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ΔI | = $\frac{1}{2}$ rule and consequences for D and B decays and ϵ'/ϵ

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It is argued that $|\Delta I| = \frac{1}{2} K$ decays are dominated by transitions to virtual diquarkantidiquark states which then evolve into mesons. Virtual diquark formation also contributes significantly to the relatively short lifetime of the $D⁰$ meson. It gives rise to a modified picture of the D^+ width and of some baryon-antibaryon decays of B mesons. The CP-violating parameter ϵ' of the K system remains small and unaffected even though the corresponding operator transforms as an isospin doublet.

Strangeness-changing nonleptonic decays are governed by the empirical $|\Delta I| = \frac{1}{2}$ rule formulated by Gell-Mann and Pais in 1954.' This rule has constituted an unsolved theoretical problem ever since. The famous currentcurrent weak-interaction Hamiltonian did not seem to give the correct answer. Using the factorization approximation^{2,3} for an estimate, much too small $|\Delta I| = \frac{1}{2}$ amplitudes and somewhat too large $|\Delta I| = \frac{3}{2}$ amplitude were obtained.³ With the advance of the standard model and QCD, hard-gluon and heavy-quark-exchange effects were then taken into account.^{4,5} These effects increase the $|\Delta I| = \frac{1}{2}$ amplitudes and decrease the $|\Delta I| = \frac{3}{2}$ amplitudes. As a result, the $|\Delta I| = \frac{3}{2}$ amplitudes calculated in factorization approximation or by using the Nambu-Goldstone realization of chiral symmetry in QCD together with QCD sum rules⁶ are now in reasonable agreement with experiment. However, the corresponding enhancement of the $|\Delta I| = \frac{1}{2}$ amplitudes turned out to be insufficient. In the factorization approximation the width for $K^0 \rightarrow (2\pi)_{I=0}$,

 $\Gamma_{(2\pi)I}^{K^0} = \Gamma_{\pi^+\pi^-}^{K^0_S} + \Gamma_{\pi^0\pi^0}^{K^0_S} - \frac{4}{3}\Gamma$

is given by

$$
\Gamma_{(2\pi)I}^{K^0} = \frac{G^2}{2} s_\theta^2 c_\theta^2 \frac{p_\pi}{8\pi m_K^2} (C^\pi)^2 ,
$$

with

$$
C^{\pi} = \frac{2a_1 - a_2}{\sqrt{3}} f_{\pi} f + (m_K^2 - m_{\pi}^2)
$$
 (1)

 $(s_{\theta} = \sin \theta, \ \theta \text{ is the Cabibbo angle, } f_{\pi} \approx 0.133 \text{ GeV})$. In this formula $f_{+} \approx 1$ describes the $K^{0} \rightarrow \pi^{-}$ form factor at

 $q^2 = m_\pi^2$. The real parameters a_1 and a_2 are combinations of the QCD coefficients⁷ $c = \approx 2.4$ and $c + \approx 0.64$:

$$
a_1 = \frac{1}{2} (1 + \xi)c_+ + \frac{1}{2} (1 - \xi)c_- ,
$$

\n
$$
a_2 = \frac{1}{2} (1 + \xi)c_+ - \frac{1}{2} (1 - \xi)c_- ,
$$
\n(2)

with $\xi = 1/N_c$ (N_c is the number of quark colors). One obtains

$$
C^{\pi}(\xi = \frac{1}{3}) \approx 4.9 \times 10^{-2} \text{ GeV}^{3} ,
$$

$$
C^{\pi}(\xi = 0) \approx 6.9 \times 10^{-2} \text{ GeV}^{3} .
$$

 $C^{\pi}(\frac{1}{3})$ is a factor 5.3 too small compared to the experimentally required value

$$
C_{\rm expt}^{\pi} \simeq 26 \times 10^{-2} \,\text{GeV}^3 \tag{3}
$$

If one uses an effective ξ close to zero as suggested from D decay studies, $8,9$ then the discrepancy between the experimental and theoretical amplitudes is still a factor 3.8. Moreover, the so-called penguin contribution⁵ is too small o remedy this situation.¹⁰ Factorization gives
 $C_{\text{Penguin}}^{\pi} \approx 2 \times 10^{-2} \text{ GeV}^3$.

