Dynamical model for composite quarks and leptons based on supersymmetric quantum chromodynamics

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In supersymmetric quantum chromodynamics (SQCD) with N "colors" and M "flavors" ($N < M$), quarks are generated as quasi-Nambu-Goldstone fermions in $\hat{c}_a \hat{w}_i$ and leptons as chiral fermions in $\epsilon^{abc}\hat{c}_a^C \hat{c}_b^C \hat{c}_c^C \hat{w}_i$ ($N = 4$) with antisymmetrized "colors" (a = three colors, i = weak isospins). Mass protection symmetry is $SU(N-1)_{L+R} \times SU(M-N+1)_L \times SU(M-N+1)_R \times [U(1)\text{'s}]$ supported by complementarity and an effective superpotential. The remaining chiral symmetries are necessarily all broken once supersymmetry is explicitly broken. Masses for quarks and leptons are found to be controlled by the scale $M_F = M_{SSB}(M_{SSB}/\Lambda_{sc})^\kappa$ with $\kappa = (M-N)/[2\rho - (M-N-2)]$ for $2\rho > M-N-2$, where M_{SSB} is the supersymmetry-breaking scale and Λ_{sc} is the scale of SQCD.

I. INTRODUCTION

In supersymmetric composite models of quarks and leptons, the lightness of quarks and leptons is linked to the Nambu-Goldstone mechanism.[1,2] In the absence of spontaneous supersymmetry (SUSY) breaking, quasi-Nambu-Goldstone fermions (QNGF's) are generated as massless particles in the Nambu-Goldstone superfields (NGS's) associated with spontaneous breakdown of global symmetry G to H (Ref. 2). If the subgroup H further includes chiral symmetry, composite fermions called chiral fermions[3] (CF's) will also be generated to satisfy anomaly-matching conditions on H (Ref. 4). Since QNGF's are controlled by the coset space G/H and CF's are controlled by the anomalies of H, the spectra of QNGF's and CF's are entirely determined by the subgroup H, which is strongly constrained by the underlying dynamics for composites.

The simplest underlying dynamics is given by supersymmetric quantum chromodynamics (SQCD) with N "colors" and M "flavors" that contains the gauge superfield $W_A^B = (\lambda, G_\mu)_A^B$ ($A,B = 1, \ldots, N$) and the matter superfields $\Phi^{(1)A}_{\ i} = (\phi^{(1)}, \psi^{(1)})_i^A$ and $\Phi^{(2)i}_{\ A} = (\phi^{(2)}, \psi^{(2)})^i_A$ ($A = 1, \ldots, N; i = 1, \ldots, M$). It possesses $G = SU(M)_L \times SU(M)_R \times U(1)_V \times U(1)_A$ with $Q_V = (1, -1, 0)$ for $(\Phi^{(1)}, \Phi^{(2)}, W)$ and $Q_A = (N-M, N, -M, 0)$ for $(\phi^{(1,2)}, \psi^{(1,2)}, \lambda, G_\mu)$, which undergoes spontaneous breakdown to $H = SU(M)_{L+R} \times U(1)_V$ for $N > M$, $H = SU(M)_{L+R} \times U(1)_A$ for $N = M$, and $H = SU(N)_{L+R} \times SU(M-N)_L \times SU(M-N)_R \times U(1)'_V \times U(1)'_A$ for $N < M$ (Refs. 5–7). Since the spontaneous SUSY breaking will be induced if $N > M$ (Refs. 5 and 6) the QNGF mechanism calls for SQCD with $N \le M$.

In SQCD with $N \le M$, if there are at least six "flavors" ($M \ge 6$) consisting of three colors of $\hat{c}_{a=1,2,3}$, one $B-L$ of $\hat{c}_0$ and two weak flavors of $\hat{w}_{i=1,2}$ with the $V-A$ coupling to the weak bosons $W^\pm$ for $\hat{w}_{Li}$, quark-lepton supermultiplets are generated as NGS's, $\hat{c}_{La,0}\hat{w}_{Li}$ and $\hat{c}_R^{Ca,0}\hat{w}_R^{Ci}$, for $N = 4$ and 6 with $M = 6$ (utilizing the minimal "flavor" number) and for $N = 4$ with $M = 8$ (utilizing two sets of the weak flavors). Dynamical examination on mass generation, however, implies phenomenologically unfavorable features:[8,9] The models contain (1) light leptoquarks in $\hat{c}_{LO}\hat{c}_R^{Ca}$ and $\hat{c}_{La}\hat{c}_R^{CO}$ associated with $[SU(4)_C]_L \times [SU(4)_C]_R \to [SU(4)_C]_{L+R}$ for $N = 4$ and 6 with $M = 6$ or (2) the appreciable mixing of quarks and leptons with their mirrors in $(\epsilon_{ijkl}\hat{w}_R^{Cj}\hat{w}_R^{Ck}\hat{w}_R^{Cl})\hat{c}_{La,0}$ and $(\epsilon^{ijkl}\hat{w}_{Lj}^A\hat{w}_{Lk}^B\hat{w}_{Ll}^C)\hat{c}_R^{Ca,0}$ (Ref. 10) for $N = 4$ with $M = 8$ (since $\hat{w}_L$ has the $V-A$ coupling).

In this paper we examine

$$G \to H = SU(N-1)_{L+R} \times SU(M-N+1)_L \times SU(M-N+1)_R \times [U(1)\text{'s}] \quad (N < M) ,$$

in which quarks and leptons are provided for $N = 4$ and their mirrors and leptoquarks turn out to be absent. The dynamical issue of supporting this breaking pattern is first examined by employing complementarity[3,11,12] and then by finding an effective superpotential that is taken to be of the Taylor-Veneziano-Yankielowicz type.[5] By using an effective Lagrangian, we find that masses of light composite fermions are characterized by the SUSY-breaking (SSB) scale M_{SSB} and the SQCD scale Λ_{sc} as $M_{SSB}(M_{SSB}/\Lambda_{sc})^\kappa$ with $\kappa = (M-N)/[2\rho - (M-N-2)]$ for $2\rho > M-N-2$.

II. SYMMETRY BREAKING

A. Complementarity

Complementarity relates two phases of massless SQCD, the Higgs phase and the confining phase,[11] and postulates that the same symmetry-breaking pattern and the same spectrum of massless particles are generated in both phases.[12] The anomaly matching that is trivial in the Higgs phase is automatic in the confining phase. In the Higgs phase, if the scalars $\phi^{(1,2)}$ develop nonvanishing vacuum expectation values (VEV's),

$$\langle \phi^{(1)}{}_i^A \rangle \,|_{\theta=0} = \langle \phi^{(1)}{}_i^A \rangle \propto \Lambda \delta_i^A \tag{1a}$$

$$(i, A = 1, \ldots, N-1)\ ,$$

$$\langle \phi^{(2)i}{}_A \rangle \,|_{\theta=0} = \langle \phi^{(2)i}{}_A \rangle \propto \Lambda \delta_i^A \tag{1b}$$

the symmetry $G \times SU(N)_{sc}^{loc}$ spontaneously breaks to

$$H_0 = SU(N-1)_{L+R} \times SU(M-N+1)_L \times SU(M-N+1)_R \times U(1)_w \times U(1)_c \times U(1)_\chi\ , \tag{2}$$

where $Q_w = [(M-N+1)Q_V - (Q_L + Q_R)]/M$, $Q_c = Q_w + Q_V + Q_{sc}$, $Q_\chi = [(M-N)(Q_L - Q_R) + (M-N+1)Q_A]/M$ with $Q_{L,R} = (M-N+1, 1-N)$ for $(i = 1, \ldots, N-1$, $i = N, \ldots, M)$, and $Q_{sc} = (1, 1-N)$ for $(A = 1, \ldots, N-1, A = N)$. In the confining phase, the same breaking $G \to H_0$ is generated by

$$\left\langle \sum_A \Phi^{(1)}{}_i^A \Phi^{(2)j}{}_A \right\rangle \Bigg|_{\theta=0} = \left\langle \sum_A \phi^{(1)}{}_i^A \phi^{(2)j}{}_A \right\rangle \propto {\Lambda_{sc}}^2 \delta_i^j \quad (i,j = 1, \ldots, N-1)\ . \tag{3}$$

