Perturbative tests of a lattice fermion proposal of Quinn and Weinstein

Geoffrey T. Bodwin
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

Eve V. Kovacs
Department of Physics, University of Illinois at Chicago, Chicago, Illinois 60680
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We discuss a proposal of Quinn and Weinstein for incorporating fermions into lattice gauge theory and analyze it in the context of weak-coupling perturbation theory. In two dimensions we find that, because of a special property of the Hamiltonian formulation, one recovers the physics of the continuum theory as the lattice spacing tends to zero. In four dimensions we find that the transverse photon develops a quadratically divergent mass. Consequently, in order to obtain a satisfactory continuum limit, one would need to augment the Quinn-Weinstein proposal through the inclusion of additional counterterms. We argue that the construction of such counterterms would entail at least a partial breaking of the chiral symmetry.

Recently, Quinn and Weinstein[1] proposed a scheme for eliminating fermion species doubling in lattice gauge theory, while preserving chiral symmetry. Their proposal is based on a Hamiltonian formulation in which the gauge has been fixed according to the condition $A_0=0$. In addition, the scheme makes use of a nonlocal lattice derivative in the fermion Hamiltonian. The crucial new feature in the Quinn-Weinstein approach is the inclusion of terms in the interaction Hamiltonian that do not derive from the form of the lattice fermion derivative. In this paper we test the suitability of this feature through explicit one-loop calculations in two and four dimensions. Some of the results derived in this paper have been summarized elsewhere.[2]

In the case of a non-Abelian gauge theory, the Quinn-Weinstein Hamiltonian is quite complicated, involving line integrals of the gauge field over all possible paths between the lattice points on which the fermion fields reside. In order to avoid these complications we specialize in this paper to the formulation that Quinn and Weinstein give for the noncompact Abelian theory (QED), as defined in Eq. (4.11) of Ref. 1. In the remainder of this paper, when we use the terms "Quinn-Weinstein proposal" or "Quinn-Weinstein formulation," we are referring to this particular noncompact formulation of QED.

One can think of the Hamiltonian formulation of lattice gauge theory as a Lagrangian formulation in which time is continuous—with the understanding that, in evaluating Feynman amplitudes, one always carries out the integration over the time components of the momenta first. We find it convenient to adopt this procedure. Hence, in the lattice amplitudes, the time components of the loop momenta in the Feynman integrals range from $-\infty$ to ∞ and the dependences of the propagators and vertices on the time components of momenta are given by the continuum expressions. All of the effects of the lattice appear in the treatment of the spatial components of the momenta. The spatial components of the momenta generally range from $-\pi/a$ to $+\pi/a$; the propagators and vertices depend on these spatial components in a way that is governed by the particular choice of lattice formulation.

In conventional formulations of lattice QED, the choice of fermion propagator determines the form of the fermion-photon vertices. That is, once one has chosen an expression for the free fermion Hamiltonian, the principle of minimal substitution, with gauge-field link variables connecting the fermion operators in fermion bilinears along paths of minimum length, determines the form of interactions. Let us denote the μth component of the Fourier transform of the fermion derivative operator by $D_\mu(k)$, so that the lattice fermion propagator is given by

$$S_F(k)=\left[\sum_\mu \gamma_\mu D_\mu(k)+m\right]^{-1} . \tag{1}$$

If $D_\mu(k)$ depends only on the μth component of k, then minimal substitution leads to the following relationships between the fermion propagator and the fermion-photon vertices through order e^2:

$$V_\mu^{(1)}(k,l)=-e\gamma_\mu \frac{D_\mu(k+l)-D_\mu(k)}{(2/a)\sin(\frac{1}{2}l_\mu a)} , \tag{2a}$$

$$V_{\mu\nu}^{(2)}(k,l_1,l_2)=-e^2\delta_{\mu\nu}\gamma_\mu \frac{D_\mu(k+l_1+l_2)-D_\mu(k+l_2)-D_\mu(k+l_1)+D_\mu(k)}{(2/a)\sin(\frac{1}{2}l_{1\mu}a)(2/a)\sin(\frac{1}{2}l_{2\mu}a)} , \tag{2b}$$

where $V_\mu^{(1)}(k,l)$ is the one-photon–fermion vertex, $V_\mu^{(2)}(k,l_1,l_2)$ is the two-photon–fermion "seagull" vertex, k is the incoming fermion momentum, the l's are the photon momenta, and μ and ν are the indices that carry the photons' polarizations. Similar relationships hold for the higher-order seagull vertices.[3]