The factorization approach works reasonably well not only for the $|\Delta I| = \frac{3}{2}$ amplitude in strange-particle decays, but also for numerous exclusive decay processes of cays, but also for numerous exclusive decay processes of D, D_s , and B mesons, ¹¹ if $\xi \approx 0$ is chosen. Why then does b, D_s , and B mesons, if $\zeta \sim 0$ is chosen. Why then does
his approximation fail so badly for the $|\Delta I| = \frac{1}{2}$ amplitude in K decays? To study this question let us imagine for a moment that diquark states (antitriplets in color) would be physical states like mesons. Recall that two quarks attract each other in an antitriplet state and repell each other in the color-symmetric sextet configuration. In

976 BERTHOLD STECH

order to calculate the fictitious decay rate of a K_S^0 into a diquark-antidiquark pair using again factorization, the weakinteraction Hamiltonian can conveniently be Fierz transformed¹² to a product of scalar and pseudoscalar currents:

$$
H_{\text{eff}} = -\frac{G}{\sqrt{2}} s_{\theta} c_{\theta} c - \epsilon_{kij} [\bar{d}_i^c (1 - \gamma_5) u_j] \epsilon_{klm} [\bar{s}_l (1 + \gamma_5) u_m^c] + \text{products of color-sextet currents} + \text{H.c.}
$$
 (4)

In Eq. (4) the (charge-conjugate) field operators are denoted by their particle names and the indices refer to color quantum numbers. The product of color-triplet currents exhibited in (4) obeys the $|\Delta I| = \frac{1}{2}$ selection rule. The field combination $\epsilon_{kij}(\bar{d}_i^c \gamma_5 u_j)$ generates a scalar antidiquark of isospin zero:

$$
\langle \left(\overline{d}\overline{u}\right)_{k'}| \epsilon_{kij} \left(\overline{d}_{i}^{c} \gamma_{5} u_{j}\right)|0\rangle = -i f_{(ud)}\left(\frac{2}{3}\right)^{1/2} v \delta_{k'k} , \qquad (5)
$$

with $v = m_{\pi}^2/(m_u^c + m_d^c) \approx 1.4$ GeV (m^c = current-quark masses).¹³ The convention used in (5) is such that if the diquark wave function would be equal to a π -meson wave function one would have $f(u_d) = f_x$.
The field combination $\epsilon_{klm}(\bar{s}_l u_m^c)$ appearing in (4) will

turn a K^0 meson into the diquark state (ud),

$$
\langle (ud)_{k'} | \epsilon_{klm} (\bar{s}_l u_m^c) | K^0 \rangle = f_0^{K^0 \to (ud)} (\frac{2}{3})^{1/2} v_K \delta_{k'k} , \quad (6)
$$

with¹³ $v_K = (m_K^2 - m_\pi^2)/(m_s^c - m_u^c) \approx v$. Again the notation is such that, in case the quark distribution in the diquark equals the quark distribution within the π meson,
one has $f_0^{K \to (ud)} = f_0^{K \to \pi^-} \approx 1$. Using (4)-(6) I find for the fictitious width of the decay of a K_S^0 meson into two diquarks a formula as given in Eq. (1) where, however, C^{π} is replaced by

$$
C^{(ud)} = \sqrt{6}c - v^2 \frac{2}{3} f_{(ud)} f_0^{(0)} (ud)
$$
 (7) $c \rightarrow (su) + \bar{d}$.

Because the force between two quarks in a color-triplet state is $\frac{1}{2}$ of the force between a quark and an antiquark in a color-singlet state, the diquark wave function is more extended than the wave function of a meson. This amounts to a reduction of $f(u_d)f_0^{K^0 \to (ud)}$ compared to $f_{\pi}f_0^{K^0 \to \pi^-}$. The reduction factor depends on the form of the interquark potential but will not be more than about 2. Including this reduction one gets from (7)

$$
C^{(ud)} \approx 51 \times 10^{-2} \,\text{GeV}^3 \tag{8}
$$

Thus the amplitude for producing a pair of diquarks is an order of magnitude larger than the direct production of two pions [and even about twice as large as the $|\Delta I| = \frac{1}{2}$ amplitude (3)].