The $U(1)_c$ charge is defined by $Q_c = Q_w + Q_V$ instead of $Q_c = Q_w + Q_V + Q_{sc}$. The spectra of massless superfields in both phases are shown in Table I. Composite fermions in $\hat{c}\,\hat{w}$ and $\hat{c}\,\hat{c}^{\,C}$ are QNGF's and other fermions are CF's. All anomaly-matching constraints are satisfied by these QNGF's and CF's.

To implement quarks and leptons, $SU(N-1)_{L+R}$ is identified with the color-$SU(3)_C$ symmetry $(N=4)$ while $SU(M-N+1)_{L,R}$ take care of the chiral version of the weak-isospin $SU(2)_w$ symmetry $(M = \text{odd} \geq 5)$. The scale of SQCD Λ_{sc} is taken to be about 1 TeV in order to account for the electroweak scale $G_F{}^{-1/2} \approx 300$ GeV. The "flavor" superfields consist of $\hat{c}_{a=1,2,3}$ and $\hat{w}_{i=1,\ldots,M-3}$. Since $\hat{c}_0$ is absent, no leptoquarks are generated. $B-L$ can be taken as $Q_c/(N-1)$. The model thus contains quarks as QNGF's in $\hat{c}_{La}\hat{w}_{Li}$ and $\hat{c}\,{}_R^{Ca}\hat{w}\,{}_R^{Ci}$ and leptons as CF's in $(\epsilon_{abc}\hat{c}\,{}_R^{Ca}\hat{c}\,{}_R^{Cb}\hat{c}\,{}_R^{Cc})\hat{w}_{Li}$ and $(\epsilon^{abc}\hat{c}_{La}\hat{c}_{Lb}\hat{c}_{Lc})\hat{W}\,{}_R^{Ci}$ (Ref. 10) without their mirrors. Generations can be ascribed to the copies of two weak flavors. Since $M < 3N = 12$ for the asymptotic freedom of SQCD, at most four generations $(M = 11)$ are allowed.

B. Effective superpotential

The symmetry breaking of $G \to H_0$ in the massless SQCD should be described by an effective superpotential W_{eff} containing composite superfields $\sum_{A=1}^{N} \Phi^{(1)}{}_i^A \Phi^{(2)j}{}_A$. An effective superpotential is taken to be of the Taylor-Veneziano-Yankielowicz type that consists of two parts:[5] $W_{eff} = W_{eff}^{(0)} + W_{eff}^{(1)}$, where $W_{eff}^{(0)}$ is responsible for the anomalous $U(1)_{anom}$ transformation $\delta\Phi^{(1,2)} = -i\Phi^{(1,2)}$. Composite superfields required are S, T, $Y^{(1,2)}$, and U denoted as

$$S = ({g_{sc}}^2/32\pi^2) W_A^B W_B^A\ , \tag{4a}$$

$$T_i^j = \Phi^{(1)}{}_i^A \Phi^{(2)j}{}_A\ , \tag{4b}$$

$$Y^{(1)}_{[i_1 \cdots i_N]} = \epsilon(A) \Phi^{(1)}{}_{i_1}^{A_1} \cdots \Phi^{(1)}{}_{i_N}^{A_N}\ , \tag{4c}$$

$$Y^{(2)[i_1 \cdots i_N]} = \epsilon(A) \Phi^{(2)i_1}{}_{A_1} \cdots \Phi^{(2)i_N}{}_{A_N}\ , \tag{4d}$$

TABLE I. (a) Transformation properties of superfields $\Phi^{(1,2)}$ and W under H in the confining phase. (x,y) in $U(1)_\chi$ denotes Q_χ for $(J=0, J=\frac{1}{2})$ of $\Phi^{(1,2)}$ and for $(J=\frac{1}{2}, J=1)$ of W. (adj denotes the adjoint.) (b) The same as in (a) but for massless superfields in the Higgs phase or in the confining phase.

(a)

Superfields	$SU(N)_{sc}^{loc}$	$SU(N-1)_{L+R}$	$SU(M-N+1)_L$	$SU(M-N+1)_R$	$U(1)_w$	$U(1)_c$	$U(1)_\chi$
$\Phi^{(1)}{}_{i=1,\ldots,N-1}^{A=1,\ldots,N}\ \hat{c}\,{}_L^A$	$\mathbf{N^*}$	$\mathbf{N-1}$	$\mathbf{1}$	$\mathbf{1}$	0	1	$(0, M-N+1)$
$\Phi^{(1)}{}_{i=N,\ldots,M}^{A=1,\ldots,N}\ \hat{w}\,{}_R^{CA}$	$\mathbf{N^*}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{M-N+1}$	-1	0	$(N-M, 1)$
$\Phi^{(2)i=1,\ldots,N-1}{}_{A=1,\ldots,N}\ \hat{c}\,{}_{RA}^C$	$\mathbf{N}$	$\mathbf{N-1^*}$	$\mathbf{1}$	$\mathbf{1}$	0	-1	$(0, M-N+1)$
$\Phi^{(2)i=N,\ldots,M}{}_{A=1,\ldots,N}\ \hat{w}_{LA}$	$\mathbf{N}$	$\mathbf{1}$	$\mathbf{M-N+1}$	$\mathbf{1}$	1	0	$(N-M, 1)$
W_A^B	**adj**	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(N-M-1, 0)$

(b)

Higgs phase	Confining	$SU(N-1)_{L+R}$	$SU(M-N+1)_L$	$SU(M-N+1)_R$	$U(1)_w$	$U(1)_c$	$U(1)_\chi$
$\hat{c}\,{}_L^{A=1,\ldots,N-1} + \hat{c}\,{}_{RA=1,\ldots,N-1}^C$	$\hat{c}_L \hat{c}\,{}_R^c$	$\mathbf{1+adj}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(0, M-N+1)$
$\hat{w}_{LA=1,\ldots,N-1}$	$\hat{c}_L \hat{w}_L$	$\mathbf{N-1}$	$\mathbf{M-N+1}$	$\mathbf{1}$	1	1	$(N-M, 1)$
$\hat{w}\,{}_R^{CA=1,\ldots,N-1}$	$\hat{c}\,{}_R^C \hat{w}\,{}_R^c$	$\mathbf{N-1}$[a]	$\mathbf{1}$	$\mathbf{M-N+1}$	-1	-1	$(N-M, 1)$
$\hat{w}_{LA=N}$	$[\hat{c}\,{}_R^C]^{N-1}\hat{w}_L^{\,a}$	$\mathbf{1}$	$\mathbf{M-N+1}$	$\mathbf{1}$	1	$1-N$	$(N-M, 1)$
$\hat{w}\,{}_R^{CA=N}$	$[\hat{c}_L]^{N-1}\hat{w}\,{}_R^{Ca}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{M-N+1}$	-1	$N-1$	$(N-M, 1)$

