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Comments on the Z electromagnetic couplings in a composite model

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We show how one can generate sizable electromagnetic couplings for the Z ($Z\gamma\gamma, ZZ\gamma$) if the neutral weak boson is a composite particle. Contrary to a recent claim, we show that a superheavy fermionic constituent gives a very suppressed contribution.

I. INTRODUCTION

Although the standard model[1] (SM) of weak interactions has scored some impressive successes—the most notable certainly being the recent discovery of the weak intermediate vector bosons—a decisive test of the theory is still missing. One would like to know more about the "inner machinery" of the theory: renormalizability and the local non-Abelian gauge nature of the model together with the existence of the elusive Higgs boson. These features are clearly the stamp of the SM and are the ones which set it apart from alternative models.[2] Some of these models[3,4] have succeeded thus far in reproducing SM results, but one expects deviations between these models beyond the Z mass.

Among the contenders, compositeness is a very popular alternative. One difficulty, however, is that we do not know what the scale of compositeness (Λ) is.[5] Nonetheless, constraints on Λ all come from tests in the lepton sector whereas there is no constraint coming from assuming compositeness for the weak bosons. Since our analysis is just concerned with a composite Z boson, this discussion about Λ is of minor importance.

We work within the framework of composite Z (Ref. 6) whose fermionic constituents[7] are held together by a hyperstrong $SU(N_H)$ force. Besides hypercolor, these constituents carry color. We also introduce an effective coupling between the Z and its constituents which has a vector part as well as an axial-vector part as in the model we comment on.[8]

II. $Z\to l\bar{l}\gamma$

It is by now completely accepted that the process $Z\to l\bar{l}\gamma$ is totally accounted for within the standard model. Bremsstrahlung is consistent with the observed events and with more data acquisition the early (disturbing) anomalous events[9] have faded away.

It is claimed by Lee in Ref. 8 that in the SM the one-loop corrections to this process only come from the generated $Z\gamma\gamma^v$ and $ZZ^v\gamma$ vertices. It is clear that the graphs contributing to the decay are exactly the same as those for $e^+e^-\to Z\gamma$. Electromagnetic corrections to this process are given in Ref. 10 whereas the weak corrections are given in Ref. 11. It should also be stressed that one cannot generate $Z\gamma\gamma^v$ or $ZZ^v\gamma$ through the W-boson loops since we do not have any C-violating term. Furry's theorem then applies and any such couplings are forbidden.

III. THE $ZZ\gamma$ VERTEX THROUGH THE FERMION TRIANGLE

We will concentrate on the $ZZ\gamma$ vertex, since in a composite picture this will be dominant over $Z\gamma\gamma$ because the latter suffers an extra electromagnetic suppression. In particular, we will show that a superheavy fermion (here superheavy refers to a mass much larger than the mass of the Z boson) gives a vanishingly small contribution to the $ZZ\gamma$ vertex *regardless of whether or not one assumes an anomaly-cancellation mechanism.* We have already presented the calculation of $ZZ\gamma$ using symmetry arguments[12] and both dimensional and Pauli-Villars regularization.[13] We have shown the existence of two moments: the electric dipole transition (EDT) which corresponds to $Z^v\to Z\gamma$, and an anapole moment which corresponds to $\gamma^v\to ZZ$. For $Z\to e^+e^-\gamma$, only the dipole transition contributes. Referring the reader to Ref. 12 for details of the calculation, we give the contribution of *each individual fermion to EDT,* which is proportional to D_Z^* of Ref. 8:

$$E^{\rho\nu\mu}[Z^{v}(k_2,\rho)\to Z(k_1,\nu)+\gamma(k,\mu)]=\frac{-eQ_f a_f v_f}{\pi^2}\epsilon^{\rho\nu\mu\alpha}k_\alpha[sI(s,M_Z{}^2)-M_Z{}^2 I(M_Z{}^2,s)]$$

$$=-\epsilon^{\rho\nu\mu\alpha}k_\alpha E_T^f\ , \tag{1}$$

with

$$s=k_2{}^2\ , \tag{2}$$

where $I(s,M_Z{}^2)$ is given in closed form in Ref. 12 and depends on the mass of the fermion f. Q_f is the charge of the fermion, and we have defined the $Z_v ff$ vertex as

$$i\gamma_v(v_f+a_f\gamma_5)\ . \tag{3}$$

It is clear from (1) that E_T^f is a transition since for $s=M_Z{}^2$ the coupling vanishes.

