Comments

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Variation of the Fermi constant and primordial nucleosynthesis

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Motivated by the dependence of the fundamental constants on the size of compact dimensions, Kolb, Perry, and Walker considered the effects on primordial nucleosynthesis of variations in some of these constants, including the Fermi constant G_F. We note that, in the standard model, a variation in G_F will necessarily imply a variation in the vacuum expectation value of a Higgs field, which in turn will affect the electron mass, quark masses (and thus the neutron-proton mass difference), and the pion mass (and thus the binding energy of the deuteron). We calculate the effect on primordial nucleosynthesis of a variation in the vacuum expectation value of the Higgs field, including all of these factors.

A recent paper by Kolb, Perry, and Walker[1] (KPW) considered the effects of changes in the fundamental constants on nucleosynthesis. They were motivated by the fact that in models with $4+D$ dimensions, in which D dimensions are compact, the fundamental constants of the four-dimensional space depend on the volume of the compact D-dimensional space. Since it is difficult, and perhaps impossible, to find a model in which four dimensions expand and the other D dimensions have fixed size, it is quite plausible that the fundamental constants will vary with time (although not necessarily with power-law behavior). Nucleosynthesis will give the strongest reliable constraints on the values of the fundamental constants in the early Universe, and thus, indirectly, on the size of extra dimensions during this epoch.

KPW first found the dependence of the primordial ^{4}He abundance on the values of Newton's constant G_N, the Fermi constant G_F, and the neutron-proton mass difference Q. They then related variations in these constants to variation of the fine-structure constant α, which was in turn related to the size of the extra dimensions.

In relating changes in G_F to changes in α, KPW noted that $G_F = g^2/8M_W^2$, where g is the SU(2) gauge coupling and M_W is the mass of the W boson. They then argued that the value of M_W is determined by the vacuum expectation value σ of a Higgs field and that since the scalar sector is "the least understood sector" they "will not consider changes in G_F due to the variation of M_W, and will assume only $\delta G_F \propto \delta g^2$."

We find this assumption to be implausible. In the standard model, $M_W^2 = \frac{1}{4}g^2\sigma^2$ and thus $G_F = 1/2\sigma^2$. As a result, the Fermi constant is completely *independent* of the gauge coupling. In the standard model, the *only* way to vary G_F is to vary σ, and this will have other implications not considered by KPW. In this Comment, we determine the effects of a variation in σ on primordial nucleosynthesis.

There are four effects on nucleosynthesis which arise if σ is varied. First, the value of G_F changes, altering all weak-interaction rates. Second, the electron mass m_e changes. Third, the quark masses change, which affects the neutron-proton mass difference. Finally, a variation in the quark masses will change the pion mass, affecting the strong nuclear force and the binding energy of the deuteron. We emphasize that these changes are, in the standard model, inevitable once G_F is varied, and must be considered along with any variation in G_F.

The first two effects are easy to determine: G_F varies as $1/\sigma^2$ and m_e varies as σ. The neutron-proton mass difference $Q \equiv m_n - m_p$ receives contributions from both the electromagnetic and weak interactions. As discussed in detail by Gasser and Leutwyler,[2] the weak contribution to Q is primarily due to the difference in the masses of the d and u quarks. The electromagnetic contribution is negative[3] (it increases m_p more than m_n) and is approximately -0.9 MeV; thus the weak contribution is approximately 2.2 MeV. Since the quark mass difference is proportional to σ, and since $Q_0 = 1.293$ MeV, we write

$$Q = -0.9 \text{ MeV} \left[\frac{\alpha}{\alpha_0} \right] + 2.193 \text{ MeV} \left[\frac{\sigma}{\sigma_0} \right] . \qquad (1)$$

We have checked that our final results are insensitive to the precise value of the electromagnetic contribution (as long as $Q_0 = 1.293$ MeV, of course).

Finally, a quark mass change of 1% will change the pion mass by 0.5%, as can be seen from the relation $(m_u + m_d)\langle \bar{\psi}\psi \rangle = f_\pi^2 m_\pi^2$. A simple bound-state model calculation (see, for example, Ref. 4) implies that the deuteron binding energy B_d depends linearly on m_π. Thus a 1% change in σ results in only a 0.5% change in B_d, which has a negligible effect on the helium abundance Y_p. We have also checked that the change in the axial-vector coupling constant due to the change in the pion mass, which can be estimated by the Adler-Weisberger relation, is completely negligible.