[a] $[\hat{c}\,{}_R^C]^{N-1} = ([\hat{c}\,{}_R^C]^{N-1})^A \propto \sum \epsilon_{i_1 \cdots i_{N-1}} (\sum \epsilon^{AA_1 \cdots A_{N-1}} \hat{c}\,{}_{RA_1}^{Ci_1} \cdots \hat{c}\,{}_{RA_{N-1}}^{Ci_{N-1}})$ and similarly for $[\hat{c}_L]^{N-1}$.

$$U^{[j_1\cdots j_{N-1}]}_{[i_1\cdots i_{N-1}]}=\epsilon_{A_1\cdots A_{N-1}A}\epsilon^{B_1\cdots B_{N-1}A}\times\Phi^{(1)A_1}_{i_1}\cdots\Phi^{(1)A_{N-1}}_{i_{N-1}}\times\Phi^{(2)j_1}_{B_1}\cdots\Phi^{(2)j_{N-1}}_{B_{N-1}}\ ,\tag{4e}$$

where the repeated indices are all summed and $\epsilon(A)$ denotes the antisymmetrization of the indices A_i. Quarks represented as QNGF's in $\hat{c}\,\hat{w}$ will be described by T and U and similarly for singlets in $\hat{c}\,\hat{c}^{\,C}$. Leptons as CF's in $[\hat{c}^{\,C}]^3\hat{w}$ are described by $Y^{(1,2)}$.

$W^{(0)}_{\rm eff}$ can be constructed from S and T only. The anomalous $\mathrm{U}(1)_{\rm anom}$ transformation of $\delta L_{\rm eff}=\delta W_{\rm eff}\,|_{\theta\theta}+\mathrm{H.c.}=2M(g_{\rm sc}{}^2/32\pi^2)F\tilde{F}$ can be translated into $\delta W_{\rm eff}=-2iMS$ since $S\,|_{\theta\theta}$ contains $F\tilde{F}$, leading to $T\partial W_{\rm eff}/\partial T=MS$. The invariance of $W_{\rm eff}\,|_{\theta\theta}$ under $\mathrm{U}(1)_A$ reads

$$[-M(S\partial/\partial S-1)+(N-M)\partial/\partial T]W_{\rm eff}=0\ .$$

By considering other invariances, one obtains

$$W^{(0)}_{\rm eff}=S\{\ln[S^{N-M}\det(T^j_i)/\Lambda_{\rm sc}{}^{3N-M}]+M-N\}\ .\tag{5}$$

The remaining $W^{(1)}_{\rm eff}$ is the function of $Z_{U,Y}=\det(T^j_i)/X_{U,Y}$:

$$W^{(1)}_{\rm eff}=S[f_U(Z_U)+f_Y(Z_Y)]\ ,\tag{6}$$

where

$$X_U=\left[\frac{1}{N_U}\epsilon(ii',jj')U^{[j'_1\cdots j'_{N-1}]}_{[i_1\cdots i_{N-1}]}U^{[j_1\cdots j_{N-1}]}_{[i'_1\cdots i'_{N-1}]}T^{j_N}_{i_N}\cdots T^{j_M}_{i_M}T^{j'_N}_{i'_N}\cdots T^{j'_M}_{i'_M}\right]^{1/2}\ ,\tag{7a}$$

$$X_Y=\frac{1}{N_Y}\epsilon(i,j)Y^{(1)}_{[i_1\cdots i_N]}Y^{(2)[j_1\cdots j_N]}T^{j_{N+1}}_{i_{N+1}}\cdots T^{j_M}_{i_M}\ ,\tag{7b}$$

with $N_U=[(N-1)!]^4[(M-N+1)!]^2$ and $N_Y=[N!]^2(M-N)!$. As is readily recognized the fields $Z_{U,Y}$ are neutral under $G\times\mathrm{U}(1)_{\rm anom}$. Since the condensation of (3) corresponds to $\langle T^j_i\rangle\,|_{\theta=0}\propto\delta^j_i$ $(i,j=1,\dots,N-1)$, the symmetry breaking $G\to H_0$ described by complementarity is realized if $W_{\rm eff}=W^{(0)}_{\rm eff}+W^{(1)}_{\rm eff}$ allows $\langle Z_U\rangle\,|_{\theta=0}\neq0$ and $\langle Z_Y\rangle\,|_{\theta=0}=0$. The formation of the condensate $\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle$ is thus essential for $G\to H_0$ in this kind of effective-Lagrangian approach. Namely, the formation of the condensate $\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle$ spontaneously chooses $\langle T^j_i\rangle\,|_{\theta=0}\propto\delta^j_i$ $(i,j=1,\dots,N-1)$ while that of $\langle Y^{(1)}_{[1\cdots N]}\rangle$ and $\langle Y^{(2)[1\cdots N]}\rangle$ spontaneously chooses $\langle T^j_i\rangle\,|_{\theta=0}\propto\delta^j_i$ $(i,j=1,\dots,N)$.[9]

Possible condensations generated by $W_{\rm eff}$ are $\pi_{ci}=\langle T^i_i\rangle\,|_{\theta=0}$ $(i=1,\dots,N-1)$, $\pi_{wi}=\langle T^{i+N-1}_{i+N-1}\rangle\,|_{\theta=0}$ $(i=1,\dots,M-N+1)$, $\pi_u=\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle\,|_{\theta=0}$, $\pi_{y1}=\langle Y^{(1)}_{[1\cdots N]}\rangle\,|_{\theta=0}$, $\pi_{y2}=\langle Y^{(2)[1\cdots N]}\rangle\,|_{\theta=0}$, and $\pi_\lambda=\langle S\rangle\,|_{\theta=0}$. The VEV's are determined by $W_{;I}=\partial W_{\rm eff}/\partial\pi_I=0$ $(I=ci,wi,u,ya,\lambda)$ with

$$W_{;ci}=[1+z_Uf'_U(z_U)+z_Yf'_Y(z_Y)](\pi_\lambda/\pi_{ci})\quad(i=1,\dots,N-1)\ ,\tag{8a}$$

$$W_{;wi}=[1+z_Yf'_Y(z_Y)](\pi_\lambda/\pi_{wi})\quad(i=1)\ ,\tag{8b}$$

$$W_{;wi=2,\dots,M-N+1}=\pi_\lambda/\pi_{wi}\quad(i=2,\dots,M-N+1)\ ,\tag{8c}$$

$$W_{;u}=-z_Uf'_U(z_U)(\pi_\lambda/\pi_u)\ ,\tag{8d}$$

$$w_{;ya}=-z_Yf'_Y(z_Y)(\pi_\lambda/\pi_{ya})\ ,\tag{8e}$$

$$W_{;\lambda}=\ln\left[(\pi_\lambda/\Lambda_{\rm sc}{}^3)^{N-M}\prod_{i=1}^{N-1}(\pi_{ci}/\Lambda_{\rm sc}{}^2)\times\prod_{i=1}^{M-N+1}(\pi_{wi}/\Lambda_{\rm sc}{}^2)\right]+f_U(z_U)+f_Y(z_Y)\ ,\tag{8f}$$