In Ref. 12 we found that

$$I(s,M_Z{}^2)=\frac{1}{S-M_Z{}^2}\left\{\frac{1}{2}-\frac{m_f{}^2}{s-M_Z{}^2}\left[S\left[\frac{M_Z{}^2}{m_f{}^2}\right]-S\left[\frac{s}{m_f{}^2}\right]\right]+\frac{M_Z{}^2}{2(s-M_Z{}^2)}\left[L\left[\frac{M_Z{}^2}{m_f{}^2}\right]-L\left[\frac{s}{m_f{}^2}\right]\right]\right\}, \tag{4}$$

with

$$S(\beta)=\int_0^1\frac{dx}{x}\ln[1-\beta x(1-x)],\quad L(\beta)=\int_0^1\frac{dx}{x}\ln[1-\beta x(1-x)]\ . \tag{5}$$

Notice that the first term in (4) is the mass-independent term, which is indicative of the presence of the anomaly. In a theory where more than one fermion is present, we have to sum over all fermions:

$$E_T=\sum_f E_T^f(m_f)\ . \tag{6}$$

In a renormalizable anomaly-free theory, such as the standard SU(2)×U(1), due to the cancellation of the anomaly, we have the condition

$$\sum_f Q_f a_f v_f=0 \tag{7}$$

(taking the color factor into account for quarks). Therefore, we agree with Lee in Ref. 8 that the mass-independent terms in E_T^f will not contribute to the total EDT E_T. However, we stress that this is only true in an anomaly-free theory where condition (7) is at work, so that in this case one may rewrite (6) as

$$E_T=\sum_f E_T^{*f}(m_f)\ , \tag{8}$$

where $E_T^{*f}(m_f)$ differs from the contribution of a single fermion E_T^f by the all-important mass-independent term

$$E_T^{*f}=\frac{-eQ_f a_f v_f}{\pi^2}[sI^*(s,M_Z{}^2)-M_Z{}^2 I^*(M_Z{}^2,s)]\ ; \tag{9}$$

however, now $I^*(s,M_Z{}^2)$ is stripped of its mass-independent term:

$$I^*(s,M_Z{}^2)=\frac{1}{S-M_Z{}^2}\left\{-\frac{m_f{}^2}{s-M_Z{}^2}\left[S\left[\frac{M_Z{}^2}{m_f{}^2}\right]-S\left[\frac{s}{m_f{}^2}\right]\right]+\frac{M_Z{}^2}{2(s-M_Z{}^2)}\left[L\left[\frac{M_Z{}^2}{m_f{}^2}\right]-L\left[\frac{s}{m_f{}^2}\right]\right]\right\}. \tag{10}$$

In brief, in an anomaly-free theory where (7) holds, one may disregard the mass-independent terms (provided one remembers to sum any amplitude over all fermions).

In a composite picture, the Z ceases to be a gauge particle; i.e., no current is gauged, and any question about the renormalizability of the model is irrelevant. After all, we only have an effective theory; Z would be like the ρ^0 in strong interactions. In this case there is no need for imposing condition (7), and, consequently, one must use the full expression for E_T^f given by (1) and (4), since the mass-independent terms do not cancel after summation. Let us now see the correct behavior of E_T^f for both small and large masses.

Using (4) and (5) we obtain the contribution of one fermion to E_T, E_T^f (that is, the contribution of one single fermion) for large masses:

$$E_T^f=-\frac{eQ_f a_f v_f}{24m_f{}^2\pi^2}(s-M_Z{}^2)\ . \tag{11}$$

This expression is true for any *single superheavy* fermion in any theory, and we see that, contrary to Ref. 8, when $m_f\to\infty$, we find a zero contribution.

Lee, on the other hand, uses $E_T^{*f}(m_f)$ (which does not include the mass-independent terms) to study the large-mass limit of the EDT. In the large-mass limit, one would find

$$E_T^{*f} = -\frac{eQ_f a_f v_f}{2\pi^2}\frac{s+M_Z{}^2}{s-M_Z{}^2} . \tag{12}$$

As we remarked earlier, this particular (misleading) expression should be used only if one has an anomaly cancellation, and, in that sense, the right quantity to study is $\sum E_T^{*f}$. Now, the fact that Eq. (12) is *mass independent* and that one must sum over all fermions the anomaly-cancellation mechanism [condition (7)] leads to the result that superheavy fermions, again, do not contribute to the EDT.

Now, it should be realized that expression (12) as it stands is inconsistent: not only does it not represent a dipole transition, but it, in fact, diverges as $s \to M_Z{}^2$. This behavior is clearly present in Ref. 8, where a look at the last formula shows the dangerous factor $1/(P_1 \cdot P_2)$. Clearly, in his conclusion, Lee has not used condition (7) which he earlier exploited to get rid of any mass-independent terms. The summation, with the use of condition (7), comes to our rescue in curing the $1/(P_1, P_2)$ behavior. In the large mass limit, the remaining mass-dependent terms in $\sum_f E_T^{*f}(m_f)$ reproduce $\sum_f E_T^f$ where E_T^f is given by (11).