The relationship between the fermion propagator and the interaction vertices is a severe constraint on the form of the fully interacting theory and has important consequences with regard to the phenomenon of spectrum doubling. In order to see this, consider the calculation of some amplitude, such as the vacuum polarization, that involves fermion loops. In general, the expression to be evaluated contains factors of the fermion propagator in the denominator and fermion-photon vertices in the numerator. Detailed power-counting arguments[3] show that, once one has made subtractions to remove the usual ultraviolet divergences, the leading contributions in the limit $a \to 0$ come from the linear regions in the neighborhoods of points at which all of the components of the lattice fermion derivative $D(k)$ vanish. We call these points the zeros of $D(k)$, and denote the ith zero by $\bar{k}_i$. [Here, we are assuming that $D(k)$ is a smooth function of the momentum. Discontinuities in $D(k)$ bring in some additional subtleties.[3]] Of course, all of the components of $D(k)$ vanish at $k=0$. It is the appearance of additional zeros of $D(k)$ for $k \neq 0$ that leads to the doubling phenomenon. For k near $\bar{k}_i$, we may approximate the fermion propagator as

$$\frac{1}{\sum_\mu \gamma_\mu D_\mu(k)+m} \approx \frac{1}{\sum_\mu c_{i\mu}\gamma_\mu(k_\mu-\bar{k}_{i\mu})+m} , \tag{3a}$$

where

$$c_{i\mu}=D'_\mu(k)\,|_{k=\bar{k}_i} . \tag{3b}$$

The coefficients $c_{i\mu}$ are related to the velocities of light for the fermion species associated with the $\bar{k}_i$. One might think that the contributions from the extra zeros in $D(k)$ could be suppressed merely by taking the coefficient $c_{i\mu}$ to infinity. However, because of relationship (2a), for k near $\bar{k}_i$ the photon-fermion vertex is given by

$$V_\mu^{(1)}(k,l) \approx -e\gamma_\mu c_{i\mu} . \tag{4}$$

Thus, numerator factors of $c_{i\mu}$ from the vertices tend to compensate for denominator factors of $c_{i\mu}$ from the propagators, and we see that the additional species can, in principle, contribute to the amplitudes regardless of their velocities of light.

In order to try to avoid this phenomenon of compensating numerator and denominator coefficients, Quinn and Weinstein abandon the minimal substitution approach that leads to (2a) and (2b). As a first step they specialize to the gauge $A_0=0$, so that the time component of the gauge field is given by the usual Gauss-law constraint equation. Only the spatial components of the fermion-photon vertices couple to a dynamical photon, which is transverse. Quinn and Weinstein assert that the forms of these spatial vertices can be chosen independently of the form of the fermion propagator. That is, they claim that the interactions need not necessarily derive from the simplest gauge-invariant combinations of fermion bilinears and link variables that one obtains from minimal substitution. In particular, they choose the vertices to be those that one would usually obtain from the "naive" lattice fermion derivative with minimal substitution:

$$V_\mu^{(1)\mathrm{QW}}(k,l)=-e\gamma_\mu\cos(k_\mu+\tfrac{1}{2}l_\mu)a , \tag{5a}$$

$$V_{\mu\nu}^{(2)\mathrm{QW}}(k,l_1,l_2)=ae^2\delta_{\mu\nu}\gamma_\mu\sin(k_\mu+\tfrac{1}{2}l_{1\mu}+\tfrac{1}{2}l_{2\mu})a , \tag{5b}$$

etc. Thus, the coefficients $c_{i\mu}$ from the vertices are all equal to unity. One could then suppress the contributions from the extra zeros in the propagator denominators merely by choosing, for the propagators, a derivative function whose slope tends to infinity near the extra zeros. As a concrete example, Quinn and Weinstein suggest the "damped SLAC" lattice derivative, which is given in configuration space by

$$D_\mu^{\mathrm{QW}}(x)=\frac{(-1)^{(x_\mu/a)}}{a^2x_\mu}e^{-(|x_\mu|/b)} . \tag{6}$$