where $z_U=Z_U\,|_{\theta=0}=\prod_{i=1}^{N-1}\pi_{ci}/\pi_u$; $z_Y=Z_Y\,|_{\theta=0}=\prod_{i=1}^{N-1}\pi_{ci}\pi_{w1}/\pi_{y1}\pi_{y2}$. The solutions should be given by $\pi_{ci}\sim\Lambda_{\rm sc}{}^2$ and $\pi_u\sim\Lambda_{\rm sc}{}^{N-1}$ and $\pi_{wi}=\pi_{y1}=\pi_{y2}=\pi_\lambda=0$. Since $\pi_{y1}=\pi_{y2}=0$ are required to be exact constraints valid even in the presence of SUSY breaking, that is, the SUSY breaking should not generate $\pi_{y1,y2}\propto M_{SSB}^{(\rm power)}$, the form of $f_Y(Z_Y)$ can be taken as

$$f_Y(Z_Y)=hZ_Y{}^{-1}\ ,\tag{9a}$$

which yields $hY^{(1)}Y^{(2)}$ like the ordinary Higgs-boson coupling. As a result, Eq. (8e) is satisfied by $\pi_{y1,y2}=0$. From $W_{;\lambda}=0$ in Eq. (8f), the singularity due to $\pi_{wi}=\pi_\lambda=0$ in $W^{(0)}_{\rm eff}$ dictates $f_U(z_U)$ [because $f_Y(z_Y)=0$ from $\pi_{y1,y2}=0$]:

$$f_U(z_U)=-\ln\left[(\pi_\lambda/\Lambda_{\rm sc}{}^3)^{N-M}\prod_{i=1}^{M-N+1}(\pi_{wi}/\Lambda_{\rm sc}{}^2)\right]+\mathrm{const}\ ,\tag{9b}$$

where const represents the terms from $\pi_{ci}/\Lambda_{\rm sc}$. Since $\pi_\lambda/\pi_{wi}=0$ from Eqs. (8b) and (8c), Eq. (9b) is consistent if

$$f_U(Z_U)=\rho\ln[Z_U-\langle Z_U\rangle\,|_{\theta=0}]+\mathrm{regular}\quad(\rho>0)\ ,\tag{10}$$

where regular stands for the regular terms such as $\ln Z_U$. The singularity due to $\pi_\lambda/\pi_{wi}=0$ is transferred to the one due to $Z_U=\langle Z_U\rangle\,|_{\theta=0}$. One observes that QNGF's in T and U are placed in the singular part $\ln(Z_U-\langle Z_U\rangle\,|_{\theta=0})$ with $\langle Z_U\rangle\,|_{\theta=0}\neq0$ while CF's are contained in the regular part $\propto Z_Y{}^{-1}$ with $\langle Z_Y{}^{-1}\rangle\,|_{\theta=0}=0$. For comparison, we note that SQCD with $N=M$ is described by $W^{(1)}_{\rm eff}=-\rho\ln Z_Y$ $(0<\rho<1)$ (Ref. 8) leading to $\pi_i\sim\Lambda_{\rm sc}{}^2$

$(i=1,\ldots,M)$ and $\pi_{y1,y2}\sim\Lambda_{sc}{}^M$ without U and SQCD with $N<M$ possessing the chiral SU$(M-N)$ symmetry is by $W_{\rm eff}^{(1)}=S[\rho\ln(Z_Y-\langle Z_Y\rangle\mid_{\theta=0})+\text{regular}]$ $(\rho>0)$ (Ref. 9) leading to $\pi_i\sim\Lambda_{sc}{}^2$ $(i=1,\ldots,N)$ and $\pi_{y1,y2}\sim\Lambda_{sc}{}^N$ without U.

It should be noted that $W_{\rm eff}^{(0)}$ correctly produces the anomaly relation of the Konishi type[5,13] in the massive SQCD with

$$W_{\rm mass}=\sum_{i=1}^{M}m_i\Phi_i^{(1)}\Phi^{(2)i}\ .$$

In the effective superpotential, $W_{\rm mass}$ is equivalent to $W_{\rm mass}^{(2)}=\sum_{i=1}^M m_iT_i^i$ and $\partial(W_{\rm eff}^{(0)}+W_{\rm eff}^{(1)}+W_{\rm mass}^{(2)})/\partial\pi_i=0$ gives

$$\pi_\lambda=m_i\pi_{wi}\ . \tag{11}$$

Therefore, the form of $W_{\rm eff}^{(0)}$ is unique in this respect.

III. EFFECTIVE POTENTIAL

Our starting Lagrangian $L_{\rm eff}$ is given by $L_{\rm eff}=L_0+[(W_{\rm eff}+W_{\rm mass}^{(2)})\mid_{\theta\theta}+\text{H.c.}]-L_{\rm mass}$ with

$$L_0=\sum_I K_I(\Omega_I,\Omega_I^*)\mid_{\theta\theta\bar\theta\bar\theta}\ , \tag{12a}$$

$$\begin{aligned}L_{\rm mass}=\Bigg[&\sum_i\mu_{iL}^2\Bigg[\sum_j(\Lambda_{sc}{}^{-2}\mid T_i^j\mid^2+\Lambda_{sc}{}^{-(4N-6)}\mid U_i^j\mid^2)+\Lambda_{sc}{}^{-2(N-1)}\mid Y_i^{(1)}\mid^2\Bigg]\\&+\sum_j\mu_{jR}{}^2\Bigg[\sum_i(\Lambda_{sc}{}^{-2}\mid T_i^j\mid^2+\Lambda_{sc}{}^{-4(N-6)}\mid U_i^j\mid^2)+\Lambda_{sc}{}^{-2(N-1)}\mid Y^{(1)j}\mid^2\Bigg]\\&+\sum_i\mu_i{}^2(T_i^i+T_i^{i*})+m_\lambda(S+S^*)\Bigg]\Bigg|_{\theta=\bar\theta=0}\ ,\end{aligned} \tag{12b}$$

where

$$U_i^j=[(N-2)!]^{-1}\sum_{\{k,l\}}\epsilon^{k_1\cdots k_{N-2}}\epsilon_{l_1\cdots l_{N-2}}U_{[k_1\cdots k_{N-2}i]}^{[l_1\cdots l_{N-2}j]}\ ,$$

$$Y_i^{(1)}=[(N-1)!]^{-1}\sum_{\{k\}}\epsilon^{k_1\cdots k_{N-1}}Y_{[k_1\cdots k_{N-1}i]}^{(1)}$$

and similarly for $Y^{(2)j}$. L_0 is the kinetic term for the composite fields $\Omega_I=S$, T, $Y^{(1,2)}$, and U taken to be $\partial^2K/\partial\Omega_I^*\partial\Omega_J=\delta_{IJ}G_I{}^{-1}(\Omega_I^*\Omega_I)$; and $L_{\rm mass}$ to the SUSY-breaking mass terms $\mu_L{}^2\phi^{(1)*}\phi^{(1)}$, $\mu_R{}^2\phi^{(2)*}\phi^{(2)}$, $\mu^2\phi^{(1)}\phi^{(2)}$, and $m_\lambda\lambda\lambda$.

