Anomaly-free version of $SU(2)\times U(1)\times U(1)'$

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The most general anomaly-free version of the $SU(2)\times U(1)\times U(1)'$ local gauge-invariant model is presented here, for two different Higgs structures (the minimal ones), as an extension of the $SU(2)\times U(1)$ Glashow-Weinberg-Salam model.

I. INTRODUCTION

The Glashow-Weinberg-Salam[1-3] (GWS) local gauge-invariant model of the electroweak interactions has been successful so far in explaining most of the experimental results related to the subject, including the weak-neutral-current results (one conspicuous exception is the $\Delta I=\frac{1}{2}$ rule).

In spite of its success this model is not thought of as the ultimate one, and a lot of different generalizations have been proposed in the last two decades to accomplish new experimental results and for aesthetic matters also. In this paper we present the most general anomaly-free version of one of such extensions, based upon the local gauge-invariant group $SU(2)\times U(1)\times U(1)'$. This model has been used extensively in the literature[4-9] for different purposes: It was used in Ref. 4 to arrange the parameters for suppressing unwanted semileptonic and nonleptonic weak decays; in Ref. 5 to arrange the parameters for canceling flavor-changing neutral currents in models without the charmed quark; in Ref. 6 as an attempt for explaining the $\Delta I=\frac{1}{2}$ rule; in Ref. 7 to arrange the parameters for explaining possible lack of evidence for parity nonconservation in the atomic physic experiments; in Ref. 8 for explaining the CERN anomalous events, and so on. (Other references to the model are enumerated in Ref. 9.) So far, in all of its applications in the literature only particular versions of the model have been used, and in most cases the version presented is not renormalizable, due to the fact that it is not possible to achieve proper cancellation of the triangular anomalies for the particular values chosen for the parameters.

II. THE MODEL

The group $SU(2)\times U(1)\times U(1)'$ is characterized for five generators, T_1, T_2, and T_3 for SU(2), Y_1 for U(1), and Y_2 for U(1)', three of which may be diagonalized simultaneously (T_3, Y_1, and Y_2). In the exact symmetrical model there are five massless gauge bosons. We also need three different coupling constants: g, $g_1/2$, and $g_2/2$ for SU(2), U(1), and U(1)', respectively.

In order to break the symmetry in the manner $SU(2)\times U(1)\times U(1)'\to U(1)_Q$, and to give masses to four out of five gauge bosons (the photon remains massless), several Higgs bosons must be introduced in the model. The most economical way for achieving it is by using (a) one SU(2) doublet ϕ_1 and one SU(2) singlet χ of complex scalar fields (six real fields) and (b) two SU(2) doublets ϕ_1 and ϕ_2 of complex scalar fields (eight real fields). In both cases ϕ_1 is the GWS Higgs multiplet. From now on we will call them versions (a) and (b), respectively.

The symmetry is spontaneously broken down to the electromagnetic one, by demanding

$$\langle\phi_i\rangle_0=(0,v_i/\sqrt{2})\ , \quad i=1,2\ , \qquad \langle\chi\rangle_0=v/\sqrt{2}\ , \tag{1}$$

for the vacuum expectation values. The most general way of writing the unbroken generator Q is

$$Q=T_3+(aY_1+bY_2)/2\ . \tag{2}$$

Equation (2) fixes the value $aY_1+bY_2=Y_{\rm GWS}$ for a given multiplet, where $Y_{\rm GWS}$ is the hypercharge for such a multiplet in the GWS model (a fixed number). So, we have the freedom to choose Y_2 fixed and Y_1 free for every multiplet. This arbitrariness introduces one more free parameter for every multiplet: the value Y_1.

