In the final dissolution of the glob, the pion momenta are distributed according to the available N-body phase space.

Absorption of the pions is handled very simply. Any pions that are within the outer radius of a glob, as seen at a given time step, are captured by the glob. The glob's mass and size are then increased accordingly. We found it important to properly account for Lorentz contraction of the globs in determining the pion capture. Otherwise, an anomolously large fraction of low rapidity pions is absorbed by high rapidity globs, due to their overlap with the low rapidity region at early times.

Collisions between the pions are handled as follows. All pairs of pions are examined at each time step, and the program calculates the dot product of the relative coordinate and the relative momentum of the pair. If the product changes sign, the particles have passed by each other, and are candidates for a collision. The impact parameter b is then calculated, and if it does not exceed a certain value b_{max} the cross section is computed. The particles scatter with probability proportional to the ratio of the computed cross section and πb_{max}^2. If the particles successfully scatter, a new direction is assigned to the relative momentum. We assume isotropic scattering, which will somewhat overestimate the importance of scattering effects.

The final procedure in the simulation is to propagate the particles by changing their spatial coordinates according to

$$r(t+\Delta t)=r(t)+(p/E)\Delta t \ . \tag{4.3}$$

It is worth mentioning at this point that the model exactly conserves the momentum and energy of the initial state. The quantities to be obtained from the model that are most interesting are the transport of momentum in the transverse direction, and the conversion of longitudinal energy to transverse energy. Concerning the transport of momentum, the model is deficient in that the pressure in the plasma phase is ignored. This is a serious shortcoming for the first stage of the expansion, but once the globs are formed their internal pressure does not play any role in the global momentum transport. As far as the exterior is concerned, the glob behaves exactly like a spherical volume of pion gas. It absorbs all pions incident on it, and it emits pions as if it were filled with a thermal distribution of them. Thus the glob mediates the transport of momentum just like the pion gas, except for the fact that the glob is massive and will be accelerated by a pressure imbalance.

V. RESULTS AND DISCUSSION

We now report the results of the simulation for a variety of initial conditions. In general, we find that the pions decouple quickly from the globs and each other. In Fig. 2 we show the fraction of pions in the expanding matter as a function of time, for two values of the multiplicity density. The lower value, $(1/a)dN/dy=4$, is about the density produced in the central region in p-p collisions at collider energies. The upper value, 40, is the density resulting from independent N-N collisions of projectile nucleons with successive target nucleons through the center of a heavy target. We assume a radius of the cylinder of 4.2 fm, corresponding to the projectile nucleus Fe with $A=56$. The initial times are determined from Eq. (2.7) to be 0.9 fm/c and 9 fm/c, respectively. For the low-multiplicity case, the conversion of matter from the plasma phase to the hadronic phase more or less agrees with the pure longitudinal expansion, Eq. (2.8). There is a slight delay in the cascade calculation, which may be attributed to the finite time for the globs to evaporate pions. In the high-multiplicity case the situation is quite different. The globs convert into pions more quickly than predicted by the one-dimensional model. Here the time scale is large enough so that the transverse expansion becomes important early in the mixed phase. Thus the density becomes low at much earlier time than would be the case for pure longitudinal expansion.

