Analysis of a quark model with charm and color for inclusive processes

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The $qq\bar{q}$ model of Mitra for meson-baryon processes is reformulated to study inclusive processes incorporating charm and color degrees of freedom. Predictions are made in terms of sum rules for various inclusive cross sections. Agreement of the sum rules with data is good for $d\sigma/dp_\perp^2$ and $d\sigma/dx$ in the available range of $p_\perp^2/x$. However, overall analysis of the flavor-symmetry breaking indicates that symmetries alone are not adequate and the underlying theory should contain a suitable mechanism to generate Regge-type behavior in the inclusive distributions. We also incorporate proton-wave-function modification due to gluons in our model.

I. INTRODUCTION

The physics of quarks has been receiving wide theoretical and experimental support during the last two decades.[1] While the discovery of color degrees of freedom[2] in e^+e^- physics has resolved their spin-statistics problem,[3] the discoveries of c and b quarks through $\psi(3.1)$ (Ref. 4) and Υ (9.4) (Ref. 5) families have established the existence of at least five species of quarks. Similarly through the field-theoretic notion of non-Abelian gluons,[6] one is able to explain the approximate scaling[7] observed in deep-inelastic lepton-hadron scattering.[8] Indeed the theory of quarks and gluons (QCD) is currently emerging as the most viable theory of strong interactions.[9] This has initiated several theoretical investigations to include QCD-motivated effects in existing quark models. In particular, the quark model of Le Yaouanc *et al.*[10] has been successful in explaining several aspects of hadron physics, viz., decays of hadrons, magnetic-moment ratios, and the like. Similarly the bag models[11] developed in the 1970s based on the confinement hypothesis played an important role in understanding the long-distance quark dynamics. More recently De Rújula, Georgi, and Glashow[12] and Isgur and collaborators[13] have systematically studied gluon-exchange effects within the quark models.

In purely hadron-hadron collisions at large $p_\perp$, two QCD-based models have emerged in recent years, viz., the quark-gluon scattering model of Feynman, Field, and Fox[14] and the constituent-interchange model (CIM) of Blankenbecler, Brodsky, and Gunion.[15] In the first class of models, the basic collision subprocesses responsible are $qq\to qq$, $qg\to qg$, and $gg\to gg$ ($q\to$quark, $g\to$gluon), while in the later class "higher-twist" subprocess $qM\to qM$ (where M denotes a meson) participates predominantly.

It is, however, not clear how these perturbative field-theoretic models will be relevant in the low-$p_\perp$ regime or in the usual fragmentation regions. In such regions, it is still meaningful to talk in terms of quark models with plausible incorporation of QCD and try to extract information about the underlying theory.[16]

A class of processes where such study can be pursued meaningfully is the inclusive processes,

$$A+B\to C+X\ .$$

Since only one particle's momentum needs to be measured, the experimentation of such processes is much easier. On the theoretical front, Feynman,[17] and Benecke, Chou, Yang, and Yen[18] advanced the hypothesis of scaling and limiting fragmentation for such processes supported by experiment. Subsequently, Mueller[19] related the inclusive distributions to a three-particle$\to$three-particle forward amplitude and then used a Regge-type theory for the latter.

The aim of the present paper is to make an analysis of this class of processes within a quark model proposed by Mitra[20] in the 1960s and later pursued by others.[21–23] The model was reformulated recently[24–26] for exclusive processes in view of the current development of quark physics.

Historically, the motivation of Mitra's model[20] was to take into account the nonadditivity idea in quark models by assuming the dominance of $qq\bar{q}$ scattering in meson-baryon processes. By additivity one means that collision processes are dominated by interaction involving two quarks at a time, the other quarks present in the projectile and the target remaining as spectators. Any deviation from this basic assumption implies nonadditivity. The necessity of nonadditivity grew in view of the limitations of additivity in explaining certain phenomena such as the sharp break in the high-energy proton-proton differential cross section[27] or in incorporating baryon-antibaryon processes[28] involving triple-quark exchanges. In Mitra's model, the nonadditivity idea was taken into account on the basis of the hierarchy of the elementarity of various hadrons. On this basis, mesons are considered more elementary[29] among hadrons as they have tighter structures than the baryons. This picture is reasonable if the force between a quark and an antiquark in a meson is appreciably stronger than that between a pair of quarks in a baryon. Indeed in QCD, the strength of the color force between a quark-antiquark pair is twice as large as the similar force between a pair of quarks inside the baryons.[30] The idea that mesons can be treated as elementary also seems to be reinforced in recent times by the pseudoscalar-emission-model results of Koniuk and

Isgur[31] and Godfrey and Isgur.[32] Under such an assumption, it is reasonable that meson-quark scattering $Mq \rightarrow Mq$ is a dominant mechanism for hadron-hadron collisions. However quarks, being the ultimate constituents of hadrons, are more elementary than mesons. Hence it is reasonable to assume that the active quark of a baryon will interact with $q\bar{q}$ composite structure of the meson so that the meson-quark scattering occurs through the three-body $qq\bar{q} \rightarrow qq\bar{q}$ processes (Figs. 1 and 2). Thus in the context of present-day hadron-hadron collisions, nonadditivity and a hierarchy basis justify the $Mq \rightarrow Mq$ transition to be the dominant mechanism in the meson-baryon processes similar to CIM of the large-$p_\perp$ regime.[15] The present model is, however, based on the quantum-mechanical discipline of nonrelativistic three-body scattering[33] rather than formal field theory. As it was constructed basically out of the available symmetries without explicit dynamics,[20] a theoretical study of the model and its experimental confrontation with recent data will, therefore, serve a useful purpose: it will provide information about when symmetries alone are adequate and when dynamics must be used.

Further, we will also endeavor here to incorporate proton-wave-function modification due to gluons in our model. To that end, we use a simple QCD-motivated ansatz as conjectured in the recent works of Lepage and Brodsky.[34]

In Sec. II we summarize the essential formalism of the model. We have considered up to the c quark and hence symmetries up to $SU(4)_f$ only. The large mass differences between the c quark and the (u,d,s) quarks indicate $SU(4)_f$ might be badly broken symmetry. As the mass differences between the b quark and (u,d,s) quarks are still larger, higher symmetries such as $SU(5)_f$ or beyond (due to the expected t quark) may not be very meaningful.

In Sec. III we discuss some of the results and their experimental comparisons. Section IV deals with summary and conclusions.

II. THE MODEL

A. The $qq\bar{q}$ wave function with spin, SU(4), and color degrees of freedom

The method of writing down the $qq\bar{q}$ wave function with spin, SU(4), and color degrees of freedom can be obtained from the procedure developed in Ref. 20. The spin part does not change; but we incorporate it to make the work self-contained. We denote the quark indices by 1 and 2, the antiquark index by 3. We shall write down the

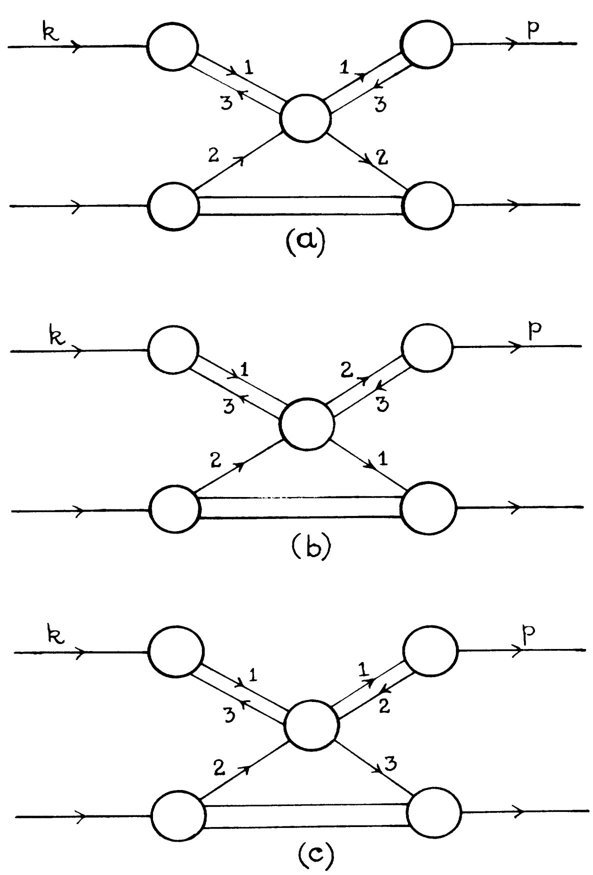

FIG. 1. Meson-baryon exclusive processes in the present model with (a) $\tilde{A}$, (b) $\tilde{B}$, and (c) $\tilde{C}$ terms of Eq. (2.58).