The quark-lepton model $(N=4)$ requires the following. (1) $\mu_i=$const and $\mu_{iL,R}=$const $(i=1,\ldots,N-1)$ for the color-SU$(N-1)$ symmetry. (2) $m_i(=m_{ci})=0$ $(i=1,\ldots,N-1)$ for the anomaly relation $m_{ci}\pi_{ci}=\pi_\lambda=0$ (Refs. 5 and 13). (3) $m_i=0$ $(i=N,\ldots,M)$ to avoid spontaneous SUSY breaking due to the instanton effect.[14] (4) $m_i=\mu_i=0$ $(i=N,\ldots,M)$ for $W_\mu^\pm$ and Z_μ based on the electroweak $[\text{SU}(2)_W]_L^{\rm loc}$ symmetry correlated to SU$(M-N+1)_L$. In addition to these constraints, the present phenomenology requires that $m_i\ll M_{\rm SSB}$ since the scalar partners of quarks and leptons have not been discovered. This requirement is consistent with the massless SQCD defined in the limit of $M_{\rm SSB}\gg m_i\to0$ but not with the massive SQCD in the limit of $m_i(\neq0)\gg M_{\rm SSB}\to0$.

The effective potential $V_{\rm eff}$ is written as

$$\begin{aligned}V_{\rm eff}=\;&G_T\left[\sum_{i=1}^{N-1}\mid W_{;ci}\mid^2+\sum_{i=1}^{M-N+1}\mid W_{;wi}\mid^2\right]+G_U\mid W_{;u}\mid^2+\sum_{a=1}^{2}G_{Y(a)}\mid W_{;ya}\mid^2\\&+G_S\mid W_{;\lambda}\mid^2+\sum_{i=1}^{M}[(\mu_{iL}{}^2+\mu_{iR}{}^2)\Lambda_{sc}{}^{-2}\mid\pi_i\mid^2+\mu_i{}^2\pi_i]+\sum_{i=1}^{N-1}(\mu_{iL}{}^2+\mu_{iR}{}^2)\Lambda_{sc}{}^{-(4N-6)}\mid\pi_u\mid^2\\&+\Lambda_{sc}{}^{-2(N-1)}\left[\left[\sum_{i=1}^{N-1}\mu_{iL}{}^2+\mu_{NL}{}^2\right]\mid\pi_{y1}\mid^2+\left[\sum_{i=1}^{N-1}\mu_{lR}{}^2+\mu_{NR}{}^2\right]\mid\pi_{y2}\mid^2\right]+m_\lambda\pi_\lambda\ .\end{aligned} \tag{13}$$

The conditions of $\partial V_{\rm eff}/\partial\pi_I=0$ are given by $\pi_{ya}=0$ and

$$G_TW_{;wi}^*(\pi_\lambda/\pi_{wi})=G_SW_{;\lambda}^*+M_{wi}{}^2+G_T'\mid\pi_{wi}\mid^2\sum_j\mid W_{;j}\mid^2\ , \tag{14a}$$

$$(1+\epsilon)G_TW_{;ci}^*(\pi_\lambda/\pi_{ci})=\eta X+(1+\epsilon)G_SW_{;\lambda}+M_{ci}{}^2+G_T'\mid\pi_{ci}\mid^2\sum_j\mid W_{;j}\mid^2\ , \tag{14b}$$

$$\epsilon G_UW_{;u}^*(\pi_\lambda/\pi_\mu)=\eta X+\epsilon G_SW_{;\lambda}-M_u{}^2-G_U'\mid\pi_u\mid^2\mid W_{;u}\mid^2\ , \tag{14b}$$

$$G_T\sum_jW_{;j}^*(\pi_\lambda/\pi_j)+\epsilon X+(N-M)G_SW_{;\lambda}^*+m_\lambda\pi_\lambda+\mid\pi_\lambda\mid^2G_S'\mid W_{;\lambda}\mid^2=0\ , \tag{14d}$$

with $\epsilon = zf'(z)$, $\eta = z[zf'(z)]'$, and $f(z) = \rho \ln(z - \langle z \rangle|_{\theta=0})$ $(z = z_U)$, where $M_{li}{}^2 = (\mu_{liL}{}^2 + \mu_{liR}{}^2)\pi_{li}{}^2 \Lambda_{sc}{}^{-2} + \mu_{li}{}^2 \pi_{li}$ $(l = w, c)$, $M_u{}^2 = \sum_{i=1}^{N-1} (\mu_{iL}{}^2 + \mu_{iR}{}^2)\pi_u{}^2 \Lambda_{sc}{}^{-(4N-6)}$, and $X = \sum G_T W_{;ci}(\pi_\lambda/\pi_{ci}) - G_U W_{;u}(\pi_\lambda/\pi_u)$.

By keeping leading terms of $(z - \langle z \rangle)^{-1}$ $(= \xi^{-1} \gg 1)$, i.e., $\epsilon \approx \rho z/\xi$ and $\eta \approx \rho z \langle z \rangle / \xi^2$, and by adopting canonical kinetic terms for composites, i.e., $G_I = f_I = \text{const}$, we find that, for real VEV's,

$$f_T(\pi_\lambda/\pi_{wi})^2 = f_S W_{;\lambda} + M_{wi}{}^2 , \tag{15a}$$

$$f_T(\rho\langle z \rangle \pi_\lambda/\xi\pi_c)^2 = [(\rho - 1) f_S W_{;\lambda} - (M_c + M_u{}^2)]/(N-2) , \tag{15b}$$

$$f_U(\rho\langle z \rangle \pi_\lambda/\xi\pi_u)^2 = [(N-1-\rho) f_S W_{;\lambda} + (N-1)(M_c{}^2 + M_u{}^2)]/(N-2) , \tag{15c}$$

$$(1+\rho) f_S W_{;\lambda} = -\left[\sum_j M_{wj}{}^2 + m_\lambda \pi_\lambda\right] + \xi[(\rho-1)(N-1)M_c{}^2 + (\rho - N + 1)M_u{}^2]/[\rho\langle z \rangle (N-2)] , \tag{15d}$$

where $\pi_c = \pi_{ci}$ and $M_c{}^2 = M_{ci}{}^2$ $(i = 1, \ldots, N-1)$. The SUSY breaking by $M_{c,u}{}^2$ is not compatible with Eqs. (15b) and (15c) because $f_S W_{;\lambda} \sim \xi M_{c,u}{}^2 \ll M_{c,u}{}^2$ while the breakings by $m_\lambda \pi_\lambda$ and $M_{wi}{}^2$ are allowed if $1 < \rho < N-1$ and $f_S W_{;\lambda} > 0$. This restriction on the allowed breakings is not specific to the canonical kinetic terms but also valid for noncanonical kinetic terms with $G_I' \neq 0$. Other general properties which do not depend on the choice of the kinetic terms are the ξ and $M_{\rm SSB}$ dependencies of π_I ($\xi \to 0$ as $M_{\rm SSB} \to 0$): (1) $\pi_{ci} \sim \Lambda_{sc}{}^2$ and $\pi_u \sim \Lambda_{sc}{}^{2(N-1)}$; (2) $\pi_{wi} \sim \xi \Lambda_{sc}{}^2$ ensured by the singular behavior of $W_{\rm eff}^{(1)}$: (3) $\pi_\lambda = \xi^2 M_{\rm SSB} \Lambda_{sc}{}^2$ for $M_{\rm SSB}$ dominated by m_λ and $\mu_{wiL,R}$ and $\pi_\lambda = \xi^{3/2} M_{\rm SSB} \Lambda_{sc}{}^2$ for $M_{\rm SSB}$ by μ_{wi}.