The parameter b controls the damping of the function. In the limit $b/a \to \infty$, $D_\mu^{\mathrm{QW}}(x)$ becomes the infinite-range Drell-Weinstein-Yankielowicz[4] derivative. Taking the one-dimensional Fourier transform of (6), we obtain $D_\mu^{\mathrm{QW}}(k)$, which is approximately equal to k_μ for $|k_\mu| \lesssim (\pi/a)-(1/b)$. As k_μ comes within order $1/b$ of $\pm\pi/a$, $D_\mu(k)$ turns over and passes through zero at $|k_\mu|=\pi/a$ with a negative slope of order b/a. Thus, by taking the limit $b/a \to \infty$, one can suppress the contributions associated with the extra zeros of $D^{\mathrm{QW}}(k)$.

At first sight, the Quinn-Weinstein choice of propagators and vertices would seem to represent a satisfactory lattice transcription. The propagators and vertices go over to the continuum expressions if one simply takes the limit $a \to 0$ without regard to any integrations over loop momenta. Also, since the current to which the transverse photon couples is itself transverse, the Hamiltonian is manifestly invariant with respect to the residual set of gauge transformations that respect the condition $A_0=0$. However, as we have already noted, the Quinn-Weinstein vertices do not satisfy the usual vector Ward

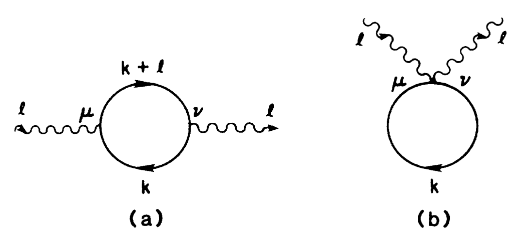

FIG. 1. The Feynman graphs that give (a) the nonseagull contribution and (b) the seagull contribution to the vacuum polarization in $O(e^2)$. A solid line represents a fermion propagator and a wavy line represents a photon propagator.

identities that derive from the lattice generalization of minimal substitution. For example, the Feynman identity (2a) does not hold. Recall that it is these Ward identities that guarantee four-vector current conservation, which in turn allows one to see that the actual degree of divergence of the vacuum polarization is not as high as the superficial degree of divergence that one obtains from simple power counting. For example, by application of four-vector current conservation, it follows that the actual degree of divergence of the vacuum polarization is only logarithmic, not quadratic, in four dimensions and that the vacuum polarization is convergent, not logarithmically divergent, in two dimensions. In the Quinn-Weinstein formulation one might worry, then, that the vacuum polarization would show the superficial degree of divergence that one obtains by simple power counting. We investigate this possibility further by explicit calculations of the vacuum polarization $\Pi_{\mu\nu}$ in two and four dimensions.

For a lattice theory in D dimensions there are, in $O(e^2)$, two graphs that contribute to $\Pi_{\mu\nu}$. These graphs, shown in Fig. 1, are (a) the nonseagull graph, whose amplitude we denote by $N_{\mu\nu}$ and (b) the seagull graph, whose amplitude we denote by $S_{\mu\nu}$:

$$N_{\mu\nu}(l)=-\mathrm{Tr}\,e^2\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\sum_\sigma\gamma_\sigma D_\sigma(k+l)}V_\mu^{(1)}(k,l)\frac{1}{\sum_\sigma\gamma_\sigma D_\sigma(k)}V_\nu^{(1)}(k+l,-l) \tag{7a}$$

and

$$S_{\mu\nu}(l)=-\mathrm{Tr}\,e^2\int\frac{d^Dk}{(2\pi)^D}\frac{1}{\sum_\sigma\gamma_\sigma D_\sigma(k)}V_{\mu\nu}^{(2)}(k,l,-l)\ , \tag{7b}$$

where l is the external momentum, and we have set the fermion mass to zero. In order to evaluate these amplitudes in the Quinn-Weinstein formulation, we substitute the appropriate temporal or spatial Feynman rules for the propagators and vertices. We will treat the cases of two and four dimensions separately.