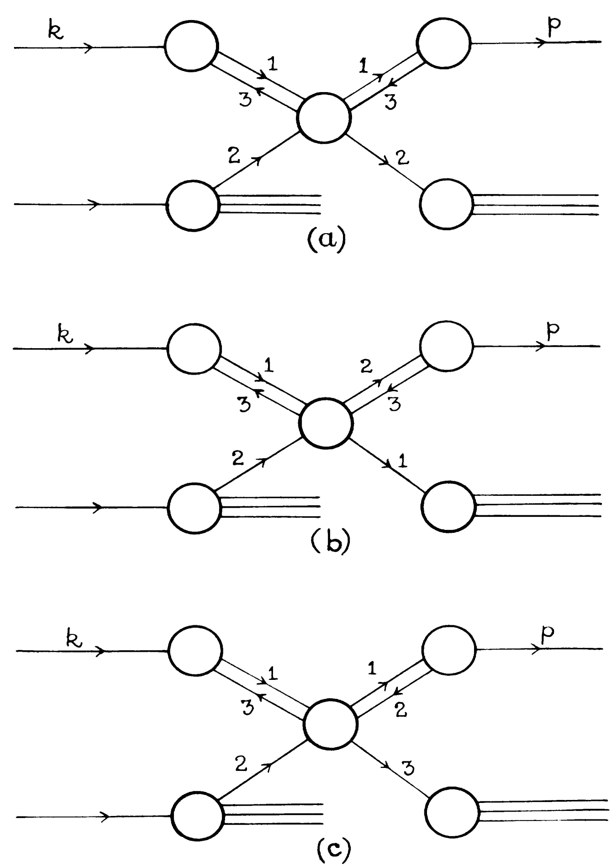

FIG. 2. Meson-baryon inclusive processes in the present model with (a) $\tilde{A}$, (b) $\tilde{B}$, and (c) $\tilde{C}$ terms of Eq. (2.58).

various functions using such a basis function to bring out the required condition of a pseudoscalar 15-plet of mesons made up of q_1 and $\bar{q}_3$ scattered on q_2.

1. Spin functions

Let α_i and β_i represent the spin-up and spin-down states for quark number 1. Appropriate to the collision of a pseudoscalar meson (P) made up of q_1 and $\bar{q}_3$ on q_2 (elastically or inelastically), the $qq\bar{q}$ wave function with even parity can be most conveniently represented by the basis function

$$\chi'(2,13)=\frac{1}{\sqrt{2}}\alpha_2(\alpha_1\beta_3-\alpha_3\beta_1)\equiv\chi \ . \tag{2.1}$$

For the odd-parity $qq\bar{q}$ wave function, on the other hand, it is most convenient to use the vector basis function

$$\boldsymbol{\chi}'(2,13)=i\boldsymbol{\sigma}^{(2)}\chi\equiv\boldsymbol{\chi} \ . \tag{2.2}$$

Since the spin-permutation operator $(12)_\sigma$ is defined as

$$(12)_\sigma=\tfrac{1}{2}(1+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}) \ , \tag{2.3}$$

the symmetric and antisymmetric spin functions will be given by

$$\chi_{s,a}=\frac{1}{\sqrt{2}}[1\pm(12)_\sigma]\chi \ , \tag{2.4}$$

and

$$\boldsymbol{\chi}_{s,a}=\frac{1}{\sqrt{2}}[1+(12)_\sigma]\boldsymbol{\chi} \tag{2.5}$$

for even- and odd-parity $qq\bar{q}$ wave functions, respectively. Besides χ and $\boldsymbol{\chi}$ defined by Eqs. (2.1) and (2.2), one has also another pair of basis functions $\boldsymbol{\chi}^s$ and $\chi_{\mu\nu}$ for a spin-quartet state of the $qq\bar{q}$ system. Here $\boldsymbol{\chi}^s$ and $\chi_{\mu\nu}$ are vector and tensor basis functions for odd- and even-parity $qq\bar{q}$ wave functions, respectively. They are defined as

$$\boldsymbol{\chi}^s=\frac{1}{\sqrt{2}}(i\boldsymbol{\sigma}^{(1)}-\tfrac{1}{2}\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\chi \ , \tag{2.6}$$

and

$$\chi_{\mu\nu}=\frac{1}{\sqrt{2}}(\sigma_\mu^{(1)}\sigma_\nu^{(2)}+\sigma_\nu^{(1)}\sigma_\mu^{(2)}-\tfrac{2}{3}\delta_{\mu\nu}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\chi \ . \tag{2.7}$$

As these two functions are totally symmetric in all particles, they correspond to the spin-quartet state of $qq\bar{q}$ system.

2. SU(4) functions

The basis of SU(4) part of the $qq\bar{q}$ wave function is chosen as

$$\phi=q_a^{(2)}(q_b^{(1)}\bar{q}_c^{(3)}-\tfrac{1}{4}\delta_{bc}q_d^{(1)}\bar{q}_d^{(3)}) \ , \tag{2.8}$$

where the SU(4) states are indicated by the subscripts a,b,c,d $(=1,2,3,4)$ and the individual particles are distinguished by the superscript (i). The corresponding symmetric and antisymmetric functions $\phi_{s,a}$ are now given by

$$\phi_{s,a}=\frac{1}{\sqrt{2}}[1\pm(12)_u^{\mathrm{SU}(4)}]\phi \ , \tag{2.9}$$

where the permutation operator $(12)_u$ in SU(4) space is given by

$$(12)_u^{\mathrm{SU}(4)}=\tfrac{1}{2}(\tfrac{1}{2}+\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)}) \ . \tag{2.10}$$

As noted earlier,[23-25] a more convenient expression for ϕ in terms of 15-plet of meson states can be obtained from the correspondence

$$(q_b^{(1)}\bar{q}_c^{(3)}-\tfrac{1}{4}\delta_{bc}q_d^{(1)}\bar{q}_d^{(3)})\leftrightarrow\frac{1}{\sqrt{2}}(\pi_\alpha\lambda_\alpha^{(1)})_c^b \ , \tag{2.11}$$

where $\lambda_\alpha^{(1)}$ $(\alpha=1,\ldots,15)$ are the SU(4) generators acting on quark number one and π_α are the corresponding meson fields. We also note that the matrices $\lambda_\alpha^{(1)}$ would represent the SU(3) Gell-Mann matrices with $\alpha=1,\ldots,8$ if transitions among (u,d,s) quarks alone are considered. Similarly, exclusion of transitions $(u\leftrightarrow s)$, $(d\leftrightarrow s)$ would reduce them to the Pauli matrices σ_α $(\alpha=1,2,3)$ representing the generators of the SU(2) isospin group.

Thus one has

$$\phi=\left[\frac{1}{\sqrt{2}}\pi_\alpha\lambda_\alpha^{(1)}\right]_c^b(q^{(2)})_a \tag{2.12}$$

which, in matrix notation, becomes

$$\phi=\frac{1}{\sqrt{2}}\pi_\alpha\lambda_\alpha^{(1)}\otimes q^{(2)} \ . \tag{2.13}$$

3. Color wave functions

In order to write the proper color wave functions, we note that the mesons are color singlets according to known phenomenology as well as QCD expectations.[30] The $qq\bar{q}$ wave functions should therefore be consistent with the boundary condition that quark number 1 and antiquark number 3 occur in the state

$$|\eta\rangle=\frac{R^{(1)}\bar{R}^{(3)}+B^{(1)}\bar{B}^{(3)}+G^{(1)}\bar{G}^{(3)}}{\sqrt{3}}$$
$$=\frac{1}{\sqrt{3}}\sum_1^3 q_\beta^{(1)}\bar{q}_\beta^{(3)} \ , \tag{2.14}$$

where the color indices β run from 1 to 3 corresponding to colors red (R), blue (B), and green (G), respectively. Hence the $qq\bar{q}$ color wave function is written as

$$\eta=\frac{1}{\sqrt{3}}\sum_\beta q_\alpha^{(2)}q_\beta^{(1)}\bar{q}_\beta^{(3)} \ . \tag{2.15}$$

The permutation operator between quark number 1 and 2 [$(12)_c$] for SU(3)$_c$ is

$$(12)_c=\tfrac{1}{2}(\tfrac{2}{3}+\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)}) \tag{2.16}$$

so that the symmetric and antisymmetric color wave functions will have the structure

$$\eta_{s,a}=\frac{1}{\sqrt{2}}[1\pm(12)_c]\eta \ . \tag{2.17}$$

For our subsequent analysis we will use the correspondence

$$\frac{1}{\sqrt{3}}\sum_{1}^{3} q_{\beta}^{(1)}\bar{q}_{\beta}^{(3)} \leftrightarrow \frac{1}{\sqrt{2}}\pi_0^c \lambda_0^{c(1)} , \tag{2.18}$$

where λ_0^c is a $SU(3)_c$-color-singlet matrix

$$\lambda_0^c = (\tfrac{2}{3})^{1/2} I \tag{2.19}$$

satisfying the general condition

$$\mathrm{Tr}(\lambda_i \lambda_j)^c = 2\delta_{ij} \quad (i=0,1,\ldots,n) \tag{2.20}$$

for any $SU(n)_f$.