This leads us to calculate the effect of low-mass fermions. We find, for $m_f \to 0$,

$$E_T^f = \frac{eQ_f v_f a_f}{\pi^2(s-M_Z{}^2)}\left[\frac{s+M_Z{}^2}{2} + \frac{M_Z{}^2 s}{s-M_Z{}^2}\ln\frac{M_Z{}^2}{s}\right] . \tag{13}$$

The first term in (13) comes from the mass-independent term in (4). This term is crucial for having $E_T^f(s=M_Z{}^2)=0$. Omission of this term leads to an inconsistent divergent contribution as $s \to M_Z{}^2$. The point we want to make is that, in this approach, it is the superlight constituents that contribute to the electric dipole transition, in complete contradiction with the result of Ref. 8.

In fact, our conclusions can be reached without carrying out any calculation. To see why a superheavy fermion does not contribute, it is best to recall the calculation of the anomalous triangle diagrams which give rise to the coupling between three vector bosons. It is best to regularize $ZZ\gamma$ using the large-mass-regulator technique.[14,15] One first considers the triangle $E_{\mu\nu\rho}(m_f)$ corresponding to a fermion of mass m_f inside the loop. This amplitude is regularized by subtracting the effect of a large mass fermion having the same couplings, thus the *regularized* vertex is

$$E_{\mu\nu\rho}^{Rf}(m_f) = E_{\mu\nu\rho}^f(m_f) - \lim_{M\to\infty} E_{\mu\nu\rho}^f(M) \tag{14}$$

(the superscript f refers to the couplings of the fermion f to the bosons).

This procedure gives the correct vector-current conservation. The subtracted term

$$\lim_{M\to\infty} E_{\mu\nu\rho}^f(M) = A^f \tag{15}$$

depends on the couplings of the particular fermion f. This is the term which breaks the axial-vector Ward identity. In an anomaly-free theory, however, these mass-independent terms cancel when summed over all fermions,

$$\sum_f A^f = 0 , \tag{16}$$

so that the whole contribution is

$$\sum_f E_{\mu\nu\rho}^{R,f}(m_f) = \sum_f E_{\mu\nu\rho}^f(m_f) . \tag{17}$$

Imagine now, as in Ref. 8, that $m_f \to \infty$ (for each species f), then from (15) and (16) we get, in the anomaly-free case,

$$\lim_{m_f\to\infty}\sum_f E_{\mu\nu\rho}^{Rf}(m_f) = \lim_{m_f\to\infty}\sum_f E_{\mu\nu\rho}^f(m_f) = \sum_f A_f = 0 ; \tag{18}$$

also, if one does not consider condition (16) but assumes that there is just one fermion, then again

$$\lim_{m_f\to\infty} E_{\mu\nu\rho}^{R,f}(m_f) = \lim_{m_f\to\infty}[E_{\mu\nu\rho}^f(m_f) - A_f] = A_f - A_f = 0 . \tag{19}$$

Therefore a superheavy fermion contributes nothing *whether or not one is dealing with an anomaly-free mechanism where condition (7), (16) is operative.*

Now we address the question how can we still have a sizable contribution if (7) holds? (In this case, this means that we are committing ourselves to an anomaly-free theory even in the prospect of a composite Z.) The answer is that one must allow a large difference between the masses of the constituents, since, if these happen to have the same mass they will contribute equally, and consequently (7) forces the total contribution to be zero. In this case it is the lightest fermions which contribute the most. However, having a composite Z, whose fermionic constituents have widely separated masses, seems to us to be an unlikely scenario. For example, this would break the custodial global weak isospin, which seems to be necessary in order to have a composite model of weak interactions which reproduces all the low-energy results of the standard model. In a composite model, the masses inside the triangle are the constituent masses which are at least of the same order of magnitude as the compositeness scale. For example, in the $SU(2)_F$ of strong interaction, $m_u \sim m_d$ is probably false as a current mass relation, but is certainly true as a constituent mass relation. This state of affairs is very much like calculating $\pi^0 \to \gamma\gamma$. The triangle diagram is in excellent agreement with experiment if we include the color factor. Another success of the triangle is that it describes rather well radiative decays of vector mesons $V \to P\gamma$ [$P=\pi,\eta$; $V=\rho,\omega,\phi,\ldots$ (Ref. 16)] and agrees very well with the vector-meson-dominance (VMD) model.[17] In fact there is a duality[18] between the two approaches if we take the quark mass to be the constituent mass: $m_u \simeq m_d \simeq 300$ MeV. This language, translated into our case, tells us that one should take m_f to be of the order of magnitude of M_Z or even $G_F \simeq 300$ GeV, for example; and most importantly one should drop condition (7). Thus, in a QCD-like theory,[4] each fermion contributes [see (11) and Ref. 19]:

$$E_f^f = -\frac{eQ_f a_f v_f}{24\pi^2 m_f^2} N_H N_C (s - M_Z^2) , \tag{20}$$

where we have introduced the color factor N_C and the hypercolor factor N_H. Moreover, a_f and v_f should be considered as strong couplings:

$$\frac{a_f^2}{4\pi} \simeq \frac{v_f^2}{4\pi} \geq 1 . \tag{21}$$

This could lead to a sizable factor in (20). We must stress that we are advocating the use of a constituent mass around the M_Z mass and not $m_f \gg M_Z$. We have already argued why it was very unrealistic to take values that are too low for m_f even though they give a larger contribution. One should realize that our treatment of the large fermionic mass is completely different from that of Lee.

It was Renard[20] who first suggested that, in a nonstandard model, $ZZ\gamma$ and $Z\gamma\gamma$ could well be large. Gounaris, Kögeler, and Schildknecht[21] proposed a VMD approach to generate such couplings and used them to evaluate $Z \to e^+e^-\gamma$. We introduced a similar idea[22] but pointed already to the use of the triangle.

Now that the anomalous Z decays seem to be accounted for by the standard bremsstrahlung, we have to put constraints on the phenomenological $ZZ\gamma$ coupling. We may require that, after all, $m_f \gg M_Z$, which pushes the scale of the interaction to the very-high-energy region. Explicit calculations[23] with condition (21) realized together with $N_H \sim N_c = 3$ and taking $M_H \sim 200$ GeV makes the contribution of $ZZ\gamma$ to the anomalous decay 10^{-6} times smaller than the standard contribution to this process. An optimistic composite model builder would not like to lose the possibility of having strong self-couplings between composite weak bosons, such as $ZY\gamma$, ZZY, ... (Y is the isoscalar partner of the W^0), even though she or he may be forced to have rather small couplings between other composites such as ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$, for example. A way out of this is then to have condition (7) realized, but just for some of the couplings. For example, in the model of Fritzsch and Mandelbaum this is easily realized if we choose the charge of one of the constituents to be $\frac{1}{2}$. In fact, we have shown[24] that this particular charge assignment is a very restrictive one, forbidding many couplings to exist (see also Ref. 25). In the case at hand, this means that by taking the mass of the constituent as equal we have the required condition (7) for $ZZ\gamma$ to vanish by choosing a special charge content for the constituents.

IV. EDT AND $g-2$

Finally, contrary to the claim in Ref. 8 that the $ZZ\gamma$ coupling would contribute to the muon anomalous magnetic moment (though tiny as the author estimates), the EDT of the Z does not lead to any *static* moment for the leptons. The reason that EDT does not lead to $g-2$ or self-mass correction is that $ZZ\gamma$ is an even-dimension operator (dim=6) which is chirality conserving.[26] The other way of seeing this is to note that $ZZ\gamma$ vanishes when the two Z have the same invariant mass. This property is crucial when we try to calculate $ll\gamma$ through the $ZZ\gamma$ vertex in the one-loop approximation. All that is induced corresponds to transition form factors. Note that in Ref. 8 this symmetry factor is lost. This is most certainly the reason why Lee finds an induced *static* moment.

V. CONCLUSION

We would like to again stress that the contribution of superheavy constituents, for generating large couplings for $ZZ\gamma$ (and for that matter $Z\gamma\gamma$) via triangle graphs, is "supersuppressed" whether or not one commits oneself to anomaly-free theories. Expressions (11), (18), and (19) are a clear indication. The inconsistency in the recent analysis by Lee lies in the fact that the author works within the context of an anomaly-free theory, where it is "vital" to sum the contribution of all fermions, but he fails to carry out this summation procedure all the way through; hence, he arrives at the wrong conclusion based on an amplitude amputated of its important mass-independent term.

Having presented the full contribution of a single fermion to $ZZ\gamma$, and paying particular attention to both light and superheavy fermions, we proposed ways of obtaining sizable trilinear couplings between neutral spin-one particles in an anomaly-free theory as well as in an anomalous one. In both cases, there is the need for strong couplings between the Z and its constituents, implemented through a hyperstrong confining force based on a large non-Abelian group. In an anomaly-free theory, however, the different fermions conspire to give a null result unless one contemplates the unlikely situation where a wide gap in the mass spectrum of the fermionic constituent exists. One is much better off if one is willing to give up the realm of an anomaly-free theory—which is possible in an effective composite model. In this case, the contributions from different fermions do add up.

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