IV. MASS SPECTRUM OF COMPOSITES

Since $W_{\rm eff}$ contains U that mixes with T by $f(Z_U)$, $W_{\rm eff}$ should guarantee the decoupling of one combination of T and U as $M_{\rm SSB} \to 0$ in agreement with complementarity. The mass matrix M for quarks in NGS's, T and U, is calculated from $\partial^2 W_{\rm eff}/\partial\tilde{\pi}_I \partial\tilde{\pi}_J$ with $\tilde{\pi}_I = \pi_I/\sqrt{f_I}$ and is found to be

$$M = -(\pi_\lambda/\tilde{\pi}_{ca}\tilde{\pi}_{wi}) \times \begin{bmatrix} 1 + \langle z \rangle \rho/\xi & -\langle z \rangle \rho \tilde{\pi}_c/\xi\tilde{\pi}_u \\ -\langle z \rangle \rho \tilde{\pi}_c/\xi\tilde{\pi}_u & \langle z \rangle \rho \tilde{\pi}_c{}^2/\xi\tilde{\pi}_u{}^2 \end{bmatrix} , \tag{16}$$

on the $(\tilde{T}, \tilde{U})$ basis, for $a = 1, \ldots, N-1$ and $i = 1, \ldots, M-N+1$. We obtain

$$m_{\rm light} = -r^2 \pi_\lambda/(1+r^2)\tilde{\pi}_{ca}\tilde{\pi}_{wi} \tag{17a}$$

$$m_{\rm heavy} = -(1+r^2)\rho\langle z \rangle \pi_\lambda/\xi\tilde{\pi}_{ca}\tilde{\pi}_{wi} \qquad (r = \tilde{\pi}_c/\tilde{\pi}_u) \tag{17b}$$

where the lighter field $\sim r\tilde{T} + \tilde{U}$ and the heavier one $\sim \tilde{T} - r\tilde{U}$. For $M_{\rm SSB} = m_\lambda$ and/or $\mu_{wiL,R}$, $m_{\rm heavy} \sim \tilde{\pi}_\lambda/\xi\tilde{\pi}_{wi} \sim M_{\rm SSB}$ while, for $M_{\rm SSB} = \mu_{wi}$, $m_{\rm heavy} \sim M_{\rm SSB}/\xi^{1/2}$. The decoupling is thus only possible for $\pi_\lambda \sim \xi^{3/2} M_{\rm SSB} \Lambda_{sc}{}^2$ $(M_{\rm SSB} = \mu_{wi})$.[14] The SUSY breaking is thus required to be dominated by $\mu_{wi}{}^2 \pi_{wi}$ with $\pi_{wi} < 0$ for $f_S W_{;\lambda} > 0$ in Eq. (15d). The μ_{wi} dominance is allowed if $\mu_{wi}{}^2 \gg \mu_{ca}{}^2, \mu_{caL,R}{}^2$ while $\mu_{wiL,R}$ and m_λ can be the same order as μ_{wi} because $\pi_{wi}/\Lambda_{sc}{}^2$, $\pi_\lambda/\pi_{wi}\Lambda_{sc} \ll 1$. Since $\mu_{wi}{}^2 \pi_{wi}$ invalidates the mass generation of $W^\pm$ and Z based on $[\mathrm{SU}(2)_W]_L^{\rm loc}$, $W^\pm$, and Z should be described by composites.

We compute $\pi_{\lambda,wi}$ generated by the SUSY breaking from μ_{wi}, which are taken to be $\mu_{wi} = \mu_w$ $(i = 1, \ldots, n) \gg \mu_{wi}$ $(i = n+1, \ldots, M-N+1)$ that give $f_S W_{;\lambda} \sim -\sum_{i=1}^n M_{wi}{}^2$. Our results[16] are $1 < \rho < min(N-1, n-1)$ leading to $n \geq 3$, $r(= \tilde{\pi}_c/\tilde{\pi}_u) = \sqrt{(N-1-\rho)/(\rho-1)}$,

$$\pi_{wi=1,\ldots,n} = -\gamma\xi|\pi_c| , \tag{18a}$$

$$\pi_{wi=n+1,\ldots,M-N+1} = \sqrt{(n-1-\rho)/n}\,\pi_{wi=1,\ldots,n} , \tag{18b}$$

$$\pi_\lambda = \pm\sqrt{(n-\rho-1)/(\rho+1)} f_T (\gamma|\pi_c|\xi/f_T)^{3/2} \mu_w , \tag{18c}$$

$$\xi^{2\rho-(M-N-2)} = C(\mu_w/\Lambda_{sc})^{2(M-N)} , \tag{18d}$$

where

$$C = \gamma^3[\gamma(n-1-\rho)\pi_c/(1+\rho)f_T]^{M-N+1} \times [n/(n-1-\rho)]^{M-N+1-n} (\Lambda_{sc}{}^2/\pi_c)^{2N+1}$$

and

$$\gamma = (\rho\langle z \rangle)^{-1}[n(\rho-1)/(N-2)(n-1-\rho)]^{1/2} .$$

The decoupling is then realized by $\rho \leq M-N-1$.

Fermion masses are calculated to be

$$m_q = f_T r^2 |\pi_\lambda|/(1+r^2)|\pi_c \pi_{wi}| , \tag{19a}$$

$$m_l = h\sqrt{f_{Y(1)} f_{Y(2)}}\,|\pi_\lambda|/|(\pi_c)^{N-1}\pi_{wi}| , \tag{19b}$$

$$m_8 = f_T \rho\langle z \rangle |\pi_\lambda|/\xi\pi_c\pi_c , \tag{19c}$$

$$m_1 = f_T \rho\langle z \rangle r^2 |\pi_\lambda|/(3+r^2)\xi\pi_c\pi_c , \tag{19d}$$

where m_q stands for quarks as QNGF's, m_l for leptons as CF's, m_8 for the color octet as QNGF's, and m_1 for the color singlet as QNGF. The mass splitting between quarks and leptons is controllable owing to the difference of QNGF's from CF's, i.e.,

$$m_q/m_l \sim f_T \Lambda_{sc}{}^{2(N-1)}/h\sqrt{f_{Y(1)} f_{Y(2)}}$$

but is not predictive. Since $\pi_{wi} \sim \xi\pi_c$, the scale for these composite fermions, M_F, is determined to be

$$M_F(\sim f_T|\pi_\lambda/\pi_c\pi_{wi}|) = M_{\rm SSB}(M_{\rm SSB}/\Lambda_{sc})^{(M-N)/[2\rho-(M-N-2)]} \tag{20}$$

with $M_{\rm SSB} = \mu_w$ and $2\rho - (M-N-2) > 0$, from which $M_F \to$ small as $\rho \to$ small (>1). For $\rho = \frac{5}{4}$ with $M = 7$ and $n = 4$, one obtains $M_F = M_{\rm SSB}(M_{\rm SSB}/\Lambda_{sc})^2$ and estimates $M_F \sim 100$ MeV if $M_{\rm SSB} = 50$ GeV and $\Lambda_{sc} = 1$ TeV

($\sim G_F^{-1/2}$), which will fit the second generation. A similar result has also been obtained in SQCD with the chiral SU($M-N$) symmetry ($N<M$) although this case is unfavorable owing to the presence of appreciable mixing with mirror quarks and leptons.[9]