4. *Space wave functions*

For positive-parity components of the space wave functions we need six pairs of wave functions defined as

$$(\psi_s^1,\psi_a^1),\ (\psi_s^2,\psi_a^2),\ (\psi_s^3,\psi_a^3)\ ,$$
$$(\psi_s^4,\psi_a^4),\ (\psi_s^{5\mu\nu},\psi_a^{5\mu\nu}),\ (\psi_s^{6\mu\nu},\psi_a^{6\mu\nu})\ .$$

These will be associated with spin functions χ_s, χ_a, and $\chi_{\mu\nu}$ as

$$(\psi_s^1,\psi_a^1),(\psi_s^2,\psi_a^2) \Longleftrightarrow \chi_s\ , \tag{2.21}$$

$$(\psi_s^3,\psi_a^3),(\psi_s^4,\psi_a^4) \Longleftrightarrow \chi_a\ , \tag{2.22}$$

$$(\psi_s^{5\mu\nu},\psi_a^{5\mu\nu}),(\psi_s^{6\mu\nu},\psi_a^{6\mu\nu}) \Longleftrightarrow \chi_{\mu\nu}\ . \tag{2.23}$$

Similarly for the negative-parity components of the $qq\bar{q}$ space wave functions, we need another set of six pairs of functions defined as

$$(\boldsymbol{\psi}_s^1,\boldsymbol{\psi}_a^1),\ (\boldsymbol{\psi}_s^2,\boldsymbol{\psi}_a^2),\ (\boldsymbol{\psi}_s^3,\boldsymbol{\psi}_a^3)\ ,$$
$$(\boldsymbol{\psi}_s^4,\boldsymbol{\psi}_a^4),\ (\boldsymbol{\psi}_s^5,\boldsymbol{\psi}_a^5),\ (\boldsymbol{\psi}_s^6,\boldsymbol{\psi}_a^6)\ .$$

They will be associated with spin functions $\boldsymbol{\chi}_s$, $\boldsymbol{\chi}_a$, and $\boldsymbol{\chi}^s$ as

$$(\boldsymbol{\psi}_s^1,\boldsymbol{\psi}_a^1),(\boldsymbol{\psi}_s^2,\boldsymbol{\psi}_a^2) \Longleftrightarrow \boldsymbol{\chi}_s\ , \tag{2.24}$$

$$(\boldsymbol{\psi}_s^3,\boldsymbol{\psi}_a^3),(\boldsymbol{\psi}_s^4,\boldsymbol{\psi}_a^4) \Longleftrightarrow \boldsymbol{\chi}_a\ , \tag{2.25}$$

$$(\boldsymbol{\psi}_s^5,\boldsymbol{\psi}_a^5),(\boldsymbol{\psi}_s^6,\boldsymbol{\psi}_a^6) \Longleftrightarrow \boldsymbol{\chi}^s\ . \tag{2.26}$$

5. *Total $qq\bar{q}$ wave functions*

Using the spin, SU(4), color, and space wave functions defined in Eqs. (2.1)–(2.26), one can write down the total $qq\bar{q}$ wave function consistent with Fermi-Dirac statistics for both positive- and negative-parity components. For the positive-parity case, it is given by

$$\begin{aligned}\psi^{(+)} = {} & (\psi_s^1\eta_s+\psi_a^1\eta_a)\chi_s\phi_a+(\psi_s^2\eta_a+\psi_a^2\eta_s)\chi_s\phi_s+(\psi_s^3\eta_s+\psi_a^3\eta_a)\chi_a\phi_s+(\psi_s^4\eta_a+\psi_a^4\eta_s)\chi_a\phi_a \\ & +(\psi_s^{5\mu\nu}\eta_s+\psi_a^{5\mu\nu}\eta_a)\chi_{\mu\nu}\phi_a+(\psi_s^{6\mu\nu}\eta_a+\psi_a^{6\mu\nu}\eta_s)\chi_{\mu\nu}\phi_s\end{aligned} \tag{2.27}$$

while for the negative-parity case, it has the representations,

$$\begin{aligned}\psi^{(-)} = {} & (\boldsymbol{\psi}_s^1\eta_a+\boldsymbol{\psi}_a^1\eta_s)\cdot\boldsymbol{\chi}_s\phi_s+(\boldsymbol{\psi}_s^2\eta_s+\boldsymbol{\psi}_a^2\eta_a)\cdot\boldsymbol{\chi}_s\phi_a+(\boldsymbol{\psi}_s^3\eta_a+\boldsymbol{\psi}_a^3\eta_s)\cdot\boldsymbol{\chi}_a\phi_a+(\boldsymbol{\psi}_s^4\eta_s+\boldsymbol{\psi}_a^4\eta_a)\cdot\boldsymbol{\chi}_a\phi_s \\ & +(\boldsymbol{\psi}_s^5\eta_s+\boldsymbol{\psi}_a^5\eta_a)\cdot\boldsymbol{\chi}^s\phi_a+(\boldsymbol{\psi}_s^6\eta_a+\boldsymbol{\psi}_a^6\eta_s)\cdot\boldsymbol{\chi}^s\phi_s\ .\end{aligned} \tag{2.28}$$

In Eqs. (2.27) and (2.28), while the spin, flavor, and color parts are explicitly given [Eqs. (2.4)–(2.7), (2.9), and (2.17)], the orbital parts are not given in any explicit forms. They will in general depend on the form of the interaction $V(p_1,p_2,p_3)$ in the $qq\bar{q}$ system since the Schrödinger equation reads,[20]

$$D(E)(\psi^{(+)}+\psi^{(-)}) = V(p_1,p_2,p_3)(\psi^{(+)}+\psi^{(-)})\ , \tag{2.29}$$

where

$$D(E) = \sum_{i=1}^{3} \frac{p_i{}^2}{2} - EM_q\ . \tag{2.30}$$

Here E is the energy and p_i $(i=1,2,3)$ are the momenta of the quark/antiquark number 1, 2, and 3 each with mass M_q. Thus to find the nature of the orbital functions, one needs the structure of the three-body potential $V(p_1,p_2,p_3)$. As in Ref. 20, their structures can be obtained in terms of spectator functions characteristic of three-body wave functions and factorable two-body forces.

Neglecting qq forces compared to $q\bar{q}$ forces, one then can write for ψ_s^1,ψ_a^1 of Eq. (2.27):

$$\psi_s^1 = D^{-1}(E)[u(p_{13})U_s^1(p_2)+u(p_{23})U_s^1(p_1)]\ , \tag{2.31}$$

$$\psi_a^1 = D^{-1}(E)[u(p_{13})U_a^1(p_2)-u(p_{23})U_a^1(p_1)]\ , \tag{2.32}$$

and similarly for other orbitals of Eqs. (2.27) and (2.28). Here $U_s^1(p)$ and $U_a^1(p)$ are the spectator functions while the potential function $u(p)$ occurs through the definition of the $q\bar{q}$ forces:

$$M_q\langle p\,|\,V\,|\,p'\rangle = -\lambda_0 u(p)u(p')\ . \tag{2.33}$$

However if the residues of these spectator functions are used as mere parameters, the dynamics of the model is completely hidden and algebraic structures of the scattering amplitudes or inclusive distributions are independent of the above assumption.