Equation (2) permits two very different situations: one for $a\neq 0$ and $b\neq 0$, and the other one for $a=0$ and $b\neq 0$. The physics for the two cases is different. From now on we will work with (2) for $b\neq 0$ and any value for a (including the value $a=0$).[10]

The covariant derivative for the model has four independent terms: the kinetic one $i\partial_\mu$ and one term for each subgroup in the Cartesian product group; that is,

$$iD_\mu=i\partial_\mu+g\mathbf{T}\cdot\mathbf{B}_\mu+\frac{g_1}{2}Y_1C_\mu+\frac{g_2}{2}Y_2C'_\mu\ , \tag{3}$$

where $\mathbf{B}_\mu$, C_μ, and C'_μ are the five gauge fields associated with SU(2), U(1), and U(1)', respectively. When we calculate the Lagrangian for the scalar sector of the theory we get

$$L_{\rm scal}=i\phi_1^\dagger D^\mu D_\mu\phi_1+i\chi^* D^\mu D_\mu\chi \quad \text{for version (a)}\ , \tag{4a}$$

$$L_{\rm scal}=i\phi_1^\dagger D^\mu D_\mu\phi_1+i\phi_2^\dagger D^\mu D_\mu\phi_2 \quad \text{for version (b)}\ . \tag{4b}$$

Spontaneous symmetry breaking in Eq. (4) produces masses for the charged weak fields $W^{\mp}=(B^1\pm iB^2)/\sqrt{2}$ given by

$$M_W{}^2=g^2v_1{}^2/4 \quad \text{for version (a)}\ , \tag{5a}$$

$$M_W{}^2=g^2(v_1{}^2+v_2{}^2)/4 \quad \text{for version (b)}\ ; \tag{5b}$$

also the neutral fields $B_\mu{}^3$, C_μ, and C'_μ mix with each other giving a 3×3 real mass matrix. In order to diagonalize such a matrix in the most general case, three Weinbergtype angles are needed: θ, ξ and ψ (θ is the Weinberg angle of the GWS model), defined as[11]

$$\sin\theta = e/g \ , \tag{6}$$

$$\tan\xi = a g_2 / b g_1 \ , \tag{7}$$

where

$$e^{-2} = a^2 g_1^{-2} + b^2 g_2^{-2} + g^{-2} \tag{8}$$

is the electric charge. The former definitions for θ and ξ are valid for both versions[12] (a) and (b). The definition for ψ depends upon the version we are working with, so we have

$$\tan(2\psi) = \frac{4M_W{}^2/\cos^2\theta}{M_Z{}^2 + M_Z'^2 - 2M_W{}^2/\cos^2\theta} \frac{\sin\theta}{\sin(2\xi)} \times (aY_{\phi_1} - \sin^2\xi) \tag{9a}$$

for versions (a) and

$$\tan(2\psi) = \frac{4M_W{}^2/\cos^2\theta}{M_Z{}^2 + M_{Z'}{}^2 - 2M_W{}^2/\cos^2\theta} \frac{\sin\theta}{\sin(2\xi)} \times \left[(aY_{\phi_1} - \sin^2\xi) - \frac{\vartheta_2{}^2}{\vartheta_1{}^2 + \vartheta_2{}^2} a(Y_{\phi_1} - Y_{\phi_2}) \right] \tag{9b}$$

for version (b). Where M_W is given by (5), M_Z and M_Z' are the masses for two weak-neutral heavy gauge bosons of the theory functions of the respective parameters for each version. Before continuing we want to emphasize that there are so far two more parameters for each version: Y_{ϕ_1} and Y_χ the free hypercharge for the Higgs bosons for version (a), and Y_{ϕ_1}, Y_{ϕ_2} the free hypercharge for the Higgs bosons for version (b). [For version (a) Y_χ can be absorbed[5] in v by the definition $V = vY_\chi$, and must be such that $Y_\chi \neq 0$. For version[13] (b), $Y_{\phi_1} \neq Y_{\phi_2}$.] So in general we have

$$M_{Z,Z'} = M_{Z,Z'}(g, g_1, g_2, v, v_1, a, b, Y_{\phi_1}, Y_\chi) \quad \text{for version (a)} \ , \tag{10a}$$

$$M_{Z,Z'} = M_{Z,Z'}(g, g_1, g_2, v_1, v_2, a, b, Y_{\phi_1}, Y_{\phi_2}) \quad \text{for version (b)} \ . \tag{10b}$$

It is important to notice that a and b in (10) are not free parameters we may play with; they must be chosen in the very moment we define the charge operator Q in Eq. (2); we are keeping them so far just for generality.[10]