The scalar masses calculated are consistent with diagrammatic argument: Attach $\mu_{iL(iR)}{}^2$ to the line for $|\phi_i^{(1)}|^2(|\phi^{(2)i}|^2)$ and $\mu_i{}^2$ to the line for $\phi_i^{(1)}\phi^{(2)i}$. For example, masses for scalar quarks $m_{\rm sq}{}^2$ and scalar leptons $m_{\rm sl}{}^2$ are calculated as

$$m_{\rm sq}{}^2=\left[c_{TU}\mu_{wiV}{}^2\pm r^2 f_T\pi_{\rm ca}{}^{-1}\left|\mu_{wi}{}^2-c_{\rm sq}\sum_{j=1}^{n}\mu_{wj}{}^2(\pi_{wj}/\pi_{wi})\right|\right]\Big/(1+r^2)\ , \tag{21a}$$

$$m_{\rm sl}{}^2=f_Y(\mu_{wiV}{}^2\pm h\mu_{wi}{}^2)/\Lambda_{\rm sc}{}^{2(N-1)}\ , \tag{21b}$$

with $c_{TU}=[(f_T r^2/\Lambda_{\rm sc}{}^2)+(\pi_u/\Lambda_{\rm sc}{}^{2(N-1)})^2]$ and $c_{\rm sq}=(\rho-1)/(N-2)(\rho+1)$, where $\mu_{wiL}=\mu_{wiR}(=\mu_{wiV})$ and $f_{Y(1)}=f_{Y(2)}(=f_Y)$ are assumed for simplicity and $\mu_{wiV}{}^2\gg\mu_{caL,R}{}^2$ are used. Since μ_{wiV} can be the similar order to μ_w, $(m_{\rm sq,sl})^2>0$ imply $(m_{\rm sq,sl})^2(\sim\mu_{wiV}{}^2\pm\mu_{wi}{}^2)\sim\mu_{wiV}{}^2\sim\mu_w{}^2$.

The model contains light neutral and color-octet QNGF's with the mass $\sim M_F$ and color-octet scalars with the mass $\sim\mu_{cL,R}{}^2,\mu_c{}^2\ll\mu_w{}^2$. The color octets can develop heavy dynamical masses ($\gtrsim$ 100 GeV) due to QCD because of larger color charges.[17] However, the neutral fermion remains as light as quarks and leptons.

V. SUMMARY

We have demonstrated that quarks and leptons are generated, respectively, as QNGF's and CF's by SQCD with $N=4$. The approximate mass-protection symmetry can be taken as $H=\mathrm{SU}(N-1)_{L+R}\times\mathrm{SU}(M-N+1)_L\times\mathrm{SU}(M-N+1)_R\times\mathrm{U}(1)_w\times\mathrm{U}(1)_c\times\mathrm{U}(1)_Y$, which is specified by the condensations $\langle\hat{c}_{La}\hat{c}\,{}_R^{Cb}\rangle|_{\theta=0}\sim\pm\delta_a^b\Lambda_{\rm sc}{}^2$ and

$$\langle[\hat{c}_L]^{N-1}[\hat{c}\,{}_R^C]^{N-1}\rangle|_{\theta=0}\sim\pm\Lambda_{\rm sc}{}^{2(N-1)}\ .$$

The massless composite superfields consist of the Nambu-Goldstone superfields associated with this breaking and the chiral superfields required by complementarity to saturate the anomalies of H (Ref. 18).

The dynamical observation depends on the effective Lagrangian of SQCD of the Taylor-Veneziano-Yankielowicz type, which can incorporate the anomaly relation of $\langle\lambda\lambda\rangle=m_i\langle\phi_i^{(1)}\phi^{(2)i}\rangle$. Composite quarks being QNGF's contained in the superfields T and U and composite leptons being CF's in the superfields $Y^{(1,2)}$ are placed in the effective superpotential symbolically specified by $S\{\ln\det(T)+\rho\ln[\det(T)-(UU)^{1/2}]+hY^{(1)}Y^{(2)}\}$. The SUSY breaking is so constrained as to be induced by the SQCD gaugino mass given by $m_\lambda\lambda\lambda$ and/or the fundamental scalar masses given by $\mu_{wi}{}^2\hat{\omega}_{Li}\hat{\omega}\,{}_R^{Ci}|_{\theta=0}$, $\mu_{wiL}{}^2|\hat{\omega}_{Li}|^2|_{\theta=\bar\theta=0}$, and $\mu_{wiR}{}^2|\hat{\omega}\,{}_R^{Ci}|^2|_{\theta=\bar\theta=0}$ that break mass protection symmetries SU$(M-N+1)_{L,R}$ and U$(1)_Y$. All chiral symmetries in G can be broken [by the term $f_S W_{;\lambda}$ in Eqs. (15a)–(15d)] even if the SUSY breaking (by m_λ and/or $\mu_{wiL,R}=$const) preserves the chiral symmetries SU$(M-N+1)_{L,R}$. This feature is in accord with the QCD result: the spontaneous breaking of all chiral symmetries because SQCD turns out to be QCD well below the scale $M_{\rm SSB}$. However, the consistency with the anomaly matching at $M_{\rm SSB}=0$ in agreement with complementarity requires that the SUSY breaking be induced by $M_{\rm SSB}=\mu_{wi}$ from $\mu_{wi}\hat{\omega}_{Li}\hat{\omega}\,{}_R^{Ci}|_{\theta=0}$, which explicitly break remaining chiral symmetries.

The successive breaking of chiral symmetries is signaled by

$$\langle T_i^j\rangle_{\theta=0}(i,j=N,\ldots,M-N+1)=\langle\hat{\omega}_{Li}\hat{\omega}\,{}_R^{Ci}\rangle|_{\theta=0}\sim-\delta_i^j\Lambda_{\rm sc}{}^2(M_{\rm SSB}/\Lambda_{\rm sc})^{2\kappa}$$

and

$$\langle S\rangle|_{\theta=0}=\langle\lambda\lambda\rangle|_{\theta=0}\sim\pm M_{\rm SSB}\Lambda_{\rm sc}{}^2(M_{\rm SSB}/\Lambda_{\rm sc})^{3\kappa}$$

for $\kappa>0$, where $\kappa=(M-N)/[2\rho-(M-N-2)]$. The consistency with the anomaly matching on H at $M_{\rm SSB}=0$ is satisfied by $\rho\le M-N-1$. The quark-lepton masses are controlled by the scale M_F (Ref. 19):

$$M_F=M_{\rm SSB}(M_{\rm SSB}/\Lambda_{\rm sc})^\kappa\quad(M_{\rm SSB}\ll\Lambda_{\rm sc})\ ,$$

while masses for scalar partners are characterized by the scale $M_{\rm SSB}$.

Other features can be summarized as follows. (i) The mass splitting between quarks and leptons is due to the difference between QNGF's and CF's; (ii) the mass splitting between up and down weak flavors is not large enough because SU$(M-N+1)_{L,R}$ can be broken by the SU$(M-N+1)$-singlet term $f_S W_{;\lambda}$ as in Eq. (15a) and the large mass splitting is only possible by $G_S W_{;\lambda}\ll M_{wi}{}^2$ subject to the fine-tuning of $m_\lambda\pi_\lambda+\sum_j M_{wj}{}^2\approx0$; (iii) at most four generations can be included and large mass splittings among generations will also depend on the fine-tuning; and (iv) the nearly masslessness of neutrinos will be ascribed to the seesaw mechanism[21] based on $\langle\nu_R^C\rangle|_{\theta=0}\sim\Lambda_{\rm sc}$, whose inclusion in $W_{\rm eff}$ is, however, difficult.