B. Calculation of the meson-quark amplitudes

The standard procedure for evaluation of the meson-quark amplitude within the spectator function technique[33] is first to obtain the space part by defining a set of parameters which represent the space overlaps of the initial to final $qq\bar{q}$ wave functions. Let these parameters be defined as $D_s^{(+)}$, $D_a^{(+)}$, $F_s^{(+)}$, $F_a^{(+)}$, $\bar{D}_s^{(+)}$, $\bar{D}_a^{(+)}$, $\bar{F}_s^{(+)}$, $\bar{F}_a^{(+)}$, $d_s^{(+)}$, $d_a^{(+)}$, $f_s^{(+)}$, $f_a^{(+)}$, which are to be associated with the positive-parity final-state $qq\bar{q}$ space wave functions ψ_s^1, ψ_a^1, ψ_s^2, ψ_a^2, ψ_s^3, ψ_a^3, ψ_s^4, ψ_a^4, $\psi_s^{5\mu\nu}$, $\psi_a^{5\mu\nu}$, $\psi_s^{6\mu\nu}$, $\psi_a^{6\mu\nu}$, respectively. Similarly we define the space overlaps of the negative-parity $qq\bar{q}$ wave function as $F_s^{(-)}$, $F_a^{(-)}$, $D_s^{(-)}$, $D_a^{(-)}$, $\bar{F}_s^{(-)}$, $\bar{F}_a^{(-)}$, $\bar{D}_s^{(-)}$, $\bar{D}_a^{(-)}$, $d_s^{(-)}$, $d_a^{(-)}$, $f_s^{(-)}$, $f_a^{(-)}$ corresponding to the space wave functions ψ_s^1, ψ_a^1, ψ_s^2, ψ_a^2, ψ_s^3, ψ_a^3, ψ_s^4, ψ_a^4, ψ_s^5, ψ_a^5, ψ_s^6, ψ_a^6, respectively. Thus, after taking into account space overlaps, $qq\bar{q} \to qq\bar{q}$ amplitudes for positive- and negative-parity components, respectively, become

$$\begin{aligned} R^{(+)} &= (D_s^{(+)}\eta_s + D_a^{(+)}\eta_a)\chi_s\phi_a + (F_s^{(+)}\eta_a + F_a^{(+)}\eta_s)\chi_s\phi_s + (\bar{D}_s^{(+)}\eta_s + \bar{D}_a^{(+)}\eta_a)\chi_a\phi_s \\ &\quad + (\bar{F}_s^{(+)}\eta_a + \bar{F}_a^{(+)}\eta_s)\chi_a\phi_a + (d_s^{(+)}\eta_s + d_a^{(+)}\eta_a)p_\mu p_\nu \chi_{\mu\nu}\phi_a + (f_s^{(+)}\eta_a + f_a^{(+)}\eta_s)p_\mu p_\nu \chi_{\mu\nu}\phi_s \,, \end{aligned} \tag{2.34}$$

$$\begin{aligned} R^{(-)} &= (F_s^{(-)}\eta_a + F_a^{(-)}\eta_s)(\mathbf{p}\cdot\boldsymbol{\chi}_s)\phi_s + (D_s^{(-)}\eta_s + D_a^{(-)}\eta_a)(\mathbf{p}\cdot\boldsymbol{\chi}_s)\phi_a \\ &\quad + (\bar{F}_s^{(-)}\eta_a + \bar{F}_a^{(-)}\eta_s)(\mathbf{p}\cdot\boldsymbol{\chi}_a)\phi_a + (\bar{D}_s^{(-)}\eta_s + \bar{D}_a^{(-)}\eta_a)(\mathbf{p}\cdot\boldsymbol{\chi}_a)\phi_s \\ &\quad + (d_s^{(-)}\eta_s + d_a^{(-)}\eta_a)(\mathbf{p}\cdot\boldsymbol{\chi}^s)\phi_a + (f_s^{(-)}\eta_a + f_a^{(-)}\eta_s)(\mathbf{p}\cdot\boldsymbol{\chi}^s)\phi_s \,, \end{aligned} \tag{2.35}$$

where $\mathbf{p}$ is the three-momentum of the final-state detected meson.

In the next step, we evaluate the spin overlaps which can be done separately for the positive- and negative-parity cases. For positive-parity functions Eq. (2.34) we take overlap with the basis function χ of Eq. (2.1) which corresponds to the initial meson-quark system. Similarly for the negative-parity functions Eq. (2.35) we must consider the overlaps with the initial state $(\mathbf{k}\cdot\boldsymbol{\chi})$, $\boldsymbol{\chi}$ being defined in Eq. (2.2) and $\mathbf{k}$ being the incident meson momentum. Ignoring the spin inelastic terms proportional to $\sigma_i^{(1)}\sigma_j^{(2)}$ that contribute only to the vector-meson production, one then has

$$\chi^\dagger \chi_a = \frac{1}{2\sqrt{2}} \,, \tag{2.36}$$

$$\chi^\dagger \chi_s = \frac{3}{2\sqrt{2}} \,, \tag{2.37}$$

$$(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger(\mathbf{p}\cdot\boldsymbol{\chi}_a) = \frac{(\mathbf{k}\cdot\mathbf{p}) + i\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{k}\times\mathbf{p})}{2\sqrt{2}} \,, \tag{2.38}$$

$$(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger(\mathbf{p}\cdot\boldsymbol{\chi}_s) = \frac{3[(\mathbf{k}\cdot\mathbf{p}) + i\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{k}\times\mathbf{p})]}{2\sqrt{2}} \,, \tag{2.39}$$

while terms like $\chi^\dagger \chi_{\mu\nu} p_\mu p_\nu$ and $(\mathbf{k}\cdot\boldsymbol{\chi})^\dagger(\mathbf{p}\cdot\boldsymbol{\chi}^s)$ being spin inelastic do not contribute to pseudoscalar mesons at all.

In the third step, we evaluate the SU(4) flavor overlaps of the resulting amplitude with the initial $qq\bar{q}$ state with respect to the basis function ϕ defined in Eq. (2.12). We should now take the SU(4) overlaps of the initial state $\pi_\alpha\lambda_\alpha^{(1)}$ [see Eq. (2.12)] with the final SU(4) states ϕ_s and ϕ_a defined in Eq. (2.9).

Thus we need to evaluate

$$\phi^\dagger\phi_s = (\pi_\beta\lambda_\beta^{(1)})^\dagger \frac{1}{\sqrt{2}}(\tfrac{5}{4} + \tfrac{1}{2}\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)})\pi_\alpha\lambda_\alpha^{(1)} \,, \tag{2.40}$$

$$\phi^\dagger\phi_a = (\pi_\beta\lambda_\beta^{(1)})^\dagger \frac{1}{\sqrt{2}}(\tfrac{3}{4} - \tfrac{1}{2}\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)})\pi_\alpha\lambda_\alpha^{(1)} \,. \tag{2.41}$$

This requires the evaluation of the products such as $\lambda_\beta^{(1)}\lambda_\alpha^{(1)}$ and $\lambda_\beta^{(1)}\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)}\lambda_\alpha^{(1)}$ in SU(4)-flavor space. To this end, we define

$$u_{\beta\alpha}^{(+)} = (if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda_\gamma^{(1)} \,, \tag{2.42}$$

$$u_{\beta\alpha}^{(-)} = (-if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda_\gamma^{(2)} \,, \tag{2.43}$$

$$D_{\alpha\epsilon}^{\beta\gamma} = d_{\beta\gamma\delta}d_{\alpha\epsilon\delta} \,, \tag{2.44}$$

$$\bar{D}_{\alpha\epsilon}^{\beta\gamma} = f_{\beta\gamma\delta}f_{\alpha\epsilon\delta} \,, \tag{2.45}$$

$$F_{\alpha\epsilon}^{\beta\gamma} = i(f_{\beta\gamma\delta}d_{\alpha\epsilon\delta} + d_{\beta\gamma\delta}f_{\alpha\epsilon\delta}) \,, \tag{2.46}$$

and note the SU(4) relations

$$\lambda_\beta^{(1)}\lambda_\alpha^{(1)} = \tfrac{1}{2}\delta_{\beta\alpha} + u_{\beta\alpha}^{(+)} \tag{2.47}$$

and

$$\begin{aligned} \lambda_\beta^{(1)}\lambda_\gamma^{(1)}\lambda_\gamma^{(2)}\lambda_\alpha^{(1)} &= \tfrac{1}{2}u_{\beta\alpha}^{(-)} + \tfrac{1}{2}\lambda_\beta^{(2)}\lambda_\alpha^{(1)} \\ &\quad + \lambda_\epsilon^{(1)}\lambda_\gamma^{(2)}(D_{\alpha\epsilon}^{\beta\gamma} - \bar{D}_{\alpha\epsilon}^{\beta\gamma} + F_{\alpha\epsilon}^{\beta\gamma}) \,. \end{aligned} \tag{2.48}$$