After some manipulation of the algebra we get the final expressions for the most general covariant derivative of the model, as a function of the real fields A_μ (the photon field), Z_μ and Z'_μ (the two heavy neutral fields), valid for both versions (a) and (b):

$$\begin{aligned} iD_\mu = {} & i\partial_\mu + g(W^\dagger_\mu T^- + \text{H.c.})/\sqrt{2} + eA_\mu Q \\ & + gZ_\mu \frac{\sin\psi}{\cos\theta} \left[Q\sin^2\theta - T_3 + \frac{\sin\theta\cot\psi}{2} [aY_1(\cot\xi + \tan\xi) - 2Y_{\text{GWS}}\tan\xi] \right] \\ & + gZ'_\mu \frac{\cos\psi}{\cos\theta} \left[Q\sin^2\theta - T_3 - \frac{\sin\theta\tan\psi}{2} [aY_1(\cot\xi + \tan\xi) - 2Y_{\text{GWS}}\tan\xi] \right] , \end{aligned} \tag{11}$$

where $T_\pm = T_1 \pm iT_2$, Y_1 is the free hypercharge parameter, and Y_{GWS} is the GWS hypercharge (a fixed value) of the multiplet on which iD_μ acts. As mentioned before, we are going to have one more free parameter for each multiplet in the theory. Notice in (11) that in the limit $\psi \to 0, \pi/2$, the covariant derivative reduces exactly to the one for the[14] GWS model, plus one extra term due to the new massive neutral current. Also in those limits we get $M_Z = M_W/\cos\theta$ (for $\psi \to 0$) and $M_{Z'} = M_W/\cos\theta$ (for $\psi \to \pi/2$) for both versions.

III. THE FIRST GENERATION

We are going to sandwich the covariant derivative (11) in between the following multiplets which exhaust the fermions of the first generation of leptons and quarks:

Multiplet[15]	$\bar{\psi}_L = (\bar{v}, \bar{e})_L$	e_R	$\bar{\psi}'_L = (\bar{u}, \bar{d})_L$	u_R	d_R
Y_{GWS}	1	-2	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$
Y_1(free)	$-Y_1^L$	Y_1^R	$-Y_1'^L$	Y_{1u}^R	Y_{1d}^R

When we calculate the Lagrangian

$$L = \bar{\psi}_L \not{D} \psi_L + \bar{e}_R \not{D} e_R + \bar{\psi}'_L \not{D} \psi'_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R \ ,$$

where $\not{D} = iD^u \gamma_\mu$, γ_μ are Dirac matrices, we get

$$L = L_{\text{kin}} + L_{\text{wk}} + L_{\text{em}} + L_{\text{neu}} \ ,$$

where L_{kin}, L_{wk}, and L_{em} are the kinetic, charged weak, and electromagnetic parts, respectively; they are exactly the same ones obtained in the GWS model. The difference with the GWS model appears in L_{neu} which we may write as

$$L_{\rm neu}=g\sum_{i=1,2}Z_i^{\mu}[\bar{e}\gamma_\mu(g_{V_i}^e-g_{A_i}^e\gamma_5)e+\bar{v}\gamma_\mu(g_{V_i}^v-g_{A_i}^v\gamma_5)v+\bar{u}\gamma_\mu(g_{V_i}^u-g_{A_i}^u\gamma_5)v+\bar{d}\gamma_\mu(g_{V_i}^d-g_{A_i}^d\gamma_5)d]\ ,$$

$Z_1=Z$, $Z_2=Z'$, and $g_{V_i}^a$, $g_{A_i}^a$ ($a=e,v,u,d$) are functions of the eight parameters, the three Weinberg angles θ,ξ,ψ, and of the five parameters Y_1^L, Y_1^R, $Y_1'^L$, Y_{1u}^R, Y_{1d}^R. At this point many authors pick up particular values for the parameters, or special relationships between them for the particular version of the model they are working with (see Refs. 4–9). Doing so, the model remains unrenormalizable due to the presence of several triangular anomalies [here there are many more anomalies than expected due to the presence of two U(1) groups]. Demanding a natural cancellation of the anomalies in the hadronic sector for the three-colored quarks with the anomalies of the leptonic sector, we get the constraints

$$\begin{aligned}&Y_1'^L=-Y_1^L/3\ ,\\&Y_1^L-Y_1^R=(4Y_{1u}^R+Y_{1d}^R-5Y_1'^L)/3\ ,\\&(Y_1^L)^2-(Y_1^R)^2=(Y_1'^L)^2-2(Y_{1u}^R)^2+(Y_{1d}^R)^2\ ,\\&2(Y_1^L)^3-(Y_1^R)^3=3(Y_{1u}^R)^3+3(Y_{1d}^R)^3-6(Y_1'^L)^3\ ,\end{aligned}\tag{12}$$

where the constraints came from the Feynman diagrams with a triangular loop of fermions (quarks or leptons), and one, two, and three gauge bosons at the vertices.