Another reason for the faster conversion could be the

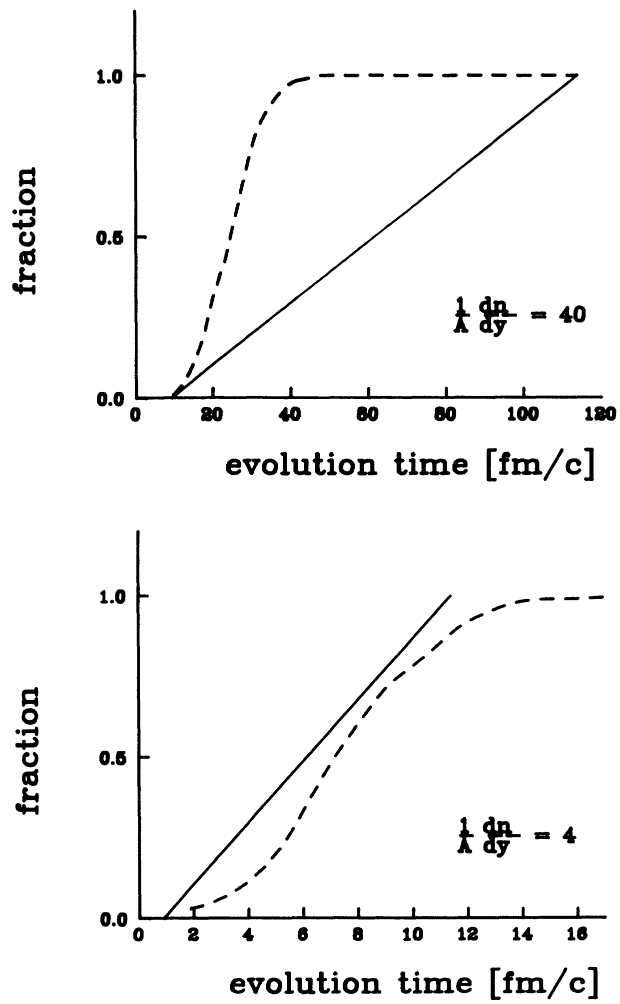

FIG. 2. Fraction of hadronic matter as a function of time for a cylinder radius of 4.2 fm and multiplicity densities $dN/dy=224$ and 2240. The solid line shows the fraction for the one-dimensional hydrodynamic expansion, Eq. (2.8). The dashed line shows the fraction of free pions as a function of time obtained in the simulation.

decoupling of the pions from the globs. This may be seen in Figs. 3 and 4, which show the radial velocity distributions of pions and globs for the case $R=6.8$ fm, $(1/A)dN/dy=8.5$. We see that the pions stream outward at high velocity, leaving the globs behind. In the one-dimensional model, the mixed phase is completely homogeneous. In the three-dimensional hydrodynamics, the transverse expansion takes place through a rarefaction shock wave, which propagates into the cylinder from its surface.[10] The shocked matter is in the purely hadronic phase, so the region containing globs does not expand radially. The very slight radial expansion in the cascade thus roughly accords with the hydrodynamics. Figure 4 also shows good agreement in the comparison with hydrodynamics for the pion radial velocity distribution.

The next point we examine is the adiabaticity of the phase transition. For low multiplicities $[(1/A)dN/dy=4]$ the globs recapture only $\frac{1}{3}$ of the pions emitted. The conditions are therefore far from adiabatic, and close to the sudden expansion limit. The total number of pions emitted is 24% larger than under adiabatic conditions. The model thus predicts an entropy increase of 24% during the phase transition. Going to much higher multiplicity $[(1/A)dN/dy=40]$, the ratio of emitted to captured pions is 1.6. The total number of pions in the final state is 16% larger than the adiabatic limit. Thus under all conditions we envisage for heavy-ion collisions, the entropy increase due to the phase transition is about 20%. The entropy increase could be substantially larger only if the plasma interface partly reflected, which does not seem likely. The entropy production could be less if the globs were more numerous, but we have not explored this in detail. In any case, the entropy production is small compared to the ratio of entropy densities in the two phases, and so the usual assumption of isentropic expansion is a reasonable first approximation.

The cascade simulation allows one to determine the freeze-out point in the expansion. The density at which this occurs is an important parameter in hydrodynamic calculations. Hydrodynamics fails when the system is

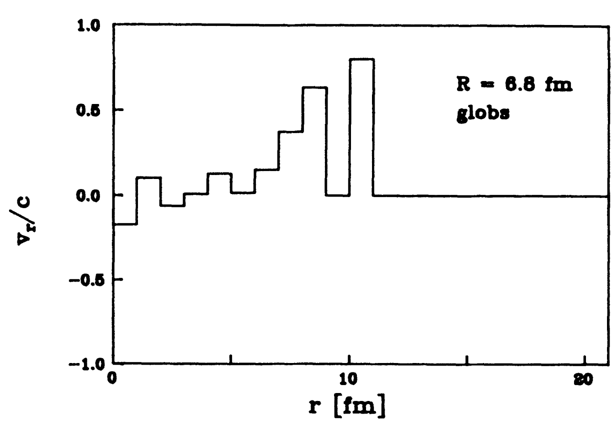

FIG. 3. Radial-velocity distribution of globs at $t=14.7$ fm/c. The initial cylinder radius is 6.8 fm and the final-state rapidity density of the pions is 1700.