Now, diquarks cannot exist in the final state. However, they can be formed as virtual states. Provided the QCD forces between quarks in a color-antitriplet state give these virtual diquarks a structure not too different from q - \bar{q} states, the above estimate teaches us the following.

 $\left| \begin{array}{c} \text{values, the above estimate teaches as the following.} \\ \text{in} \quad | \Delta I | = \frac{1}{2} \quad \text{strangeness-changing processes the} \end{array} \right|$ formation of a virtual state consisting of a pair of diquarks —or ^a system closely resembling this state—is very much favored by the weak Lagrangian. After its formation in K decays this state will evolve with probability one to π mesons. This picture of a decay to virtual intermediate diquark states then accounts for the enhancement

of $|\Delta I| = \frac{1}{2}$ amplitudes in a qualitative way. The fictious decay width calculated above can even be viewed as an estimate of the *inclusive* parity-changing $|\Delta I| = \frac{1}{2} K_S^0$ decay width which describes at the same time the exclusive process $K_S^0 \rightarrow (2\pi)_{I=0}$ since no other final state is energetically available.¹⁴

(ii) It appears that factorization is indeed a useful approximation if appropriate (real or virtual) intermediate states are considered.

A direct test of the picture given here will not be easy. Forthcoming lattice-gauge-theory calculations¹⁵ may hopefully give the correct number for the $|\Delta I| = \frac{1}{2}$ matrix element. But much additional effort will have to be made to extract from such calculations the physical origin of the enhancement. Similar remarks apply to present efforts to calculate the $|\Delta I| = \frac{1}{2}$ enhancement in a $1/N_c$ expansion.¹⁶

If the above understanding of strange-particle decays is correct, important consequences for D and B decays can be expected. In order to obtain a rough estimate of the special inclusive decay width of a D meson which is due to the formation of intermediate diquark states, I consider the reaction

$$
c \rightarrow (su) + \bar{d}
$$

Here (su) is a scalar diquark generated by the scalar current appearing in the charm analogue of the Hamiltonian (4). Choosing in what follows the constituent masses $m_c = 1.55$, $m_s = 0.4$, $m_d = m_u = 0.3$, $m_{(su)} = 0.7$ GeV, and $f_{(su)} = (1/\sqrt{2})f_K$, $c = (m_c) = 1.6$, $\theta = 0$, one gets

$$
\Gamma_{c \to (su)+d}^{(su)} \approx 33 \times 10^{10} \text{ sec}^{-1} . \tag{9}
$$

Let us compare this number with corresponding estimates of semileptonic decays, and of nonleptonic decays due to the direct generation of mesons by vector and axial-vector currents. The semileptonic decay width from free quark decay $c \rightarrow s e^- \overline{v}$ using the above mass values for c and s is

$$
\Gamma_{c \to s e^{-\bar{v}_e}} \approx 19 \times 10^{10} \,\text{sec}^{-1} \tag{10}
$$

in good agreement with the data for D^0 and D^+ decays. About the same number is also obtained by summing up the exclusive widths $D \rightarrow (\overline{K}, \overline{K}^*, \rho, \pi) e^{-} \overline{v}_e$ calculated with a more detailed model.¹⁷

For a crude estimate of the inclusive width due to the current-generated mesons π^+ , ρ^+ , \bar{K}^0 , \bar{K}^{0*} , I consider the decays

$$
x \to \pi^+, \rho^+ + s, \ c \to \overline{K}^0, \overline{K}^{*0} + u \ , \qquad (11)
$$

36

$$
\Gamma_{c \to \pi^+, \rho^+ + s}^{(ud)} \approx a_1^2 \times 69 \times 10^{10} \text{ sec}^{-1} ,
$$

\n
$$
\Gamma_{c \to \overline{K}^0, \overline{K}^{*0} + u} \approx a_2^2 \times 74 \times 10^{10} \text{ sec}^{-1} .
$$
 (12)

These numbers agree approximately with the summed-up exclusive two-body D-decay modes calculated in much more detail in Ref. 11. Moreover, the multiplication of the semileptonic width by 3 (the number of colors) gives \approx 57 × 10¹⁰ sec⁻¹, a number somewhat below the values appearing in (12). This is quite understandable since the attractive force between quarks increases the transition probability.