The set of equations (12) have, for a unique solution,[16]

$$\begin{aligned}&Y_1'^L=-Y_1^L/3\ ,\quad Y_1^R=2Y_1^L\ ,\\&Y_{1u}^R=-4Y_1^L/3\ ,\quad Y_{1d}^R=2Y_1^L/3\ ,\end{aligned}\tag{13}$$

which means that out of the original five free parameters for the first generation, we have only one left to play with: namely, Y_1^L. Now $g_{V_i}^a$ and $g_{A_i}^a$ may be written as

$$\begin{aligned}&g_{A_1}^e=g_{A_1}^d=-g_{A_1}^u=-g_{A_1}^v=-g_{V_1}^v\equiv g_1=-\frac{\sin\psi}{4\cos\theta}\{1-\sin\theta\cot\psi[aY_1^L(\tan\xi+\cot\xi)+\tan\xi]\}\ ,\\&g_{V_1}^e=\frac{\sin\psi}{4\cos\theta}\{1-4\sin^2\theta+3\sin\theta\cot\psi[aY_1^L(\tan\xi+\cot\xi)+\tan\xi]\}\ ,\\&g_{V_1}^u=-\frac{\sin\psi}{4\cos\theta}\{1-\tfrac{8}{3}\sin^2\theta+\tfrac{5}{3}\sin\theta\cot\psi[aY_1^L(\tan\xi+\cot\xi)+\tan\xi]\}\ ,\\&g_{V_1}^d=\frac{\sin\psi}{4\cos\theta}\{1-\tfrac{4}{3}\sin^2\theta+\tfrac{1}{3}\sin\theta\cot\psi[aY_1^L(\tan\xi+\cot\xi)+\tan\xi]\}\ ,\end{aligned}\tag{14}$$

and

$$g_{V_2,A_2}^a=g_{V_1,A_1}^a(\psi\to\psi+\pi/2)\ .$$

Notice that the functions in (14) reduce exactly to the ones in the standard model for $\psi=\pi/2$ in $g_{V_1}^a,g_{A_1}^a$ and for $\psi=0$ in $g_{V_2}^a,g_{A_2}^a$. According to (9) we have two ways of getting such limits: a natural one by demanding $M_{Z'}\gg M_Z$ be valid for both versions and a mathematical one by demanding the numerator in (9) to become zero by the relationship $aY_{\phi_1}=\sin^2\xi$ in (9a), and the appropriate one in (9b). The phenomenological analysis of the two neutral currents has attractive features and will be published elsewhere. The main fact is that the model $SU(2)\times U(1)\times U(1)'$ can do exactly the same as the GWS model does, and even better.

IV. FURTHER CONSTRAINTS

When introducing the second and third generations of quarks and leptons, we may keep the sequential model by demanding

$$Y_1^L(\text{1st generation})=Y_1^L(\text{2nd generation})=Y_1^L(\text{3rd generation})$$

or we may not. We have seen no reason for discarding the sequential model. But, are there further constraints between the free hypercharge parameters remaining: namely, Y_{ϕ_1}, Y_χ, and Y_1^L for version (a) and Y_{ϕ_1}, Y_{ϕ_2}, and Y_1^L for version (b)? Yes, if we demand the fermionic masses to be generated by Yukawa-type terms, as in the GWS model.