This, in turn, yields

$$\phi^\dagger\phi_s = \frac{1}{\sqrt{2}}\pi_\beta^\dagger\pi_\alpha[\tfrac{5}{4}(\tfrac{1}{2}\delta_{\beta\alpha} + u_{\beta\alpha}^{(+)}) + \tfrac{1}{4}u_{\beta\alpha}^{(-)} + E_{\alpha\beta}] \tag{2.49}$$

and

$$\phi^\dagger\phi_a = \frac{1}{\sqrt{2}}\pi_\beta^\dagger\pi_\alpha[\tfrac{3}{4}(\tfrac{1}{2}\delta_{\beta\alpha} + u_{\beta\alpha}^{(+)}) - \tfrac{1}{4}u_{\beta\alpha}^{(-)} - E_{\alpha\beta}] \,, \tag{2.50}$$

where we have defined

$$E_{\alpha\beta} = \tfrac{1}{4}\lambda_\alpha^{(1)}\lambda_\beta^{(2)} + \tfrac{1}{2}\lambda_\epsilon^{(1)}\lambda_\gamma^{(2)}(D_{\alpha\epsilon}^{\beta\gamma} - \bar{D}_{\alpha\epsilon}^{\beta\gamma} + F_{\alpha\epsilon}^{\beta\gamma}) \,. \tag{2.51}$$

In the fourth step one has to determine the color overlaps. To this end we need to evaluate

$$\eta^\dagger\eta_s = (\pi_0^c\lambda_0^{c(1)})^\dagger \frac{1}{\sqrt{2}}(\tfrac{4}{3} + \tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)})(\pi_0^c\lambda_0^{c(1)}) \,, \tag{2.52}$$

$$\eta^\dagger\eta_a = (\pi_0^c\lambda_0^{c(1)})^\dagger \frac{1}{\sqrt{2}}(\tfrac{2}{3} - \tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)})(\pi_0^c\lambda_0^{c(1)}) \,. \tag{2.53}$$

Since Eqs. (2.52) and (2.53) involve only color-singlet mesons π_0^c with $SU(3)_c$-singlet operator $\lambda_0^{c(1)}$, one has

$$\eta^\dagger\eta_s = \frac{\sqrt{2}}{3}\pi_0^{c\dagger}\pi_0^c(\tfrac{4}{3} + \tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)}) \,, \tag{2.54}$$

$$\eta^\dagger\eta_a=\frac{\sqrt{2}}{3}\pi_0^{c\dagger}\pi_0^c(\tfrac{2}{3}-\tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)}) . \tag{2.55}$$

For subsequent algebraic analysis, we define

$$S^c=\tfrac{4}{3}+\tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)} , \tag{2.56}$$

$$T^c=\tfrac{2}{3}-\tfrac{1}{2}\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)} . \tag{2.57}$$

Using these definitions and collecting all the results of spin, SU(4), and color overlaps, the meson-quark amplitude can be written by the following scalar:

$$\tilde{O}=\pi_\beta^\dagger\pi_\alpha[\tilde{A}(\tfrac{1}{2}\delta_{\beta\alpha}+u_{\beta\alpha}^{(+)})+\tilde{B}u_{\beta\alpha}^{(-)}+\tilde{c}E_{\alpha\beta}] , \tag{2.58}$$

where

$$\tilde{A}=\frac{1}{24\sqrt{2}}(3A^{(+)}+\bar{A}^{(+)}+3PA^{(-)}+P\bar{A}^{(-)}) , \tag{2.59}$$

$$\tilde{B}=\frac{1}{24\sqrt{2}}(3B^{(+)}-\bar{B}^{(+)}+3PB^{(-)}-P\bar{B}^{(-)}) , \tag{2.60}$$

$$\tilde{C}=\frac{4}{24\sqrt{2}}(3B^{(+)}-\bar{B}^{(+)}+3PB^{(-)}-P\bar{B}^{(-)}) , \tag{2.61}$$

with the following definitions:

$$P=\mathbf{k}\cdot\mathbf{p}+i\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{k}\times\mathbf{p}) , \tag{2.62}$$

$$A^{(\pm)}=5(F_s^{(\pm)}T^c+F_a^{(\pm)}S^c)+3(D_s^{(\pm)}S^c+D_a^{(\pm)}T^c) , \tag{2.63}$$

$$\bar{A}^{(\pm)}=3(\bar{F}_s^{(\pm)}T^c+\bar{F}_a^{(\pm)}S^c)+5(\bar{D}_s^{(\pm)}S^c+\bar{D}_a^{(\pm)}T^c) , \tag{2.64}$$

$$a^{(\pm)}=5(f_s^{(\pm)}T^c+f_a^{(\pm)}S^c)+3(d_s^{(\pm)}S^c+d_a^{(\pm)}T^c) , \tag{2.65}$$

$$B^{(\pm)}=(F_s^{(\pm)}T^c+F_a^{(\pm)}S^c)-(D_s^{(\pm)}S^c+D_a^{(\pm)}T^c) , \tag{2.66}$$

$$\bar{B}^{(\pm)}=(\bar{F}_s^{(\pm)}T^c+\bar{F}_a^{(\pm)}S^c)-(\bar{D}_s^{(\pm)}S^c+\bar{D}_a^{(\pm)}T^c) , \tag{2.67}$$

$$b^{(\pm)}=(f_s^{(\pm)}T^c+f_a^{(\pm)}S^c)-(d_s^{(\pm)}S^c+d_a^{(\pm)}T^c) . \tag{2.68}$$

Equation (2.58) represents the meson-quark amplitude in the model, which is our main result.

In $SU(4)_f$, Eq. (2.58) has three terms proportional to $\lambda^{(1)},\lambda^{(2)}$ and $\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)}$ representing the flavor-exchange effects. In $SU(3)_c$ it has, on the other hand, two terms proportional to 1 and $\boldsymbol{\lambda}^{c(1)}\cdot\boldsymbol{\lambda}^{c(2)}$ due to Eqs. (2.56) and (2.57).

From group-theoretical point of view, we note that in $SU(4)_f$ $qq\bar{q}$ has a decomposition

$$(4\otimes4\otimes\bar{4})_f=4_a\oplus36\oplus4_b\oplus20 , \tag{2.69}$$

while in $SU(3)_c$ it has a similar decomposition

$$(3\otimes3\otimes\bar{3})_c=3_a\oplus6\oplus3_b\oplus15 . \tag{2.70}$$

However, since $q\bar{q}$ mesons are color singlet according to QCD,[30] Eq. (2.70) is reduced to

$$[3_c\otimes(3\otimes\bar{3})_c]_{\text{singlet}}=3_c . \tag{2.71}$$

Thus for color-singlet $q\bar{q}$ mesons, only the color elastic amplitude $3_c\to3_c$ operates instead of several nonsinglet-color transitions allowed by Eq. (2.70).

