Cosmic texture and the microwave background

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(Received 20 March 1987)

The isotropic cosmic microwave background may come from the antipode of a spatially closed, but $\Omega < 1$ textured universe.

I. FOREWORD

In a previous paper I described the special properties of cosmological textures.[1] Here I will demonstrate how a textured universe can offer a new interpretation of a long-standing cosmological puzzle: the isotropy of the cosmic microwave background (CMB).

There are two aspects of this puzzle. One is that the isotropy involves correlations between spacetime regions which have never been in causal contact, otherwise known as the horizon problem. The CMB hitting us is said to have originated on a receding spherical-surface now $\sim 15\times 10^9$ light years away, the surface of last scattering, Fig. 1. It was emitted when light decoupled from an ion plasma at $t_{\rm dec}\sim 4\times 10^5$ yr after the hot bang. Letting $t_0\sim 15\times 10^9$ yr denote the present age of the Universe, then points which were in causal contact at decoupling could not have a presently observed angular separation greater than $\delta\theta\sim 2(t_{\rm dec}/t_0)^{1/3}\sim 3°$, but at angles larger than this it is known that $\delta T/T<3\times 10^{-5}$. The observed isotropy of the CMB therefore violates the very basic notion of causality. At present, the only satisfactory solution to this horizon problem is the inflation hypothesis.[2–4] One of several problems with inflation is that it predicts a perfectly flat universe, $\Omega_0=1$, while the age and the observed mass density of the Universe are consistent with $\Omega_0\approx 0.2$ (Ref. 5).

The other side of the CMB isotropy puzzle is that fluctuations in the baryon density at decoupling are expected, from both grand-unified-theory (GUT) baryosynthesis and inflation, to follow the photon temperature fluctuations adiabatically. The observed upper bound on $\delta T/T$ constrains the adiabatic baryon fluctuations to be too small to account for the observed large-scale structure of the Universe.[6,7] Largely for this reason, it is widely accepted that the dark matter is nonbaryonic.

Here I propose that the CMB hitting us from all directions originated in the same causally connected region, on the other side of a spatially closed, textured universe. This solves the horizon problem. Other benefits are that an $\Omega_0\approx 0.2$ universe is perfectly admissible, and the constraint on adiabatic baryon fluctuations at decoupling is relaxed.

After reviewing briefly the meaning of texture in Sec. II, in Sec. III I use the texture hypothesis and the observational fact of isotropy to calculate the radius of our Universe. In Sec. IV, I conclude with various comments.

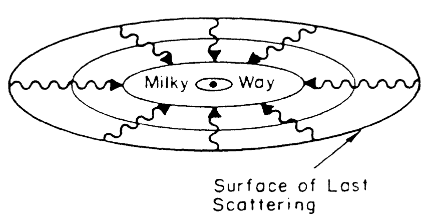

FIG. 1. Surface of last scattering.

II. GLOBAL TEXTURES IN COSMOLOGY

A texture is a stable solution to the classical field equations of a field theory with a spontaneously broken non-Abelian global symmetry on a three-sphere, where the group G and its unbroken subgroup H have the property that $\pi_3(G/H)\neq 1$. This somewhat dizzying statement is explained in Ref. 1.

Here we need only know that a texture contributes a winding energy density $\rho\omega$ to Einstein's equation for a closed, spatially homogeneous universe. $\rho\omega$ red-shifts as $\sim a^{-2}(t)$, where $a(t)$ is the radius of the three-sphere. Einsteins's equation with zero cosmological constant becomes

$$\left(\frac{\dot a}{a}\right)-\frac{8\pi G}{3}(\rho+\rho_\omega)=-\frac{1}{a^2}$$

or

$$\left(\frac{\dot a}{a}\right)^2\frac{8\pi G}{3}\rho=\frac{\gamma-1}{a^2}\ ,$$

where ρ is the ordinary energy density of radiation and matter and γ is the model-dependent *texture constant.* Making the usual definitions of the Hubble parameter $(H\equiv\dot a/a)$ and Ω $(\Omega\equiv\rho/\rho_{\rm CT}\equiv 8\pi G\rho/3H^2)$, this can be written as

$$H^2(1-\Omega)=\frac{\gamma-1}{a^2}\ , \tag{2.1}$$

which shows clearly that in this closed, textured universe it is possible to have $\Omega<1$ as long as $\gamma>1$. A value of $\gamma\sim 1$ corresponds to a symmetry breaking at the Planck scale. Since this is the scale at which quantum gravity effects are important, and very little is known about them, it cannot be stressed enough that this effect is highly conjectural.