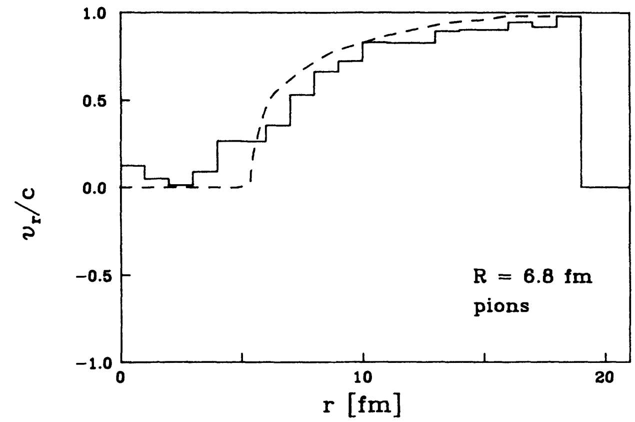

FIG. 4. Radial-velocity distribution of pions, under the same conditions as in Fig. 6, is shown as the histogram. The dashed curve is the hydrodynamic prediction, using the calculation method of Refs. 3 and 4.

rarified beyond the point where collisions have ceased. One way to examine the freeze-out is to compare the characteristic time for expansion of the system with the collision time. We define an expansion time $t_{\rm exp}$ as

$$t_{\rm exp}=\frac{\langle V\rangle}{d\langle V\rangle/dt}\ , \tag{5.1}$$

where $\langle V\rangle$ is the volume of the system. For presenting the results of the simulation, we have defined the volume as

$$\langle V\rangle=\Delta y\,\tau\sum_i r_1^2(i)\exp[-(r_i/R)^2]\ . \tag{5.2}$$

Initially, the expansion is longitudinal and the expansion time equals the proper time τ. The system does not freeze-out until radial expansion becomes significant. Then the expansion time falls below τ; for a spherical expansion $t_{\rm exp}\tau/3$. In Fig. 5 we display the expansion time

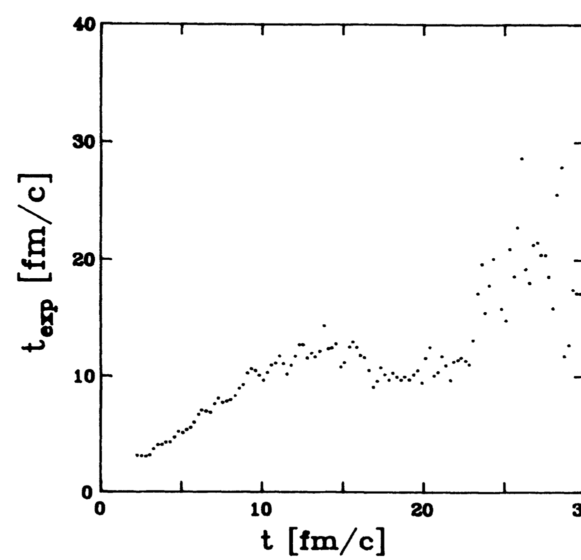

FIG. 5. Expansion time of the system, computed according to Eq. (5.1). Conditions are $dN/dy=1713$ and $R=6.8$ fm.

as a function of time for the same conditions as in Figs. 3, and 4. The simulation begins when the system enters the mixed phase, at $t=\tau_q=2.7$ fm/c. The expansion is longitudinal until about 14 fm/c, after which t_{exp} falls below t. The expansion time is roughly constant until about 22 fm/c. The definition (5.2) of the volume is not useful for longer times. Under conditions of pure longitudinal expansion, the conversion to a pure hadronic phase would not occur until $t=33$ fm/c. Thus the radial expansion is important during the mixed-phase period, and certainly invalidates a one-dimensional description of this period.

To see the freeze-out point of this system we display the average collision time, defined as the ratio of the number of pions to the collision rate of the pions:

$$t_{\text{coll}}=\Delta t\frac{\text{number of pions}}{\text{number of pions colliding in }\Delta t}\ . \tag{5.3}$$

This is shown in Fig. 6. The time is short until just beyond 20 fm/c, when it grows very rapidly. It exceeds the expansion time at about 22 fm/c, so we may take this to be the freeze-out point. We could also determine the freeze-out point as the average time of the last collision made by a pion in the final state. This measure also gives 22 fm/c for the freeze-out time. We thus see that the freeze-out occurs virtually as soon as the plasma phase disappears. There is no time at which the system looks like an interacting hadronic gas.