Let us work with version (a). Masses for the electron and the two quarks in the first generation may be generated by

$$\begin{aligned}L_{\rm int}=&G_e(\bar{\psi}_L\phi_1e_R+\text{H.c.})\\&+(G_d\bar{\psi}'_L\phi_1d_R+G_\mu\bar{\psi}'_L\tilde{\phi}_1u_R+\text{H.c.})\ ,\end{aligned}\tag{15}$$

where $\tilde{\phi}_1=-iT_2\phi_1^*$. Several comments are in order here. The main fact is that (15) has to be SU(2) invariant, as it is indeed; but it also must be U(1) and U(1)′ invariant, and it is this last invariance that imposes the new restriction, which is, for (15),

$$Y_{\phi_1}=-Y_1^L\ ,\tag{16}$$

where some relationships from (13) have been used. Equivalently, we may replace the entire expression in (15) by

$$\begin{aligned}L'_{\rm int}=&G'_e(\chi\bar{\psi}_L\phi_1e_R+\text{H.c.})\\&+(G'_d\chi\bar{\psi}'_L\phi_1d_R+G'_\mu\chi\bar{\psi}'_L\tilde{\phi}_1u_R+\text{H.c.})\end{aligned}\tag{17}$$

and the constraint now is

$$Y_{\phi_1}+Y_\chi=-Y_1^L\ .\tag{18}$$

It is important to notice that we cannot mix any one term in (15) with any one in (17) because it will imply $Y_\chi = 0$ and then we will not get the appropriate symmetry-breaking pattern.[13]

The only reason we see for preferring (15) as a mass term over (17) is because it is simpler, easiest to handle, and we do not have to worry about Y_χ in the entire calculation (as mentioned before, it will disappear out of the algebra when we define $V = vY_\chi$).

So, for version (a) with a mass term given by (15), we will have at the end only one free parameter to play with for the entire model, which is Y_1^L (Y_χ will remain hidden).

For version (b) we also have two alternatives for L_{int}: One is relation (15) and the other is $L'_{\text{int}} = L_{\text{int}}(\phi_1 \to \phi_2)$. Again, we cannot mix terms between the two alternatives because it will imply $Y_{\phi_1} = Y_{\phi_2}$ with the same consequences as before.[13] For each one of the two Yukawa Lagrangians for version (b) we will have the constraints $Y_{\phi_1} = -Y_1^L$, Y_{ϕ_2} free for L_{int} and $Y_{\phi_2} = -Y_1^L$, Y_{ϕ_1} free for L'_{int}. Both ways are totally equivalent. In any case we will have two free parameters to play with for version (b): Y_1^L and one of the Y_{ϕ_i}. After all, we notice that version (a) has the fewest number of parameters: only one.

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[10]For example, $a = 0$ and $b = 1$ is the model in Ref. 5; $a = b = 1$ is the model in Ref. 6, etc.

[11]The physical fields are related to the mathematical ones by the relationship

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta\sin\xi & \cos\theta\cos\xi & \sin\theta \\ \cos\psi\cos\xi + \sin\theta\sin\psi\sin\xi & -\sin\xi\cos\psi + \sin\psi\sin\theta\cos\xi & -\sin\psi\cos\theta \\ -\sin\psi\cos\xi + \cos\psi\sin\theta\sin\xi & \sin\psi\sin\xi + \cos\psi\sin\theta\cos\xi & -\cos\psi\cos\theta \end{pmatrix} \begin{pmatrix} C_\mu \\ C'_\mu \\ B^3_\mu \end{pmatrix} .$$

[12]For the particular model $a = 0$, $b \neq 0$, $\tan\xi = \sin\xi = 0$ ($\cos\xi = 1$) and we need only two Weinberg angles θ and ψ in order to diagonalize the mass matrix.

[13]If $Y_\chi = 0$ in version (a) or $Y_{\phi_1} = Y_{\phi_2}$ in version (b), the symmetry-breaking pattern will be SU(2)×U(1)×U(1)′ $\to U(1)_{Q'} \times U(1)_{Q''}$ instead of the desired one. This model could be suitable for dealing with the possible fifth force, as long as we assign a dynamical role to the scalar Higgs field.

[14]The covariant derivative for the GWS model is

$$iD_\mu = i\partial_\mu + g(W_\mu^\dagger T^- + \text{H.c.})/\sqrt{2} + eA_\mu \hat{Q} + \frac{gZ_\mu}{\cos\theta}(\hat{Q}\sin^2\theta - T_3) ,$$

where $\hat{Q} = T_3 + Y_{\text{GWS}}/2$.

[15]As usual $\psi'_L = (1-\gamma_5)\psi'/2; \psi'_R = (1+\gamma_5)\psi'/2$ for any Dirac field ψ'.

[16]Notice the unsuspected result $Y_1(\text{free}) = -Y_{\text{GWS}} Y_1^L$.