In two dimensions, it is a simple matter to carry out a complete analytic evaluation of $\Pi_{\mu\nu}^{\mathrm{QW}}(l)$ in the limit $a\to0$. We consider first the nonseagull part $N_{\mu\nu}^{\mathrm{QW}}(l)$. Power-counting arguments show that the small regions of integration $|k_1\pm\pi/a|\lesssim(1/b)$ yield contributions that are suppressed by at least one factor of a/b relative to the leading contribution, so we can discard these regions. For k_1 outside the small regions we can linearize the spatial paths of the propagator denominators by using

$$D_\mu^{\mathrm{QW}}(k_\mu)\approx k_\mu\ . \tag{8}$$

Then, for the approximate form of the integrand that follows from (8), we can extend the range of integration to $k_1=\pm\pi/a$, making a relative error of order a/b. Next, we rationalize the fermion denominators and use Feynman parameters to obtain an expression for $N_{\mu\nu}^{\mathrm{QW}}(l)$ that is already quite similar to the continuum expression for $\Pi_{\mu\nu}(l)$:

$$N_{\mu\nu}^{\mathrm{QW}}(l)=-\mathrm{Tr}\frac{e^2}{(2\pi)^2}\int_0^1dx\int_{-\pi/a}^{\pi/a}dk_1\int_{-\infty}^{\infty}dk_0\frac{(\not k+\not l)V_\mu^{(1)\mathrm{QW}}(k,l)\not k V_\nu^{(1)\mathrm{QW}}(k+l,-l)}{(k^2+2xk\cdot l+xl^2)^2}\ . \tag{9}$$

Now we perform the usual shift of integration variables to eliminate the cross term in the denominator. Since (9) is, at worst, logarithmically divergent, we can ignore the shift in the limits of integration. Then, evaluating the numerator traces and retaining only terms even in k, we obtain

$$N_{\mu\nu}^{\mathrm{QW}}(l)=\frac{e^2}{(2\pi)^2}\int_0^1dx\int_{-\pi/a}^{\pi/a}dk_1\int_{-\infty}^{\infty}dk_0\frac{2[2k_\mu k_\nu-k^2\delta_{\mu\nu}-x(1-x)(2l_\mu l_\nu-l^2\delta_{\mu\nu})]}{[k^2+x(1-x)l^2]^2}$$
$$\times C_\mu(ka+(\tfrac{1}{2}-x)la)C_\nu(ka+(\tfrac{1}{2}-x)la)\ , \tag{10}$$

where

$$C_\mu(ka)=\begin{cases}1 & \text{for } \mu=0\ ,\\ \cos k_\mu a & \text{for } \mu\neq0\ .\end{cases} \tag{11}$$

For the numerator terms in (10) that contain no powers of k the integrals are convergent. In the continuum limit the dominant contribution for this part of the amplitude comes from k of order l, so the arguments of the C_μ's vanish. Hence, we set the C_μ's equal to unity, obtaining the usual continuum result

$$[N_{\mu\nu}^{\mathrm{QW}}(l)]_{\text{no }k\text{'s}}=-\frac{e^2}{2\pi}\left[\frac{-2l_\mu l_\nu}{l^2}+\delta_{\mu\nu}\right]\ . \tag{12}$$

For the terms in (10) with two powers of k in the numerator, it appears that the contribution is logarithmically divergent. In a regulator scheme that respects four-vector current conservation, one can use the fact that $\sum_\sigma l_\sigma \Pi_{\sigma\nu}(l)=0$ to show that the sum of the seagull and nonseagull contributions to $\Pi_{\mu\nu}(l)$ is actually convergent. As we have already mentioned, relationships such as (2a) and (2b) do not hold in the Quinn-Weinstein case, and, hence, we have no *a priori* assurance that the amplitude is finite. However, if we simply follow our prescription for the Hamiltonian formulation and proceed with the k_0 integration in (10) for this part of the amplitude, then it turns out that the remaining integration over k_1 is finite. This fortuitous occurrence appears to be a quirk of two dimensions and, as we shall see, it is not repeated in four dimensions. A straightforward contour integration over k_0 yields

$$[N_{\mu\nu}^{\rm QW}(l)]_{\text{two } k\text{'s}} = -(-1)^\mu \frac{e^2}{2\pi} \int_0^1 dx \int_{-\pi/a}^{\pi/a} dk_1 \frac{x(1-x)l^2 C_\mu^2(ka+(\frac{1}{2}-x)la)\delta_{\mu\nu}}{2[k_1^2+x(1-x)l^2]^{3/2}} \ . \tag{13}$$