III. THE POINT OF LAST SCATTERING

Though it is quite proper to be wary, let us suspend our disbelief for the moment and explore the consequences of texture. For most sizes of the Universe the horizon problem is still there, but for the spatial three-sphere in which the CMB decoupled at the antipode the radiation would be naturally isotropic (Fig. 2). The apparent violation of causality is not real because the radiation hitting us from all directions actually came from the same spatial region. In fact, given the texture hypothesis, one can take the view that the isotropy of the CMB gives us all the information we need to calculate the size of the Universe, which is done below, for both $\Omega_0=1$ and $\Omega_0<1$.

In the following we parametrize the three-sphere with three angular coordinates ϵ,θ,ϕ, and use the metric

$$d\tau=dt^2-a^2(t)[d\epsilon^2+\sin^2\epsilon(d\theta^2+\sin^2\theta\, d\phi^2)] ,$$

$$0\le\epsilon\le\pi,\quad 0\le\theta\le\pi,\quad 0\le\phi\le 2\pi .$$

It is also assumed that matter has dominated over radiation since decoupling.

A. The $\Omega_0=1$, $\gamma=1$ universe

The scale factor evolves as

$$a(t)=a_0\left[\frac{t}{t_0}\right]^{2/3} . \tag{3.1}$$

Taking the Sun's position in the Hubble flow, corrected for its peculiar velocity, as the point where $\epsilon=0$, then the light which decoupled at $t_{\rm dec}$, will have come from the antipode if

$$\pi=\int_{t_{\rm dec}}^{t_0}\frac{dt}{a(t)}=\frac{3t_0}{a_0}[1-(t_{\rm dec}/t_0)^{1/3}] .$$

The present size of our Universe then must be

$$a_0=\frac{3t_0}{\pi}[1-(t_{\rm dec}/t_0)^{1/3}] .$$

Now it is reasonable to ask, how sensitive is the isotropy to this radius? At decoupling, the size of the causal horizon in ϵ units is

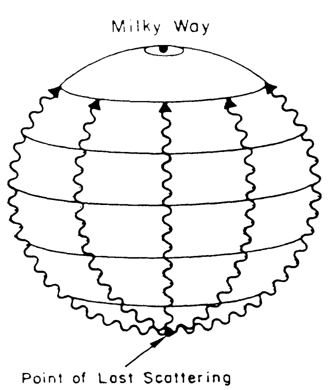

FIG. 2. Point of last scattering.

$$\int_0^{\epsilon_{\rm dec}}d\epsilon=\int_0^{t_{\rm dec}}\frac{dt}{a(t)}=\frac{3t_0}{a_0}(t_{\rm dec}/t_0)^{1/3} ,$$

and since we can expect the CMB to be thermalized on this scale at decoupling, it will be isotropic when measured today if

$$a_0=\frac{3t_0}{\pi}[1-(t_{\rm dec}/t_0)^{1/3}\pm(t_{\rm dec}/t_0)^{1/3}] .$$

Finally, using the empirical value $H_0=(h/9.78\times10^9\ \rm yr)$, with $\frac{1}{2}<h<1$, and $t_{\rm dec}=4\times10^5$ yr, the radius becomes

$$\frac{5.9}{h}\lesssim\frac{a_0}{10^9\ \text{light years}}\lesssim\frac{6.2}{h} .$$

From Eq. (3.1), the time interval over which the CMB remains isotropic is

$$\Delta t=\frac{\pi}{2}\Delta a\approx\frac{0.4\times10^9}{h}\ \text{yr} .$$

As the Universe was coming into, or as it will pass out of, this interval, the nonisotropy would exist only at large angles.

It is important to note that $\Omega=1$ is a very special case, requiring $\gamma=1$ exactly and leaving a_0 undetermined by Eq. (2.1). There seems to be no physical reason that a_0 should have the above value, so the isotropy appears to be merely a result of chance. It will be somewhat different in the next example.

B. The $\Omega_0<1$, $\gamma>1$ universe

First, I would like to put forward an argument that the value of a_0 is not as arbitrary as in the previous example. Equation (2.1) implies

$$aH=\left[\frac{\gamma-1}{1-\Omega}\right]^{1/2} . \tag{3.2}$$

Let us assume that texture formation at the Planck scale gives $\gamma-1\sim1$. From this it follows that early in the history of the Universe, when $1-\Omega\ll1$, we had $a\gg1/H$, but as $1-\Omega\to1$, we have $a\to1/H\sim t$. The assumption on $\gamma-1$ is therefore enough to deduce that a must evolve to a value which is reasonably close to that which gives an isotropic CMB.