We further note that the double-exchange terms $(\lambda_\alpha^{(1)}\lambda_\beta^{(2)})_f$ or $(\boldsymbol{\lambda}^{(1)}\cdot\boldsymbol{\lambda}^{(2)})_c$, which represent higher-order flavor and color exchanges, are expected to be suppressed in the present model.[20] Under this approximation, Eqs. (2.63)–(2.68) reduce to

$$A^{(\pm)}=\tfrac{2}{3}[5(F_s^{(\pm)}+2F_a^{(\pm)})+3(2D_s^{(\pm)}+D_a^{(\pm)})] , \tag{2.72}$$

$$\bar{A}^{(\pm)}=\tfrac{2}{3}[3(\bar{F}_s^{(\pm)}+2\bar{F}_a^{(\pm)})+5(2\bar{D}_s^{(\pm)}+\bar{D}_a^{(\pm)})] , \tag{2.73}$$

$$a^{(\pm)}=\tfrac{2}{3}[5(f_s^{(\pm)}+2f_a^{(\pm)})+3(2d_s^{(\pm)}+d_a^{(\pm)})] , \tag{2.74}$$

$$B^{(\pm)}=\tfrac{2}{3}[(F_s^{(\pm)}+2F_a^{(\pm)})-(2D_s^{(\pm)}+D_a^{(\pm)})] , \tag{2.75}$$

$$\bar{B}^{(\pm)}=\tfrac{2}{3}[(\bar{F}_s^{(\pm)}+2\bar{F}_a^{(\pm)})-(2\bar{D}_s^{(\pm)}+\bar{D}_a^{(\pm)})] , \tag{2.76}$$

$$b^{(\pm)}=\tfrac{2}{3}[(f_s^{(\pm)}+2f_a^{(\pm)})-(2d_s^{(\pm)}+d_a^{(\pm)})] . \tag{2.77}$$

C. Application to inclusive processes

In order to apply the model to inclusive process $A+B\to C+X$, as a first step, we need to consider the exclusive meson-baryon amplitude

$$A_{\text{if}}=\langle\psi_i\,|\,\tilde{O}\,|\,\psi_f\rangle . \tag{2.78}$$

Here $\tilde{O}$ is as defined in Eq. (2.58), ψ_i and ψ_f are initial and final $3q$ composites of the baryons. ψ's have the following spin-, flavor-, space-cum-color structures in $\mathbf{20}_m$ of $SU(4)_f$:

$$\psi_{20_m}=\psi^s\eta^a\left[\frac{\chi'\phi'+\chi''\phi''}{\sqrt{2}}\right] , \tag{2.79}$$

where ψ^s and η^a, respectively, denote the symmetric space and the antisymmetric color components of the baryon wave functions. Similarly, (χ',χ'') are the conventional mixed symmetric spin states while (ϕ',ϕ'') are the similar quantities in $SU(4)_f$ space.

As a second step, we take modulus square of the exclusive amplitude Eq. (2.78), sum over a complete set of states, and obtain the inclusive distribution

$$\Delta^{1/2}(2\pi)^3(2E_p)\frac{d^3\sigma}{dp^3}\sim\langle\psi_i\,|\,\tilde{O}^\dagger\tilde{O}\,|\,\psi_i\rangle , \tag{2.80}$$

where E_p and p are the energy and momentum of the detected particle, and

$$\begin{aligned}\Delta^{1/2}&\equiv\Delta^{1/2}(s,m_a{}^2,m_b{}^2)\\&=(s^2+m_a{}^4+m_b{}^4-2sm_a{}^2-2s_{m_b}{}^2-2m_a{}^2m_b{}^2)^{1/2} ,\end{aligned} \tag{2.81}$$

m_a,m_b being the masses of the particles A and B, while $\sqrt{s}$ is the center-of-mass energy. If masses are neglected compared with the c.m. energy in the high-energy range, one has

$$(2\pi)^3(2E_p)\frac{d^3\sigma}{dp^3}\sim\frac{1}{s}\langle\psi_i\,|\,\tilde{O}^\dagger\tilde{O}\,|\,\psi_i\rangle . \tag{2.82}$$

Equation (2.82) shows that for equal c.m. energies, $\langle\psi_i\,|\,\tilde{O}^\dagger\tilde{O}\,|\,\psi_i\rangle$ will measure the relative strengths of various single-particle distributions:

$$\langle\psi_i\,|\,\tilde{O}^\dagger\tilde{O}\,|\,\psi_i\rangle\sim\frac{2E_p}{\sigma_T}\frac{d^3\sigma}{dp^3} . \tag{2.83}$$

As a consequence, the inclusive cross section $\sigma(A+B\to C+X)$ for pseudoscalar meson production will have the following simple structure in the model:

$$\sigma(A+B\to C+X)=A_q u+B_q v+C_q w \; , \qquad (2.84)$$

where u, v, and w are certain calculable flavor factors and A_q, B_q, and C_q are functions of the residues $(D^{(\pm)}_{s,a}, F^{(\pm)}_{s,a}, \bar{D}^{(\pm)}_{s,a}, \bar{F}^{(\pm)}_{s,a}, d^{(\pm)}_{s,a}, f^{(\pm)}_{s,a})$, incident and final meson momenta, and the scattering angle.

It is interesting to note that the 24 space wave functions occurred in Eqs. (2.27) and (2.28) yield only three model parameters in the inclusive cross sections Eq. (2.84). Hence it is expected to have phenomenological utility, while keeping its results as model independent as possible, even within the general framework.

D. Gluons in the model

Gluons are important ingredients of the current theory of quarks (QCD). Besides their traditional role in explaining approximate scaling,[7] they also modify the hadron wave functions. Recently, Lepage and Brodsky[34] have proposed that gluon exchange generates a spin-flavor antisymmetric component in the baryon wave function in addition to the standard symmetric part. More recently[35,36] such modifications have been considered within the framework of quark models. In the present context, if QCD generates a spin-flavor-antisymmetric part besides the spin-flavor-symmetric state Eq. (2.79) the baryon wave function assumes the structure

$$\psi=\eta^a\left[\cos\phi\,\psi^s\frac{\chi'\phi'+\chi''\phi''}{\sqrt{2}}+\sin\phi\,\psi^a\frac{\chi'\phi''-\chi''\phi'}{\sqrt{2}}\right] , \qquad (2.85)$$

where the angle ϕ is a measure of the quark-gluon interaction at finite energies, and $\psi^a\neq\psi^s$ implies that the space wave function of the baryon is modified consistent with Fermi-Dirac statistics. As the second term of Eq. (2.85) has its origin in perturbative QCD, *a priori* there is no reason to believe that it is applicable to inclusive processes where momentum transfer is not very large. We therefore assume it to be a simple QCD-motivated ansatz within the framework of nonrelativistic quark models. Under this assumption, Eq. (2.83) is modified to

$$\cos^2\phi\langle\psi_q\,|\,\tilde{O}^{\dagger}\tilde{O}\,|\,\psi_q\rangle+\sin^2\phi\langle\psi_g\,|\,\tilde{O}^{\dagger}\tilde{O}\,|\,\psi_g\rangle \sim \frac{2E_p}{\sigma_T}\frac{d^3\sigma}{dp^3} , \qquad (2.86)$$

where

$$\psi_q=\eta^a\psi^s\frac{\chi'\phi'+\chi''\phi''}{\sqrt{2}} \qquad (2.87)$$

and

$$\psi_g=\eta^a\psi^a\frac{\chi'\phi''-\chi''\phi'}{\sqrt{2}} . \qquad (2.88)$$

The inclusive cross section in the model is then modified from Eq. (2.84) to

$$\begin{aligned}\sigma(A+B\to C+X)=&(A_q\cos^2\phi+A_g\sin^2\phi)u\\&+(B_q\cos^2\phi+B_g\sin^2\phi)v\\&+(C_q\cos^2\phi+C_g\sin^2\phi)w \; ,\end{aligned} \qquad (2.89)$$

where A_g,B_g,C_g contain the modifications of the original residue functions A_q,B_q,C_q due to gluons. In Sec. III we will discuss some of the consequences of the Eqs. (2.84) and (2.89).

III. PREDICTIONS OF THE MODEL

Using Eq. (2.58) in Eq. (2.83) and neglecting the $\tilde{C}$ term in leading order, we can obtain a large number of relations among various inclusive cross sections $(2E_p/\sigma_T)d^3\sigma/dp^3$, $d\sigma/dp_\perp^2$ or $d\sigma/dx$ (E_p = energy of the detected particle, σ_T = total cross section). Denoting the cross sections as $\sigma(A+B\to C+X)$, we record a few of them that can be tested with available data.[37,38]

TABLE I. Comparison of Eqs. (3.1) and (3.2) with data (Ref. 37) in terms of $d\sigma/dp_\perp^2$.