This integral is manifestly convergent and dominated by k of order l. Thus, to obtain the leading contribution in the limit $a \to 0$, we can set the C_μ's equal to unity. The remaining integral is easily evaluated and yields

$$[N_{\mu\nu}^{\rm QW}(l)]_{\text{two } k\text{'s}} = -(-1)^\mu (e^2/2\pi)\delta_{\mu\nu} \ . \tag{14}$$

We now consider the seagull contributions. Since, in the Hamiltonian formulation, the seagull vertices are identically zero when any of their indices are temporal, there is no contribution $S_{00}(l)$. For $S_{11}(l)$, we follow the same procedure as in the nonseagull case and arrive at the result

$$S_{11}^{\rm QW}(l) = -\frac{e^2}{(2\pi)^2} \int_{-\pi/a}^{\pi/a} dk_1 \int_{-\infty}^{\infty} dk_0 \frac{2ak_1 \sin k_1 a}{k_0^2+k_1^2} \ . \tag{15}$$

The k_0 and k_1 integrations are easily performed and yield

$$S_{11}^{\rm QW}(l) = -2e^2/\pi \ . \tag{16}$$

We now collect the various contributions to $\Pi_{\mu\nu}^{\rm QW}(l)$, which are given in (12), (14), and (16):

$$\Pi_{\mu\nu}^{\rm QW}(l) = \begin{cases} -\dfrac{e^2}{\pi}\left[2\delta_{\mu\nu} - \dfrac{l_\mu l_\nu}{l^2}\right] & \text{for } \mu \text{ and } \nu=1 \ ; \\[2ex] -\dfrac{e^2}{\pi}\left[\delta_{\mu\nu} - \dfrac{l_\mu l_\nu}{l^2}\right] & \text{otherwise} \ . \end{cases} \tag{17}$$

From (17) we see that in the Quinn-Weinstein formulation one obtains the usual continuum results for Π_{00}, Π_{01}, and Π_{10}. For the purely spatial component Π_{11} the coefficient of $\delta_{\mu\nu}$ is twice as large in the Quinn-Weinstein formulation as in the continuum theory. However, this has no physical consequences, since, in $A_0=0$ gauge in two dimensions, there is no transverse photon to couple to the spatial components of the vacuum polarization. Note that the divergence of the vacuum polarization is nonzero, as we had anticipated from the fact that the vector Ward identities do not hold in the Quinn-Weinstein formulation.

Now let us turn to the vacuum polarization in four dimensions. Here we do not wish to carry out a complete calculation. Instead, we examine only the most divergent parts of the Feynman integrals, namely, the parts that go as a^{-2}, in order to see if the usual cancellation of these terms persists in the Quinn-Weinstein formulation. Thus, in the Feynman integrands, we drop all dependence on the external momentum l. As in two dimensions, the small regions of integration $|k_i \pm \pi/a| \lesssim (1/b)$ give a contribution that is suppressed by a factor of a/b relative to the leading contribution, so we can drop that part of the integration. If we were attempting to calculate the subleading divergences and the finite part, greater care would be required here. The approximation (8) is valid outside the small regions of integration, and, for the approximate form of the integrand, we can extend the range of integration to $k_i = \pm\pi/a$, making a relative error of order a/b. Hence, after performing the usual manipulations, we find that the leading UV divergence for the nonseagull graph is given by

$$N_{\mu\nu}^{\rm QW}(\text{UV}) = -e^2 \int_{-\pi/a}^{\pi/a} \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk_0}{(2\pi)} \frac{4(2k_\mu k_\nu - k^2\delta_{\mu\nu})C_\mu(ka)C_\nu(ka)}{(k_0^2+\mathbf{k}^2)^2} \ . \tag{18}$$