The next property we examine is the average number of collisions of the final-state pions. This is shown as a function of dN/dy in Fig. 7. Under typical conditions of expected multiplicity, there are only 1–2 collisions per pion in the final state. There is little opportunity in the final state to convert longitudinal energy into transverse energy. The low number of π-π collisions means that the final-state pions to a large extent preserve a view of the surface of the quark-gluon plasma.

We finally examine the transverse momentum of the final-state pions. This is the observable that is directly measured and should have some bearing on the high-density dynamics. The prediction of the cascade simulation is shown in Fig. 8, compared with hydrodynamic calculations[4] under the same conditions ($R=4.2$ fm). The transverse momentum is very insensitive to multiplicity in the cascade calculation. Essentially all of the transverse momentum is present in the initial pion emission from the glob surface. This might have been anticipated from the previous discussion; the π-π collisions have only a small effect on increasing the collective momentum.

The cascade results agree rather well with the hydro-

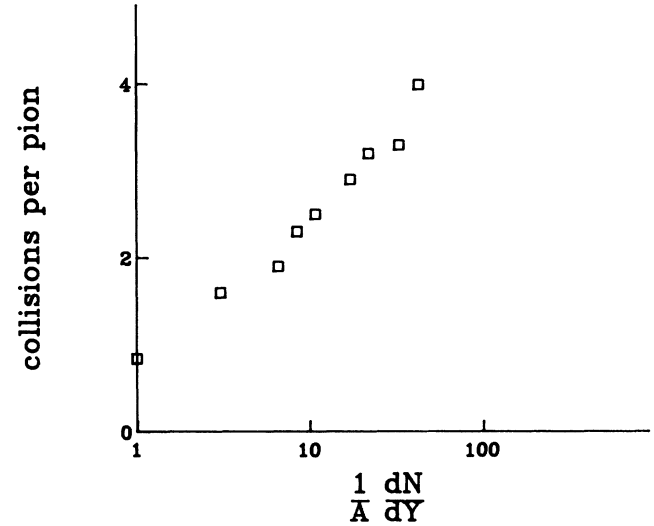

FIG. 7. Average number of collisions of a final-state pion, as a function of the assumed multiplicity density. The radius of the cylinder is taken as 4.2 fm.

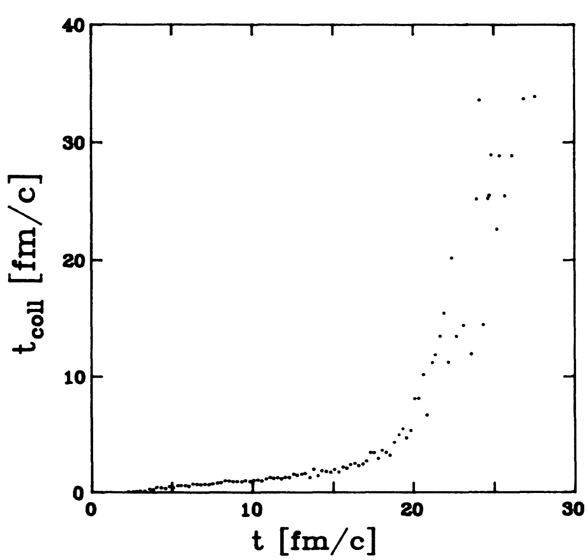

FIG. 6. Collision time of pions under conditions of Fig. 5.