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[1]H. Terazawa, Prog. Theor. Phys. **64**, 1763 (1980); W. A. Bardeen and V. Visnjic, Nucl. Phys. **B194**, 422 (1982); H. Miyazawa, in *Proceedings of the 4th INS Winter Seminar on the Structure of Quarks and Leptons,* Tokyo, 1981, edited by M. Yasuè (INS, University of Tokyo, Tokyo, 1982), p. 71.

[2]W. Buchmüller, S. T. Love, R. D. Peccei, and T. Yanagida, Phys. Lett. **115B**, 233 (1982); W. Buchmüller, R. D. Peccei, and T. Yanagida, *ibid.* **124B**, 67 (1983).

[3]G. 't Hooft, in *Recent Developments in Gauge Theories,* proceedings of the Cargese Summer Institute, 1979, edited by G. 't Hooft *et al.* (NATO Advanced Study Institute Series, Series B: Physics, Vol. 59) (Plenum, New York, 1980), p. 135.

[4]R. Barbieri, A. Masiero, and G. Veneziano, Phys. Lett. **128B**, 179 (1983); O. W. Greenberg, R. N. Mohapatra, and M. Yasuè, Phys. Rev. Lett. **51**, 1737 (1983); R. N. Mohapatra, J. C. Pati, and M. Yasuè, Phys. Lett. **151B**, 251 (1985); H. Yamada and M. Yasuè, *ibid.* **175B**, 169 (1986).

[5]T. R. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. **B218**, 493 (1983); G. Veneziano, Phys. Lett. **124B**, 357 (1983).

[6]D. Amati, G. Rossi, and G. Veneziano, Nucl. Phys. **B249**, 1 (1985), and references therein.

[7]T. R. Taylor, Phys. Lett. **125B**, 185 (1983); **128B**, 403 (1983); J. M. Gérard and H. P. Nilles, *ibid.* **129B**, 243 (1983).

[8]A. Masiero and G. Veneziano, Nucl. Phys. **B249**, 593 (1985); A. Masiero, R. Pettorino, M. Roncadelli, and G. Veneziano, *ibid.* **B261**, 633 (1985).

[9]M. Yasuè, Phys. Rev. D **35**, 355 (1987).

[10]Antisymmetrization with respect to "colors" is understood, i.e.,

$$\epsilon_{ijkl}(\hat{\omega}\,{}^{Cj}_{R}\hat{\omega}\,{}^{Ck}_{R}\hat{\omega}\,{}^{Cl}_{R})\hat{c}_{La,0}=\epsilon^{ABCD}\epsilon_{ijkl}(\hat{\omega}\,{}^{Cj}_{RA}\hat{\omega}\,{}^{Ck}_{RB}\hat{\omega}\,{}^{Cl}_{RC})(\hat{c}_{La,0})_D\ .$$

[11]E. Fradkin and S. H. Shenkar, Phys. Rev. D **19**, 3682 (1979); T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).

[12]S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B173**, 208 (1980); T. Matsumoto, Phys. Lett. **97B**, 131 (1980).

[13]K. Konishi, Phys. Lett. **135B**, 439 (1984). See also T. E. Clark, O. Piguet, and K. Sibold, Nucl. Phys. **B159**, 1 (1979).

[14]The instanton-induced amplitude for the $(N-1)$-massless $\psi^{(1,2)}$ and the gaugino λ is

$$\left\langle \left|\prod_{i=1}^{N-1}\phi_i^{(1)}\phi^{(2)i}\right|\lambda\lambda\right\rangle \propto \prod_{i=N}^{M} m_i\ ,$$

which indicates $\langle\lambda\lambda\rangle\neq 0$ contradicting $\pi_\lambda(=\langle\lambda\lambda\rangle)=m_i\pi_i=0$ $(i=N,\ldots,M)$ and spontaneous SUSY breaking is implied (Ref. 6).

[15]For $M_{\rm SSB}=m_\lambda$ and/or $\mu_{wiL,R}$, if one abandons the use of complementarity, the anomaly matching will be satisfied by introducing additional composite superfields $\hat{c}\,{}^{*}_{L}\hat{\omega}\,{}^{*}_{L}$ and $\hat{c}\,{}^{C*}_{R}\hat{\omega}\,{}^{C*}_{R}$ that cancel the effect from the extra quarks. The complex conjugate superfields $\Phi^{(1,2)*}$ should be L handed and are constructed as $\bar{D}\,\bar{D}[\overline{W}\,\overline{W}\exp(g_{\rm sc}V)\Phi^{(1,2)*}]=(G_{\mu\nu}G^{\mu\nu}\phi^{(1,2)*}+\cdots,\ \sigma_\rho\partial^\rho(G_{\mu\nu}\sigma^{\mu\nu}\lambda\phi^{(1,2)*})+\cdots)$, where $\bar{D}$ is the covariant derivative and V is the vector superfield, from which $\overline{W}$ is given by $DD\bar{D}V$. See S. Takeshita, Prog. Theor. Phys. **71**, 376 (1984).

[16]In the leading approximation, these solutions give [the right-hand side of Eq. (15d)]$=0$, which is actually of order $\mu_w{}^2\xi$. Its inclusion is possible by the modification of $f(z)=\ln(z-\langle z\rangle)^\rho \rightarrow f(z)=\ln\{(z-\langle z\rangle)^\rho[1+a_1(z-\langle z\rangle)]\}$ with $a_1\sim\mu_w{}^2$.

[17]W. J. Marciano, Phys. Rev. D **21**, 2425 (1980); S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B169**, 373 (1980).

[18]One of the dynamical differences between the Nambu-Goldstone and chiral superfields lies in the residual four-fermion couplings induced by the noncanonical kinetic terms. For explicit discussions, see, for example, R. R. Volkas and G. C. Joshi, Phys. Rev. D **35**, 1050 (1987).

[19]The scale M_F is calculated in other dynamical models: $M_F\sim M_{\rm SSB}$(SUSY mass)$/\Lambda_{\rm sc}$ for Nambu-Goldstone fermions in Ref. 20(a) and $M_F\sim M_{\rm SSB}{}^2/\Lambda_{\rm sc}$ if $M_{\rm SS}\gg$SUSY mass or $M_F\sim M_{\rm SSB}$(SUSY mass)$/\Lambda_{\rm sc}$ if $M_{\rm SSB}\ll$SUSY mass for QNGF's generated by F-type interactions of the Nambu–Jona-Lasinio type in Ref. 20(b). For models with a local SUSY, $M_F\lesssim\Lambda_{\rm sc}(\Lambda_{\rm sc}/M_{\rm Pl})^{n_\psi}$ with $n_\psi\geq 1$ is suggested in Ref. 20(c).

[20](a) H. Terazawa, in *Flavor Mixing in Weak Interactions,* proceedings of the Europhysics Topical Conference, Erice, Italy, 1984, edited by L. L. Chau (Plenum, New York, 1984), p. 655; (b) S. Takeshita and M. Yasuè, Phys. Lett. **164B**, 61 (1985); Phys. Rev. D **34**, 1847 (1986). See also J. F. Nieves, *ibid.* **33**, 1762 (1986); (c) J. C. Pati, M. Cvetič, and H. S. Sharatchandra, Phys. Rev. Lett. **58**, 851 (1987).

[21]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity, proceedings of the Supergravity Workshop,* Stony Brook, 1979, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number in the Universe,* Tsukuba, Ibaraki, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK, Tsukuba-gun, Ibaraki-ken, Japan, 1979).