(Our convention for this discussion in four dimensions is to use Greek letters to denote indices that run from 0 to 3, Latin letters to denote indices that run from 1 to 3, and boldface letters, denote three-vectors.) Using the symmetry of the integrand under $k_\mu \to -k_\mu$, we see that in the numerator of the integrand we can make the replacement

$$k_\mu k_\nu \to k_\mu^2 \delta_{\mu\nu} \ . \tag{19}$$

Thus $N_{\mu\nu}^{\rm QW}(\text{UV})$ is proportional to $\delta_{\mu\nu}$. It is now a simple matter to carry out the k_0 integration by contour methods. For the temporal component we find that

$$N_{00}^{\rm QW}({\rm UV})=0\ . \tag{20}$$

That is, the leading divergent cancels for the temporal component, just as in the two-dimensional case. For the spatial components we have

$$N_{ij}^{\rm QW}({\rm UV})=-e^2\delta_{ij}\int_{-\pi/a}^{\pi/a}\frac{d^3k}{(2\pi)^3}\frac{2(k_i^2-\mathbf{k}^2)\cos^2(k_ia)}{(\mathbf{k}^2)^{3/2}}\ . \tag{21}$$

The integrand is negative definite, and the integral is quadratically divergent. One can analyze the leading divergence in the seagull graph in a similar fashion (of course, there is no seagull contribution to $\Pi_{00}^{\rm QW}$):

$$S_{ij}^{\rm QW}({\rm UV})=-e^2a\delta_{ij}\int_{-\pi/a}^{\pi/a}\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk_0}{(2\pi)}\frac{4k_i\sin(k_ia)}{k_0^2+\mathbf{k}^2}=-e^2a\delta_{ij}\int_{-\pi/a}^{\pi/a}\frac{d^3k}{(2\pi)^3}\frac{2k_i\sin(k_ia)}{(\mathbf{k}^2)^{1/2}}\ . \tag{22}$$

In this case the integrand is positive definite and the integral is quadratically divergent. Combining the (21) and (22) and making a change of variables to scale the lattice spacing a out of the integral, we obtain the leading UV divergence for the spatial components of the complete vacuum polarization:

$$\Pi_{ij}^{\rm QW}({\rm UV})=\frac{e^2}{4\pi^3}\delta_{ij}\frac{1}{a^2}\int_{-\pi}^{\pi}d^3k\left[\frac{(\mathbf{k}^2-k_i^2)\cos^2k_i}{(\mathbf{k}^2)^{3/2}}-\frac{k_i\sin k_i}{(\mathbf{k}^2)^{1/2}}\right]\ . \tag{23a}$$

The second term in the integrand of (23a), which comes from the seagull contribution, tends to cancel the first term, which comes from the nonseagull contribution. Indeed, if we had been computing in a conventional lattice scheme, these terms would have canceled precisely. Consider, for example, the effect of using the naive derivative, rather than the Quinn-Weinstein derivative. Everywhere in the integrand, k_i and k_j would be replaced by $\sin k_i$ and $\sin k_j$, respectively. Then, one could show, by integrating the seagull term by parts, that the seagull and nonseagull contributions would cancel. In the Quinn-Weinstein expression, the cancellation is incomplete; numerical evaluation of the integral yields the result

$$\Pi_{ij}^{\rm QW}({\rm UV})\approx\frac{e^2}{4\pi^3}\delta_{ij}\frac{1}{a^2}(-51.4)\ . \tag{23b}$$

Thus, the spatial components of the vacuum polarization develop a leading divergence that is quadratic, rather than logarithmic. Usually one can associate the leading divergence in the vacuum polarization with a multiplicative renormalization of the coupling constant. However, the presence of a dimensionful (quadratic) divergence precludes this possibility. Furthermore, the divergence is not an unphysical artifact: $\Pi_{ij}^{\rm QW}({\rm UV})$ has a transverse part, which is proportional to $\delta_{ij}-k_ik_j/\mathbf{k}^2$, and hence couples to the transverse photon. Thus, one must interpret (23) as giving rise to a quadratically divergent mass for the dynamical transverse photon.