We thus calculate the helium abundance as a function of $\Delta\sigma \equiv (\sigma - \sigma_0)/\sigma_0$ and of $\Delta\alpha \equiv (\alpha - \alpha_0)/\alpha_0$. It is found that the functional dependence is linear and is given by

$$Y_p = 0.240 - 0.31 \left[\frac{\sigma - \sigma_0}{\sigma_0} \right] + 0.38 \left[\frac{\alpha - \alpha_0}{\alpha_0} \right] . \qquad (2)$$

This fits our calculation to an accuracy of better than 0.001 for $|(\sigma - \sigma_0)/\sigma_0| < 0.1$ and $|(\alpha - \alpha_0)/\alpha_0| < 0.1$. If one requires that $\Delta Y_p < 0.01$, to agree with the success of standard nucleosynthesis, then, for fixed α, $\Delta\sigma < 0.032$ at the time of nucleosynthesis. For fixed σ, we see that $\Delta\alpha$ must be less than 0.026. As stated above, the only effect of a change in α is in the electromagnetic contribution to Q, whereas a change in σ has a larger effect on Q and also affects m_e and G_F. These three effects tend to cancel, making the effect of a change in α somewhat larger.

In order to consider the size of the compact dimensions, we have to know how σ is related to the radius of these dimensions, R_D. In models in which the weak scale is determined via dimensional transmutation,[5] σ is either linearly related to $M_{\rm Pl}$ or inversely related, such as $M_I^2/M_{\rm Pl}$, where $M_I \sim 10^{11}$ GeV. This gives $\sigma/\sigma_0 = (R_D/R_{D0})^3$ and $(R_D/R_{D0})^{-3}$, respectively. If the only consequences of changing the radii of the compact dimensions are on α, σ (and thus G_F), and $M_{\rm Pl}$ (and thus G_N), then we can calculate the maximum allowed change in R_D between the nucleosynthesis era and the present. We find that $|1 - R_D/R_{D0}|$ must be less than 0.004 or 0.008 for the two models above, in rough agreement with KPW. However, since this analysis ignores possible variations of Yukawa couplings, the strong coupling constant, etc., any firm conclusions about the size of higher dimensions is somewhat premature.

We have argued that the KPW assumption that $\delta G_F \propto \delta g^2$ is not plausible in the standard model and that the only way to vary G_F is to vary σ, the vacuum value of the Higgs field. This will lead to other effects, such as a change in m_e and a change in $Q \equiv m_n - m_p$. We have seen that the success of standard nucleosynthesis forces σ to be less than 3% different during nucleosynthesis from what it is today, if all other parameters are fixed. This forces the radii of extra dimensions to be less than $\sim 1\%$ different during this epoch from their current values.

Note added in proof. After this work was accepted for publication, we learned of related work by J. D. Barrow [Phys. Rev. D **35**, 1805 (1987)]. He argued that stronger constraints than those of KPW can be obtained by considering a variation of the "strong coupling constant." We have two comments on his work. First, he considered only variations in the strong-interaction coupling between hadrons, and not in the QCD SU(3) quark-gluon coupling. It is the latter coupling which will vary as the size of the compact dimensions. As a nonperturbative effect, the hadronic coupling is probably quite insensitive to the value of the QCD coupling at a given scale; in any event, the relationship between the two coupling constants is quite unclear. Second, a change in the QCD coupling will necessitate a variation in the QCD scale parameter Λ. This will, in turn, dramatically affect the neutron, proton, and pion masses, etc. Thus, we do not see how one can consider variation in the QCD coupling constant without including these additional effects, nor do we see how to relate this variation to the variation in the hadronic coupling constant considered by Barrow.

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[1]E. Kolb, M. Perry, and T. Walker, Phys. Rev. D **33**, 869 (1986).

[2]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[3]KPW assumed that the electromagnetic contribution was of similar size to the total contribution, and thus $Q/Q_0 \sim \alpha/\alpha_0$. This is true; however, the fact that the electromagnetic contribution is negative implies that $Q/Q_0 \sim -\alpha/\alpha_0$. While this will reverse the slope of the lines in their final figure, it will not affect the bounds they obtained on R_D/R_{D0}.

[4]P. C. W. Davies, J. Phys. A **5**, 1296 (1972).

[5]A. B. Lahanas and D. V. Nanopoulos, Phys. Rep. **145**, 1 (1987).