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Stability of the vacuum in scalar field models in 1+1 dimensions

A. Harindranath and J. P. Vary
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
and Physics Department, Iowa State University, Ames, Iowa 50011
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Utilizing the recently proposed discretized light-front quantization method we evaluate the critical coupling for vanishing mass gap (with respect to the perturbative vacuum) for two interacting scalar field models in 1+1 dimensions. Our extrapolated results for the critical coupling are compared with results based on the equal-time formulation.

I. INTRODUCTION

This paper is the third in a series[1,2] which attempts to study the structure of interacting scalar-field-theory models in 1 + 1 dimensions in a discretized light-front quantization (DLFQ) scheme.[3,4] Our major goal in these efforts is to explore the validity of light-front quantization in general and the DLFQ method in particular by comparing results for "physical" observables with results obtained in the equal-time formulation. The present effort addresses critical phenomena in scalar field models.

In a previous work[1] we studied the spectrum of $\lambda(\phi^4)_2$ theory in the disordered phase of the theory. Starting from a normal-ordered Hamiltonian we investigated the effect of imposing a finite renormalization on the mass gap with respect to the perturbative vacuum. We found that the results with and without finite mass renormalization lead to the same estimate of the critical coupling where the square of the invariant mass of the lowest excitation vanishes. (See Fig. 3 in Ref. 1.) We quoted our results for a single value of the dimensionless light-front momentum operator $K(=16)$. In this work we give an estimate of the critical coupling in the continuum limit ($K\rightarrow\infty$). We compare our value with those extracted by other means in the equal-time formulation and find reasonable agreement. For comparison and contrast we consider a different model: $(\phi^4)_2$ theory together with the ϕ field coupled to a constant external field, thus breaking the $\phi\rightarrow-\phi$ symmetry explicitly. As is well known,[5] the vacuum of the resulting Hamiltonian is unique. We study this model after introducing a shift and verify the nonvanishing of the mass gap for physical values of the coupling.

This paper is organized as follows. In Sec. II we discuss $(\phi^4)_2$ theory in the disordered phase. Section III deals with the external field problem. Our conclusions are presented in Sec. IV.

II. $(\phi^4)_2$ THEORY IN THE DISORDERED PHASE

The spectrum of $(\phi^4)_2$ theory in the disordered phase has been investigated in Ref. 1. The construction of the mass operator and the method of solution has been described in that work. In the present work we are interested in the critical bare coupling for which the mass gap vanishes. To achieve this we fix the mass parameter m in the Lagrangian and diagonalize the invariant-mass matrix. Since the perturbative vacuum is decoupled from the rest of the Fock-space states once one neglects the zero modes, the lowest eigenvalue of the mass operator gives the square of the mass gap with respect to the perturbative vacuum. For an initial guess for the bare coupling λ we iterate the equation for the lowest eigenvalue M_1,

$$M_1(m^2,\lambda)=0\ ,$$

for fixed $m^2=1.0$ until convergence is achieved within three significant figures.

We calculate the critical coupling as a function of the dimensionless momentum operator K. At $K=1$ we have a single state containing a single particle carrying momentum K. At $K=2$ we have one state where a single particle carries momentum K and a second state where two particles each carry momentum $K/2$. Since the mass operator only connects states that differ in particle number 0 or 2, these states are decoupled and the mass gap never vanishes for fixed m. We have performed calculations for even values of K up to 20. Using the last four data points we extrapolate to $K=100$ using techniques described in Ref. 6. The extrapolated value of the critical coupling is ≈ 33.

The critical coupling for $(\phi^4)_2$ theory has been investigated by several authors. Using the Hartree approximation Chang[7] finds that the disordered phase can be mapped into the ordered phase for $\lambda=54.27$. However, it is not clear that this self-dual condition also represents the critical coupling. Using the coupled cluster method Funke *et al.*[8] find $22.8<\lambda_{\text{critical}}<51.6$. Using a momentum-space discretization technique, Kroger *et al.*[9] find a vanishing mass gap for $\lambda\approx 36$. Thus our result in the light-front formulation is consistent with similar estimates from equal-time formulations.