Process / $P_{\rm lab}$ (GeV/c) / $p_\perp^2$ [(GeV/c)^2]	$d\sigma/dp_\perp^2$ in mb/(GeV/c)^2: $K^-p\to\bar{K}^0x$ 10.1	$\pi^+p\to\pi^+x$ 8	$K^+p\to K^0x$ 8.2	$\pi^-p\to\pi^-x$ 16	$K^-p\to\pi^-x$ 9	Eq. (3.1) LHS	Eq. (3.1) RHS	Eq. (3.2) LHS	Eq. (3.2) RHS
0.0	34	414.15	8.0	295	130	887.5	896.3	925.62	896.3
	±0	±12.6	±0.4	±5	±15	±15.13	±25.2	±19.68	±25.2
0.1	22	153.1	5.2	120	22.5	361.63	350.2	367.03	350.2
	±3	±17.6	±0.2	±10	±3.5	±30.06	±41.2	±31.09	±41.2
0.2	12.5	74.05	3.3	50	5	151.03	173.1	151.56	173.1
	±1.5	±6.25	±0.3	±2	±1	±6.09	±15.5	±6.31	±15.5
0.3	8	42.7	2.25	28	2.3	84.7	101.4	84.72	101.4
	±1	±2.5	±0.25	±2	±0.7	±6.08	±7.0	±6.22	±7.0
0.4	4.85	25.1	1.45	18	0.35	54.45	59.9	54.11	59.9
	±0.35	±2.5	±0.15	±1	±0.21	±3.05	±5.7	±3.06	±5.7
0.5	3.1	20	1.0	12.5		37.81	46.2		
	±0.3	±0	±0.1	±0.5		±1.53	±0.6		

TABLE II. Comparison of Eq. (3.1) with data (Ref. 37) in terms of $d\sigma/dx$.

Process / P_{lab} (GeV/c) / x	$d\sigma/dx$ (mb) $K^-p\to\bar{K}^0x$ 10.1	$\pi^+p\to\pi^+x$ 8	$K^+p\to K^0x$ 8.2	$\pi^-p\to\pi^-x$ 8	Eq. (3.1) LHS	RHS
0.1	12±1	15.53±1.41	8.25±0.25	15.8±0.36	49.98±1.16	55.06±4.82
0.2	11.5±0.5	12.18±0.18	7.5±0.5	13.24±0.18	42.06±0.7	47.36±1.36
0.3	9.5±0.25	10.23±0	7.0±0.5	11.3±0	36.09±0.16	39.46±0.5
0.4	8±0.5	8.83±0	5.5±0.5	9.35±0.18	29.77±0.7	33.65±1.0
0.5	5.75±0.25	7.59±0.53	4.25±0.25	7.59±0.18	24.1±0.62	26.68±1.56
0.6	4.25±0.25	7.42±0.36	3.25±0.25	7.42±0.36	23.28±1.16	23.34±1.22

A. Sum rules

1. Sum rules in $d\sigma/dp_\perp^2$ involving purely light-quark transitions

We have

$$3\sigma(\pi^-p\to\pi^-x)+\tfrac{5}{16}\sigma(K^+p\to K^0x)=2\sigma(K^-p\to\bar{K}^0x)+2\sigma(\pi^+p\to\pi^+x)\ , \qquad (3.1)$$

$$3\sigma(\pi^-p\to\pi^-x)+\tfrac{5}{16}\sigma(K^-p\to\pi^-x)=2\sigma(\pi^+p\to\pi^+x)+2\sigma(K^-p\to\bar{K}^0x)\ . \qquad (3.2)$$

In Table I we present a comparison of our predictions, Eqs. (3.1) and (3.2) with recent experimental results.[37] It shows that the overall agreement with data is good.

2. Sum rules in $(d\sigma/dx)$ involving purely light-quark transitions

Equation (3.1) can be tested in terms of $d\sigma/dx$ as well (Table II) using recent data.[37] Here too the overall agreement is good within the range of x under consideration 3.

3. SU(4) sum rules in $(2E_p/\sigma_T)d^3\sigma/dp^3$ involving charmed-particle production.

We record below some of the testable sum rules containing transitions from light (u,d,s) to charm quark (c):

$$\sigma(\pi^-p\to\pi^0x)=\tfrac{8}{19}\sigma(K^+p\to K^+x)+\tfrac{1}{4}\sigma(\pi^-p\to D^-x)+\sigma(\pi^+p\to\pi^0x)\ , \qquad (3.3)$$

$$\tfrac{5}{19}\sigma(K^+p\to K^+x)+3\sigma(\pi^-p\to\pi^-x)=2\sigma(\pi^-p\to D^-x)+2\sigma(\pi^+p\to\pi^+x)\ , \qquad (3.4)$$

$$\tfrac{147}{152}\sigma(K^+p\to K^+x)+\tfrac{1}{4}\sigma(\pi^+p\to\pi^+x)+\tfrac{1}{4}\sigma(\pi^-p\to D^-x)=\tfrac{19}{16}\sigma(K^+p\to K^0x)+\tfrac{3}{8}\sigma(\pi^-p\to\pi^-x)\ , \qquad (3.5)$$

$$\tfrac{1}{6}\sigma(K^-p\to F^-x)+\tfrac{1}{3}\sigma(K^-p\to\pi^0x)+\tfrac{1}{2}\sigma(\pi^-p\to D^-x)=\sigma(K^-p\to\eta x)\ , \qquad (3.6)$$

$$\tfrac{1}{2}\sigma(K^-p\to F^-x)+\tfrac{1}{6}\sigma(\pi^-p\to K^0x)+\tfrac{1}{3}\sigma(K^-p\to\pi^0x)=\sigma(K^-p\to\eta x)\ , \qquad (3.7)$$

$$\tfrac{1}{3}\sigma(\pi^+p\to\pi^0x)+\sigma(\pi^+p\to\eta x)=\tfrac{1}{6}\sigma(K^-p\to F^-x)+\tfrac{1}{3}\sigma(K^+p\to K^0x)\ . \qquad (3.8)$$

A comparison of the sum rules with experiment[37,38] is made in Table III at average $E_{\mathrm{c.m.}}\sim 5.5$ GeV and average $P_{\mathrm{lab}}\sim 15.0$ GeV/c. It is found, the agreement is good for Eqs. (3.3)–(3.5), tolerable for Eqs. (3.6) and (3.7), while poor for Eq. (3.8). Thus our results indicate that symmetries alone are not adequate, and dynamics of the underlying theory must be put to accommodate such symmetry-breaking effects within the model.

We however note that for the production of charmed mesons alot of phase space is needed because for every charmed meson in the final state, there must also be an anticharm state. So one would need at least 3 GeV just to reach threshold. Thus the comparison at higher energies ($E_{\mathrm{c.m.}}\gg 3$ GeV) would hopefully be more meaningful than the present attempt.

4. SU(3) sum rules in $(2E_p/\sigma_T)d^3\sigma/dp^3$ involving purely light-quark transitions

Equations (3.1) and (3.2) can also be tested in terms of $(2E_p/\sigma_T)d^3\sigma/dp^3$. This has been done in Table IV at average $E_{\mathrm{c.m.}}\sim 5.5$ GeV and $P_{\mathrm{lab}}\sim 15.0$ GeV/c. The agreement is again good. We also list below a few more sum rules which have been tested with available data[38] in Table IV at the same $E_{\mathrm{c.m.}}$ and P_{lab}:

$$\sigma(\pi^-p\to\pi^0x)=\tfrac{1}{4}\sigma(K^-p\to\bar{K}^0x)+\tfrac{1}{2}\sigma(K^+p\to K^0x)+\sigma(\pi^+p\to\pi^0x)\ , \qquad (3.9)$$

$$\sigma(\pi^-p\to\pi^0x)=\tfrac{8}{19}\sigma(K^+p\to K^+x)+\tfrac{1}{4}\sigma(K^-p\to\bar{K}^0x)+\sigma(\pi^+p\to\pi^0x)\ , \qquad (3.10)$$

$$3\sigma(\pi^-p\to\pi^-x)+\tfrac{5}{19}\sigma(K^+p\to K^+x)=2\sigma(K^-p\to\bar{K}^0x)+2\sigma(\pi^+p\to\pi^+x)\ , \qquad (3.11)$$

$$\sigma(K^-p\to\eta x)=\tfrac{2}{3}\sigma(K^-p\to\bar{K}^0x)+\tfrac{1}{3}\sigma(K^+p\to K^0x)\ , \tag{3.12}$$

$$4\sigma(\pi^+p\to\eta x)+\tfrac{4}{3}\sigma(\pi^+p\to\pi^0x)=\sigma(K^-p\to\pi^-x)+\sigma(K^-p\to\eta x)\ , \tag{3.13}$$

$$\sigma(K^+p\to K^+x)=\tfrac{19}{16}\sigma(K^+p\to K^0x)\ , \tag{3.14}$$

$$\sigma(K^-p\to K^-x)+2\sigma(\pi^+p\to\pi^0x)=\tfrac{3}{2}\sigma(K^-p\to\bar{K}^0x)+\sigma(K^+p\to K^+x)\ , \tag{3.15}$$

$$\tfrac{1}{3}\sigma(K^-p\to K^-x)+\tfrac{1}{3}\sigma(\pi^-p\to\pi^-x)+\tfrac{1}{24}\sigma(K^+p\to K^0x)=\sigma(K^-p\to\eta x)\ , \tag{3.16}$$

$$4\sigma(\pi^+p\to\eta x)+\tfrac{4}{3}\sigma(\pi^+p\to\pi^0x)=\sigma(K^+p\to K^0x)+\sigma(K^-p\to\eta x)\ . \tag{3.17}$$