Now we will calculate what γ and a_0 must be to explain the present isotropy. The solution to Eq. (2.1) is

$$a(\beta)=\frac{H_0{}^2\Omega_0a_0{}^3}{2(\gamma-1)}[\cosh(\beta\sqrt{\gamma-1})-1] , \tag{3.3}$$

$$t(\beta)=\frac{H_0{}^2\Omega_0a_0{}^3}{2(\gamma-1)^{3/2}}[\sinh(\beta\sqrt{\gamma-1})-\beta\sqrt{\gamma-1}] , \tag{3.4}$$

from which Eq. (3.1) can be recovered in the limit $\gamma\to1$. As before, the CMB will have come from the antipode if

$$\pi=\int_0^{\pi}d\epsilon=\int_{t_{\rm dec}}^{t_0}\frac{dt}{a(t)}=\beta_0-\beta_{\rm dec} .$$

Putting this in Eq. (3.4) and using Eq. (2.1) gives

$$\frac{2}{\Omega_0}1=\cosh(\pi\sqrt{\gamma-1}) , \tag{3.5}$$

with which we can calculate γ from the observed value of Ω_0. The radius of the Universe is then obtained by evaluating Eq. (3.2) at the present:

$$a_0 \frac{1}{H_0}\left[\frac{\gamma-1}{1-\Omega_0}\right]^{1/2} . \quad (3.6)$$

The value of β_{dec} can be calculated from Eqs. (3.4) and (3.5) by assuming $\Omega_{dec}=1$ and that $\beta_{dec}\sqrt{\gamma-1}$ is small. The result is $\beta_{dec}=3t_{dec}/a_{dec}$. A further approximation, that $a(t)$ obeyed Eq. (3.1) for most of the time since decoupling, gives

$$\beta_{dec}\approx 0.1 .$$

Since $2\beta_{dec}$ is the angular size, on the three-sphere, of a causally connected region at decoupling, upper and lower limits on γ and a_0 can be found. For $\Omega_0=0.2$, these are

$$1.75 \lesssim \gamma \lesssim 1.85 ,$$

$$\frac{9.47}{h} \lesssim \frac{a_0}{10^9 \text{ light years}} \lesssim \frac{10.08}{h} .$$

This value of γ is consistent with the comment made in the beginning of this section. From Eq. (3.5) we can get the time interval over which the CMB remains isotropic:

$$\Delta t = a_0 \Delta\beta = 2\beta_{dec} a_0 \approx \frac{2}{h}\times 10^9 \text{ yr} .$$

IV. EPILOGUE

Evidently, in this textured cosmology the CMB is not a good indicator of the isotropy and homogeneity of the Universe. The horizon problem does not exist in this model. In addition, though we have no reason to doubt the validity of the cosmological principle on grand scales, the texture hypothesis relaxes the rigid CMB constraint on models for the formation of structure. More precisely, abiabatic density fluctuations at decoupling may be larger than previous interpretation of the isotropic CMB would admit. This weakens the argument that the dark matter be nonbaryonic.

Another amusing consequence of the texture hypothesis is that, though the CMB was not isotropic in the past, it has become isotropic at the present; the surface of last scattering collapsed into one causal region. How will the CMB appear in the future? If we take $\Omega_0=0.2$ as correct, then our Universe is now entering upon the free expansion phase of an open Robertson-Walker universe. In free expansion $a = t\sqrt{\gamma-1}$ and the proper distance a light ray travels is

$$l = a(t)\int d\epsilon = \frac{a(t)}{\sqrt{\gamma-1}}\int \frac{dt}{t} \sim t \ln t ,$$

which increases, by a logarithmic factor, relative to the Hubble expansion. The CMB will therefore slowly become less isotropic. In Sec. III B the time over which this occurs was calculated. The logarithmic behavior is reflected by the fact that this is much longer than the interval calculated for $\Omega_0=1$. Extrapolated into the distant future, the CMB will go through cycles of isotropy and nonisotropy, with intervals ever increasing.[8]

Some final remarks about the formation of texture are appropriate. I showed in Ref. 1 that for $\gamma \sim 1$ the energy scale at which the texture appears is the Planck mass. An important question is to what extent can processes presumed to occur in the regime of quantum gravity be taken seriously. I do not know the answer to this question. Another point to make is that since texture is a topological defect coming from a broken global symmetry, there will be three massless Goldstone bosons. These may have cosmological problems. On the positive side, the texture scale is at one on the fundamental scales of nature, so unnatural fine-tunings are not necessary to achieve some of the most important features of the observed Universe.

ACKNOWLEDGMENT

This work was supported by Department of Energy Contract No. DE-AC03-76SF00515.

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