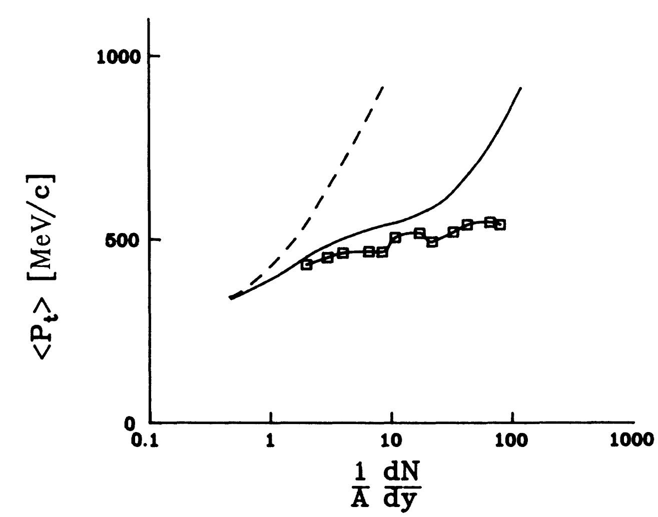

FIG. 8. Transverse momentum of pions, as a function of multiplicity, for an initial cylinder of radius 4.2 fm. The dashed and solid lines show the result of hydrodynamic calculations assuming no phase transition and a transition temperature of 200 MeV, respectively. The squares show the results of the present model.

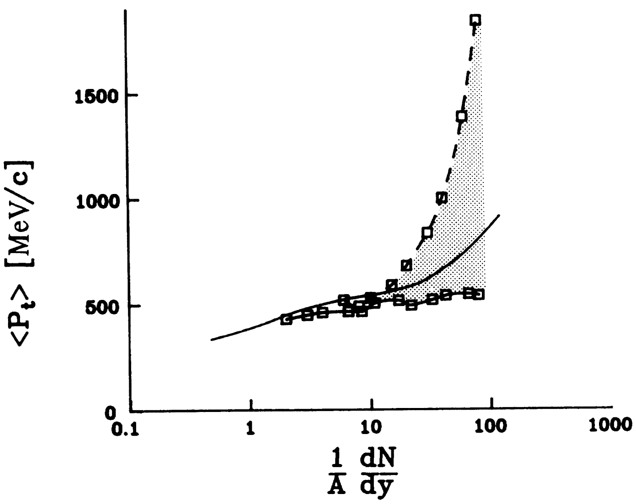

FIG. 9. Effect of pressure in the pure-plasma phase on the transverse momentum. An upper bound on the additional transverse momentum from the radial expansion of the globs is shown by the upper dashed curve.

dynamic calculation for multiplicities up to $(1/A)dN/dy \sim 20$. Thus the treatment of the hadronic gas in the final state is not important for calculating the transverse momentum. At very high multiplicities the results disagree. This is due to our neglect of the transverse momentum developed before the time τ_q, when the interior is in the pure plasma phase. We can estimate the effect of the neglected plasma pressure on the momentum of the globs at τ_q, if we are willing to ignore the transverse expansion beforehand. This calculation is done in the Appendix. We repeated the cascade calculation, including the transverse momentum from Eq. (A3) in the initial conditions on the globs at τ_q. The resulting transverse momentum in the final state is shown as the upper curve in Fig. 9. Since the expansion before τ_q has been neglected, the calculation overestimates the generated transverse momentum. Thus the actual momentum will be between the limits shown as the shaded area in the Figure. This is quite consistent with the hydrodynamic predictions, despite the evident shortcoming of hydrodynamics in the hadronic expansion phase.

VI. CONCLUSIONS

If there is a phase transition to a quark-gluon plasma, the dynamics of the hot matter is determined almost entirely by what happens in the plasma phase and at the interface with the vacuum, according to our microscopic model. Collisions between the pions in the hadronic final state are sparse and do not affect the global energy and momentum transport. This is perhaps encouraging, suggesting that the final-state hadrons may give a direct view of the plasma interface.

We also found that the numbers of pions created exceeds the entropy-conserving amounts by about 20%. When one considers that the ratio of entropy densities in the plasma and the hadronic phases is ~ 13, the entropy production in the phase transition is minor and arguments based on entropy conservation remain very useful.

Finally, the method described here should allow other interesting observables to be calculated. In particular, the correlation between pions due to their Bose statistics can be calculated with a model of the pion sources. The bremsstrahlung radiation is another interesting observable. It is directly proportional to the number of collisions, and the yield of bremsstrahlung exceeds the background of photons from other sources at small transverse momentum.