A comparison of (12) with the estimate (9) shows now that the generation of virtual diquark states is important in D decays, but not as dominant as in K decays. The reason is that the dependence of this rate on the mass of the decaying quark is only linear while it is proportional to the third power in the usual way of factorization.

The total width for the D^0 meson as estimated from (9), (10), and (12) is now

$$
\Gamma_{D^0} \approx (19 \times 2 + 69a_1^2 + 74a_2^2 + 33) \times 10^{10} \text{ sec}^{-1} \ . \tag{13}
$$

With $a_1(m_c) \approx 1.3$ and $a_2(m_c) \approx -0.55$ (Ref. 11) one obtains $\Gamma_{D^0} \approx 210 \times 10^{10}$ sec⁻¹. This number compares well with the experimental width¹⁸ Γ_{p}^{expt} = (226 ± 10) × 10¹⁰ \sec^{-1}

In D^+ decays the two processes (11) with coefficients a_1 and a_2 , respectively, lead to the same final states and a_1 and a_2 , respectively, lead to the same final states and therefore interfere. $11,19$ Using again (9)–(12) the total width is

$$
\Gamma_{D} \approx [19 \times 2 + (\sqrt{69}a_1 + \sqrt{74}a_2)^2 + 0.85 \times 33]
$$

×10¹⁰ sec⁻¹. (14)

The last term is again due to (9). To account for the Fermi statistics of the two antidown quarks a 15% reduction 'has been assumed. (The reduction factor would be $\frac{3}{4}$ if only S -wave states were essential.) Equation (14) gives $\Gamma_{D^+} \approx 103 \times 10^{10}$ sec⁻¹ in agreement with the experimental number¹⁸ $\Gamma_{D^+}^{exp} = (97 \pm 5) \times 10^{10}$ sec⁻¹.

The astonishingly good agreement with the data of the simple estimates (9) - (14) performed here is probably fortuitious. However, the overall consistency of the picture is remarkable. The short lifetime of the D^0 (short compared to expectations based on short-distance QCD factors only) is now better understood. Previously, it was thought to arise from an accumulation of weak annihilation processes. Although there is still some room for weak annihilation, it is now seen that transitions to virtual diquark states play an important role. These states turn into the final decay products by quark annihilation and creation processes and by quark rearrangement, but at a time scale characteristic of strong interaction.

In D^+ decays, the new contribution is even more important for the lifetime because of the destructive interference of the a_1 and a_2 amplitudes. It diminishes the $D⁺/D⁰$ lifetime ratio which would otherwise be more pronounced. The previously held opinion of a "normal" decay of the D^+ is now in doubt.

In view of these new insights do we have to modify the

treatment of exclusive two-body D and D_s decays? I think not very much: the exotic intermediate states will mainly turn into multimeson states and hardly form energetic two-meson states. However, channels with small kinetic-energy release (such as the famous decay channel $D^0 \rightarrow \phi \overline{K}^0$ and channels with small amplitudes may be fed by these and other intermediate processes as discussed in Ref. 11. Conceivably, the values of the parameters a_1 and a_2 may be affected from such rescattering processes, too.

A new situation arises in B decays. Here the pair of intermediate diquarks will practically always turn into baryon-antibaryon pairs since enough energy is available. Energetic two-body decays to mesons such as $B \to D^*\pi$ calculated in Ref. 11 are certainly not affected by the new mechanism. The inclusive decay width which arises from the processes $b \rightarrow (cd)\overline{u}, b \rightarrow (cs)\overline{c}$ is

$$
\Gamma_b^{(cd),(cs)} \approx 1.6 \times 10^{10} \text{ sec}^{-1}
$$
 (15)

 $(m_b = 5, m_{(cd)} = 1.9, m_{(cs)} = 2.1$ GeV and $|V_{bc}| = s_y$ =0.05, $f_{(cd)} = f_{(cs)} = f_K/\sqrt{2}$ has been used for this estimate).

