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Brief Reports

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Lunar laser ranging and laboratory Eötvös-type experiments

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In order to fully utilize 1-cm-accuracy lunar laser ranging observations to test metric gravitational theory, laboratory Eötvös-type experiments which test the equality of free-fall for different materials, iron and silicate type, at the level $|\delta a/a| \leq 5\times 10^{-13}$ are required.

Lunar laser ranging can detect a very small difference between the acceleration rate of Earth and the Moon in the Sun's gravitational field. If these bodies are assumed to accelerate anomalously in a gravitational field,

$$\mathbf{a}_E = \left[\frac{M_G}{M_I}\right]_E \mathbf{g}_S \equiv (1+\Delta_E)\mathbf{g}_S \ , \tag{1a}$$

$$\mathbf{a}_M = \left[\frac{M_G}{M_I}\right]_M \mathbf{g}_S \equiv (1+\Delta_M)\mathbf{g}_S \ , \tag{1b}$$

then the Earth-Moon distance will be perturbed (polarized toward the Sun) by an amount[1]

$$\begin{aligned} x(t) &= g_S(\Delta_E-\Delta_M)\frac{3\omega-\Omega}{\Omega(\omega-\Omega)(2\omega-\Omega)}\cos(\omega-\Omega)t \\ &\simeq 1.83\times 10^{12}(\Delta_E-\Delta_M)\cos(\omega-\Omega)t \text{ cm} \ . \end{aligned} \tag{2}$$

M_G and M_I are the gravitational and inertial masses of the bodies, g_S is the Sun's gravitational acceleration field at Earth, ω is the Moon's sidereal angular rate around Earth, and Ω is Earth's sidereal angular rate around the Sun.

If almost two decades of observations using laser ranging between several Earth stations and lunar reflectors can place a 1-cm limit on experimental uncertainty in the amplitude of the Earth-Moon range of frequency $\omega-\Omega$ (Refs. 2–4) then an Eötvös-type experiment has been performed with these bodies which becomes the highest accuracy experiment of this type:

$$|\Delta_E-\Delta_M| \lesssim 5.5\times 10^{-13} \ . \tag{3}$$

Laboratory Eötvös-type experiments of recent times have quoted accuracies between 10^{-11} (Ref. 5) and 10^{-12} (Ref. 6).

The other interpretation of the observation that Earth and the Moon accelerate toward the Sun at a common rate is as a test of post-Newtonian gravity. In the parametrized post-Newtonian (PPN) theory of metric gravity, celestial bodies with appreciable gravitational self-energy will accelerate anomalously in a gravitational field[7,8]

$$\frac{M_G}{M_I} = 1+(4\beta-3-\gamma)\frac{U_G}{Mc^2} \ . \tag{4}$$

U_G is the gravitational self-energy of the body, Mc^2 is the body's total mass energy, and β and γ are two PPN coefficients which label post-Newtonian degrees of freedom of metric gravitational theories.[9] In Einstein's standard general-relativity metric theory of gravity $\gamma=\beta=1$.

Interpreting (3) as a test of gravitational theory, and using for the value of Earth's gravitational self-energy

$$(U_G/Mc^2)_E = 4.6\times 10^{-10} \ ,$$

one obtains the constraint on post-Newtonian gravity:

$$|4\beta-3-\gamma| \leq 1.2\times 10^{-3} \ . \tag{5}$$

Since γ has been measured by solar-system radar ranging experiments to be

$$|1-\gamma| \leq 1\times 10^{-3}$$

lunar laser ranging can be considered to be the β coefficient experiment (β parametrizes the nonlinearity of the gravitational theory):

$$|1-\beta| \leq 3\times 10^{-4} \ . \tag{6}$$

Our best constraint on the nonlinear structure of gravitational theory comes from the lunar ranging observations.

The purpose of this paper is to point out that the lunar laser ranging observations when improved to the 1-cm accuracy cannot simultaneously be used for both interpretations—as a constraint on the composition dependence of free-fall rates of bodies which occurs in nonmetric theories of gravity and as a test of post-Newtonian metric gravity. To fully utilize the lunar laser ranging data to test gravitational theory at these new levels of precision it is necessary that laboratory Eötvös-type experiments be performed at improved levels of accuracy so that the question of composition dependence of acceleration in gravitational fields (which could cancel gravitational self-energy effects) can be eliminated from the interpretation of the lunar laser ranging observations.

From the observed densities of Earth and the Moon, 5.52 and 3.34 g/cm^3, respectively, one can model the entire Moon and Earth's crust as silicate materials and Earth's core as iron. The ideal laboratory Eötvös-type experiment for our purposes would then be comparison of acceleration of iron and silicate materials at the $|\delta a/a| < 5\times 10^{-13}$ level. However, since most models of composition dependence of free-fall rates (dependence on the neutron/proton ratio, nuclear-binding energy or nuclear electrostatic energy, etc.) vary continuously with atomic number, laboratory experiments which compare aluminum and gold whose atomic numbers differ by a factor of 4 or 5 greater than Earth and the Moon's mean atomic number difference, are adequate at the level $|\delta a/a| < 2\times 10^{-12}$, if the model assumptions mentioned above are accepted.

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[4] A private communication from C. O. Alley indicates that a new analysis of lunar laser ranging data to date may permit setting a 1-cm accuracy on the appropriate range amplitude has prompted this paper.

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[9] We neglect in (4) other PPN coefficients which have been constrained by other experiments and observations to be insignificant for the purposes of this observation.