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APPENDIX: HYDRODYNAMIC CALCULATION OF TRANSVERSE MOMENTUM

We derive a relation between the pressure in the medium and the transverse momentum generated as follows. The cylinder of high-energy-density matter is bisected along its axis by a plane. We then integrate the local conservation law for momentum over a half-space bordered by the plane. This tells us that the rate at which momentum is generated in half-space is equal to the pressure across the dividing plane. We also integrate over time, and that gives us the total momentum generated in the half-space. Thus if we know the pressure on the plane as a function of position and time, we can calculate the momentum as

$$p_n = \int \int P \, dA \, dt \; . \tag{A1}$$

Here p_n is the component of momentum normal to the plane contained in one of the half-spaces, and P is the pressure on the plane. The relation of this momentum to the observed transverse momentum in the final state depends on the velocity distribution of the final-state particles at the breakup. If the thermal component of the velocity is small, the particles will move in a radial direction and the momentum (A1) directly relates to the individual particle momenta. The relation is

$$\langle p_\perp \rangle = \frac{\pi}{2} \langle p_n \rangle = \frac{\pi}{N} p_n \; , \tag{A2}$$

where N is the number of particles in the half-space. The other extreme is when the thermal momentum of the final-state particles is large compared to the collective momentum from (A1) and (A2). In that case, the coherent momentum combines in quadrature with the thermal momentum and the effect on the average perpendicular momentum is of second order in the ratio.

For the case at hand, we evaluated the pressure from Eq. (2.4), assuming the temperature follows the one-dimensional hydrodynamics: $T \sim t^{-1/3}$ from a time $t=0$ to $t=\tau_q$. This will certainly overestimate the pressure integral: the earliest times give the largest contribution to the integral, but some finite time is needed to establish hydrodynamic equilibrium. Evaluation of the integral yields the following formula for the coherent transverse momentum:

$$\langle p_\perp \rangle = \frac{(3.6)^2 dN/dy}{\pi c_{qg} R^3 T_c^2} \left[\frac{3}{4} - \frac{c_{qg} - c_\pi}{4c_{qg}} \right] . \tag{A3}$$

[1]E. V. Shuryak, Phys. Rep. **61**, 71 (1980); E. V. Shuryak and O. Zhirov, Phys. Lett. **89B** 253 (1980); Yad. Fiz. **28**, 485 (1978) [Sov. J. Nucl. Phys. **28**, 247 (1978)].

[2]L. van Hove, Phys. Lett. **118B**, 138 (1982).

[3]H. von Gersdorff, L. McLerran, M. Kataja, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986).

[4]M. Kataja, P. V. Ruuskanen, L. McLerran, and H. von Gersdorff, Phys. Rev. D **34**, 2755 (1986).

[5]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[6]B. R. Martin, D. Morgan, and G. Shaw, *Pion-Pion Interactions*

[7]L. Rosselet *et al.*, Phys. Rev. **D 15**, 574 (1977).

[8]P. Estabrooks and A. D. Martin, Nucl. Phys. **B79**, 301 (1974); S. Protopopescu, *et al.*, Phys. Rev. D 7, 1279 (1973).

[9]W. Hoogland *et al.*, Nucl. Phys. **B69**, 266 (1974).

[10]G. Baym, B. Friman, J. Blaizot, M. Soyeur, and W. Czyz, Nucl. Phys. **A407**, 541 (1983).

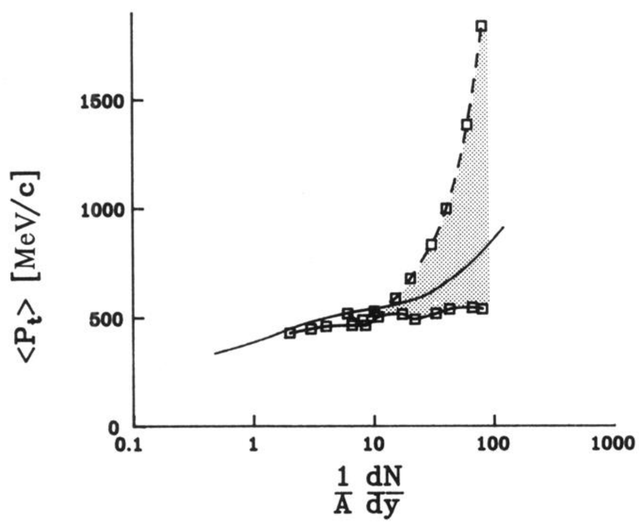

FIG. 9. Effect of pressure in the pure-plasma phase on the transverse momentum. An upper bound on the additional transverse momentum from the radial expansion of the globs is shown by the upper dashed curve.