III. EXTERNAL-FIELD PROBLEM

In the preceding section we have utilized the vanishing map gap with respect to the perturbative vacuum as a criterion for the vacuum instability of the model. It is of interest to see what happens if we perform similar calculations for a model for which it is known that the vacuum is unique. It has been shown by Simon[5] that the vacuum is unique for the model described by the Hamiltonian density

$$\mathcal{H}=N\left[\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{1}{4!}\lambda\phi^4+\tfrac{1}{2}\phi^2-B\phi\right],$$

with $\lambda>0, B\neq 0$, in $1+1$ dimensions. We study the mass gap of this model in this section.

Let us start with the Lagrangian density

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}m^2\phi^2-\frac{\lambda}{4!}\phi^4+B\phi\ .$$

Thus

$$-\mathcal{L}_{\text{int}}=a\phi^4+b\phi^2+c\phi\ ,$$

where $a=\lambda/4!$, $b=\frac{1}{2}m^2$, and $c=-B$. The DLFQ scheme cannot be directly applied to this model since the $B\phi$ term spoils the conservation of P^+ (nonconservation of momentum in the external field). However, we can do the following.[10] Start with $-\mathcal{L}_{\text{int}}$ and introduce a constant shift in the field $\phi\rightarrow\phi+\chi$, where χ is a constant c-number field. Then

$$-\mathcal{L}_{\text{int}}=a\phi^4+4a\chi\phi^3+(6a\chi^2+b)\phi^2+(4a\chi^3+2b\chi+c)\phi+a\chi^4+b\chi^2\ .$$

Now choose

$$4a\chi^3+2b\chi=-c=B\ .$$

Then

$$-\mathcal{L}_{\text{int}}=\frac{\lambda}{4!}\phi^4+\frac{\lambda}{3!}\phi^3+\left[\frac{1}{2}m^2+\frac{\lambda}{4}\right]\phi^2$$

plus constants where we have chosen $\chi=1.0$. Now we can construct the light-front Hamiltonian for this model and look for vanishing mass gap in the parameter space $0\leq\lambda\leq\infty$. Recall that the Hamiltonian is unbounded for $\lambda<0$. The expressions for ϕ^2 and ϕ^4 in the DLFQ are given in Ref. 1. The expression for ϕ^3 is given in Ref. 2. The mass operator can now connect states that differ in particle number by 0, 1, and 2. Thus we can search for vanishing mass gap already at $K=2$. At $K=2$ we have

$$\begin{vmatrix} m^2+\frac{\lambda}{2}-\alpha & \frac{\lambda}{\sqrt{4\pi}} \\ \frac{\lambda}{\sqrt{4\pi}} & 4\left[m^2+\frac{\lambda}{2}\right]+\frac{\lambda}{4\pi}-\alpha \end{vmatrix}=0\ .$$

Here α is the eigenvalue of the square of the mass operator M^2. For $m^2=1$ we get $\lambda_{\text{critical}}=-1.535$ or -2.7134. Thus the vacuum is stable for physical values of λ at $K=2$.

This is to be contrasted with an interaction Lagrangian

$$-\mathcal{L}_{\text{int}}=\tfrac{1}{2}m^2\phi^2+\frac{\lambda}{3!}\phi^3+\frac{\lambda}{4!}\phi^4$$

for which at $K=2$ we get

$$\begin{vmatrix} m^2-\alpha & \frac{\lambda}{\sqrt{4\pi}} \\ \frac{\lambda}{\sqrt{4\pi}} & 4m^2+\frac{\lambda}{4\pi}-\alpha \end{vmatrix}=0\ .$$

Now for $m^2=1.0$ the mass gap vanishes for $\lambda=7.66$.

The differences in the behavior of the mass gap for the two Lagrangians can be solely attributed to the presence of the coupling term $(\lambda/4)\phi^2$ in the first case which acts as a mass term.

It is of concern whether the behavior of the first Lagrangian for $K=2$ is maintained at higher K values since the continuum limit is given by $K\rightarrow\infty$. We have found that $\lambda_{\text{critical}}=-1.653$, -1.708, and -1.745 for $K=4$, 6, and 8, respectively, which is assuring for the correct behavior as K increases.

IV. CONCLUSIONS

We have investigated the stability of the vacuum in two interacting scalar field models in the DLFQ method. In the case of a vacuum instability our estimate of the critical coupling for vanishing mass gap with respect to the perturbative vacuum agrees with those from equal-time formulations. In the model where the vacuum is known to be stable the DLFQ method again provides a consistent result by yielding a nonvanishing mass gap.

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