We now summarize our results and conclusions for the calculations presented in this paper. We have investigated the vacumn polarization in the Quinn-Weinstein formulation by direct calculation in both two and four dimensions. In two dimensions the vacuum polarization actually turns out to be finite, rather than logarithmically divergent. The temporal components reproduce the continuum result. For the spatial components, the term proportional to δ_{ij} appears with a coefficient that is twice the continuum one. However, this is of no physical consequence, since the Quinn-Weinstein approach is formulated in $A_0=0$ gauge: in two dimensions there is no transverse photon to couple to the spatial components of the current. The correctness of the two-dimensional result seems to rely on properties of the Hamiltonian formulation that are peculiar to two dimensions. Thus, we have investigated the four-dimensional theory as well. There we have computed the most divergent part of the spatial components of the vacuum polarization and find that it is given by

$$\Pi_{ij}^{\rm QW}\sim\delta_{ij}a^{-2} \tag{24}$$

in the limit $a\to 0$. This divergent quantity leads to a mass renormalization for the transverse photon. Hence, one would not obtain continuum electrodynamics in the limit $a\to 0$. One could, in principle, recover the correct continuum limit by adding to the Hamiltonian a Pauli-Villars fermion, whose mass would tend to infinity as $a\to 0$. Of course, this would defeat the original purpose of the Quinn-Weinstein proposal by explicitly breaking the chiral invariance of the full Hamiltonian. However, even in the presence of a Pauli-Villars fermion, the part of the Hamiltonian corresponding to the light fermion would possess a γ_5 invariance. This invariance would protect $\bar\psi\psi$, the chiral order parameter of the light fermion, from developing a vacuum expectation value in perturbation theory. Consequently, the Quinn-Weinstein scheme with a Pauli-Villars fermions might be useful in studying the spontaneous breaking of chiral symmetry. However, the presence of a massive fermion would not be compatible with the full set of Ward identities of a theory of chiral fermions, such as the electroweak theory.

As an alternative to the Pauli-Villars scheme, one could introduce a photon-mass counterterm, which would be adjusted in the limit $a\to 0$ so as to keep the photon massless. Presumably, in the absence of the usual vector Ward identities, it would also be necessary to include a counterterm corresponding to the logarithmic divergence in the light-by-light scattering graph and a counterterm corresponding to any subleading divergence in the vacuum-polarization graph that could not be absorbed into the coupling-constant renormalization. An investigation of axial currents would require the introduction of counterterms to control divergences in the graphs with axial-vector vertices as well. The choice of counterterms would be constrained by the requirement that they restore the usual vector Ward identities (four-vector current conservation). On the basis of well-known results from continuum physics,[5] we would expect any counterterm scheme that respects the vector Ward identities to lead to at least a partial breakdown of chiral symmetry. For example, one could satisfy the vector Ward identities by defining the counterterms through subtractions at zero external momentum. Then, in order to control infrared divergences in the fermion loops, one would need to introduce a fermion mass, which would explicitly break the chiral symmetry. (In fact, such a counterterm procedure would be equivalent to Pauli-Villars regularization, since in an infinite-mass Pauli-Villars loop one can always neglect the external momenta.) In practice, the implementation of a counterterm program in connection with a numerical simulation would require the adjustment, as the continuum limit is approached, of counterterm coefficients that compensate for the quadratic divergence in the photon mass and for the subleading divergences. Since an explicit fermion mass term, such as a Wilson term, leads only to a linear divergence in the continuum limit, it appears that the Quinn-Weinstein approach would offer no computational advantage over the Wilson formulation in numerical simulations.

In general we expect the phenomenon of a quadratically divergent photon-mass renormalization in four dimensions to appear whenever one constructs a lattice theory whose propagators and vertices do not respect vector-current conservation at the four-vector level. One need not employ strict minimal substitution in order to achieve four-vector current conservation. For example, even if one does not choose straight-line paths for the links that connect fermion fields in fermion bilinears, the usual vector Ward identities still hold, and one obtains four-vector current conservation.[6] The crucial point is that there is no guarantee of four-vector current conservation if one introduces couplings to the gauge field by the *ad hoc* inclusion of various interaction terms. It is clear, from our present investigation, that if one attempts to solve the fermion doubling problem by modifying the principles that usually constrain the structure of a gauged field theory, then one runs the risk of losing the properties that make gauge theories attractive in the first place.

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