The result (15) refers to the directly generated 0^+ diquark states (cd) , (cs) only. The corresponding branching ratio is about 2%. However, the actual branching ratio for the decay of B mesons to baryon-antibaryon states will be much bigger, since transitions to $0⁻$ and higher excited diquark states can be reached in these decays, and because also the usually considered decay mechanism will contribute to baryon production as described by Bigi.²⁰ It is foreseeable that in the future B meson decays to baryon-antibaryon states will be an important tool for the study of diquark and subsequent triquark formation. These studies can also be decisive for determining $|V_{ub}| = s_{\beta}$ from $b \rightarrow (ud)\overline{u}$ and $b \rightarrow (us)\overline{c}$ reactions and for the detection of CP violation.

Finally, I discuss the implication of the above picture for the connection of ϵ' (the parameter controlling the CP violation in the $K \rightarrow 2\pi$ decay amplitude) with the quark mixing angles β , γ , δ' .

xing angles $\beta, \gamma, \delta'.$
The quantity ϵ' is determined by the imaginary part of the $K^0 \rightarrow 2\pi$ matrix element of the effective $|\Delta S| = 1$ Hamiltonian. The effective $|\Delta S| = 1$ Hamiltonian of the standard model has been calculated by Gilman and Wise⁷ in terms of operators Q_1 to Q_6 . Until now calculations of the $K^0 \rightarrow 2\pi$ matrix elements of these operators have been considered very doubtful because of the lack of understanding of the $|\Delta I| = \frac{1}{2}$ amplitude. Now one may note that only the operators Q_1 to Q_4 can generate spin-zero diquarks. Q_1 and Q_2 can do this very efficiently as was shown in this paper. The operator Q_5 can generate mesons and spin-one diquarks. However, the coefficient of Q_5 in the Hamiltonian is very small and the (axial-)vector matrix elements containing diquarks are not enhanced significantly. Finally, the operator Q_6 is quite different from Q_1 and Q_2 . The matrix elements of Q_6 can be factorized with meson states only and not with color-triplet diquarks. For a calculation of ϵ' , it seems therefore well justified to use for the contribution from Q_1 to Q_4 the exberimental $|\Delta I| = \frac{1}{2}$ amplitude and for the contribution from Q_6 the factorization result. In view of the arguments

978 BERTHOLD STECH

given previously, factorization of the Q_6 matrix element should be as good as for the $|\Delta I| = \frac{3}{2}$ amplitude. The coefficients in front of the operators Q_i depend on the top-quark mass. I use values relevant for $m_t \approx 40$ GeV. The contribution to ϵ' from the operators Q_1 to Q_4 is small,

$$
\mathrm{Re}(\epsilon'/\epsilon) \simeq \pm 1.3 s_{\beta} s_{\gamma} s_{\delta'}
$$

and can be neglected here.²¹

The matrix element of Q_6 calculated via factorization is very sensitive to f_K/f_π and to the scalar $\pi\pi$ form factor. Using values as in Ref. 22 one finds, for ϵ'/ϵ ,

$$
Re(\epsilon'/\epsilon) \approx 9s_{\beta} s_{\gamma} s_{\delta'} . \tag{16}
$$

Equation (16) can be used to determine qualitatively the product $s_{\beta}s_{\delta}$, independent of any theory for the K^0 - \overline{K} ^t mixing parameter ϵ . It is also useful to test models for the mass matrices, 2^{3-26} such as the one which connects all mixing angles with (maximal) CP violation of the mass

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matrix.²⁴

In conclusion, the striking $|\Delta I| = \frac{1}{2}$ enhancement in strangeness-changing decays appears to be due to the special properties of the operators Q_1 and Q_2 of the effective Hamiltonian. These operators can efficiently generate virtual diquark-antidiquark states which then evolve into hadrons. In D and B decays transitions to corresponding virtual states are of significance for the total decay widths, but have presumably little effect on most of the two-body decay rates. In the K^0 system the matrix element of the operator Q_6 determines the CP-violating parameter ϵ' . Usual factorization should here be as good as it is for the ' ΔI = $\frac{3}{2}$ amplitudes or for matrix elements describing energetic D decays, even though Q_6 is an $I = \frac{1}{2}$ operator.

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