A study of Table IV reveals that the agreement is good for Eqs. (3.9)–(3.11), tolerable for the Eqs. (3.12)–(3.14), and poor for the Eqs. (3.15)–(3.17). Once again it indicates the limitation of symmetries and calls for suitable dynamics in the model.

5. SU(2) sum rule in $(2E_p/\sigma_T)d^3\sigma/dp^3$

We also compare the following SU(2) sum rule with data[38] in Table IV:

$$\tfrac{7}{2}\sigma(\pi^+p\to\pi^+x)+\tfrac{1}{2}\sigma(\pi^-p\to\pi^-x)+\tfrac{5}{2}\sigma(\pi^-p\to\pi^0x)=14\sigma(\pi^+p\to\pi^0x)\ . \tag{3.18}$$

The agreement is tolerable. However, certain degree of symmetry breaking still needs to be incorporated for better agreement with data.

6. Quantitative estimations of SU(4), SU(3), and SU(2) symmetry breakdown in the sum rules

In order to find the clues of the possible symmetry-breaking mechanisms we estimate the measures of breakdown of the symmetries at the sum-rule level. This can be obtained by putting a few typical sum rules in terms of suitable ratios.

For SU(4) breaking, we have

$$\frac{\tfrac{1}{4}\sigma(\pi^-p\to D^-x)}{\sigma(\pi^-p\to\pi^0x)-\tfrac{8}{19}\sigma(K^+p\to K^+x)-\sigma(\pi^+p\to\pi^0x)}=1\ , \tag{3.3$'$}$$

$$\frac{2\sigma(\pi^-p\to D^-x)}{\tfrac{5}{19}\sigma(K^+p\to K^+x)+3\sigma(\pi^-p\to\pi^-x)-2\sigma(\pi^+p\to\pi^+x)}=1\ , \tag{3.4$'$}$$

$$\frac{\tfrac{1}{4}\sigma(\pi^-p\to D^-x)}{\tfrac{19}{16}\sigma(K^+p\to K^0x)+\tfrac{3}{8}\sigma(\pi^-p\to\pi^-x)-\tfrac{147}{152}\sigma(K^+p\to K^+x)-\tfrac{1}{4}\sigma(\pi^+p\to\pi^+x)}=1\ , \tag{3.5$'$}$$

$$\frac{\tfrac{1}{6}\sigma(K^-p\to F^-x)+\tfrac{1}{2}\sigma(\pi^-p\to D^-x)}{\sigma(K^-p\to\eta x)-\tfrac{1}{3}\sigma(K^-p\to\pi^0x)}=1\ , \tag{3.6$'$}$$

$$\frac{\tfrac{1}{2}\sigma(K^-p\to F^-x)}{\sigma(K^-p\to\eta x)-\tfrac{1}{6}\sigma(\pi^-p\to K^0x)-\tfrac{1}{3}\sigma(K^-p\to\pi^0x)}=1\ , \tag{3.7$'$}$$

$$\frac{\tfrac{1}{6}\sigma(K^-p\to F^-x)}{\tfrac{1}{3}\sigma(\pi^+p\to\pi^0x)+\sigma(\pi^+p\to\eta x)-\tfrac{1}{3}\sigma(K^+p\to K^0x)}=1\ . \tag{3.8$'$}$$

For SU(3) breaking, we have

$$\frac{\tfrac{5}{16}\sigma(K^-p\to\pi^-x)}{2\sigma(\pi^+p\to\pi^+x)+2\sigma(K^-p\to\bar{K}^0x)-3\sigma(\pi^-p\to\pi^-x)}=1\ , \tag{3.2$'$}$$

$$\frac{\sigma(K^-p\to\eta x)}{\tfrac{2}{3}\sigma(K^-p\to\bar{K}^0x)+\tfrac{1}{3}\sigma(K^+p\to K^0x)}=1\ , \tag{3.12$'$}$$

$$\frac{\sigma(K^-p\to\pi^-x)+\sigma(K^-p\to\eta x)}{4\sigma(\pi^+p\to\eta x)+\tfrac{4}{3}\sigma(\pi^+p\to\pi^0x)}=1\ , \tag{3.13$'$}$$

$$\frac{\sigma(K^-p\to\eta x)}{4\sigma(\pi^+p\to\eta x)+\tfrac{4}{3}\sigma(\pi^+p\to\pi^0x)-\sigma(K^+p\to K^0x)}=1\ . \tag{3.17$'$}$$

TABLE III. Test of SU(4) sum rules in terms of $(2E_p/\sigma_T)d^3\sigma/dp^3$ at $E_{\rm c.m.}\sim 5.5$ GeV and $P_{\rm lab}\sim 15.0$ GeV/c. We compare the left-hand side (LHS) and right-hand side (RHS) of each of Eqs. (3.3)–(3.8).

Equation number	LHS (mb)	RHS (mb)	LHS/RHS
(3.3)	38.2±1.5	40.78±0.1	0.94
(3.4)	124.85±0.9	94.81±1.0	1.32
(3.5)	23.07±0.13	22.39±0.31	1.03
(3.6)	8.42±0.59	4.6±1.8	1.83
(3.7)	9.0±0.67	4.6±1.8	1.96
(3.8)	13.47±0.33	2.06±0.08	6.54

In Table V we evaluate the experimental values of the above ratios. It is evident from Table V that the ratios [Eqs. (3.3′)–(3.8′)] measuring the relative strengths of the charm-changing to the charm zero transitions are in the range from 3.5×10^{-2} to 3.3×10^{-4} while those of strangeness-changing transitions to the strangeness zero transitions Eqs. (3.2′), (3.12′), (3.13′), and (3.17′) are in the range 0.1–0.7. Evidently, SU(4) is broken worse than SU(3), and charm transitions are suppressed over others by factors 10^3–10^4 consistent with other analyses.[39]

TABLE IV. Test of SU(3) sum rules in terms of $(2E_p/\sigma_T)d^3\sigma/dp^3$ at $E_{\rm c.m.}\sim 5.5$ GeV and $P_{\rm lab}\sim 15.0$ GeV/c.

Equation number	LHS (mb)	RHS (mb)	LHS/RHS
(3.1)	123.68±0.95	109.7±1.64	1.13
(3.2)	130.11±1.21	109.7±1.64	1.2
(3.9)	40.78±0.26	38.2±1.5	1.07
(3.10)	38.2±1.5	42.64±0.18	0.89
(3.11)	124.85±0.9	109.3±1.64	1.14
(3.12)	6.98±0.27	4.6±1.8	1.52
(3.13)	53.87±1.33	31.2±2.8	1.73
(3.14)	11.6±0	7.16±0.21	1.62
(3.15)	80.1±0.9	22.78±0.48	3.52
(3.16)	16.55±0.33	4.6±1.8	3.6
(3.17)	53.87±1.33	10.63±1.97	5.1
(3.18)	281.7±5.65	502.6±1.4	0.56

In SU(3) itself there are sum rules involving diffractive transitions (no quantum numbers are exchanged) and charge-exchange transitions. In order to study the relative strengths of such transitions, we recast some of the sum rules in the following ratios:

$$\frac{3\sigma(\pi^-p\to\pi^-x)+\frac{5}{19}\sigma(K^+p\to K^+x)-2\sigma(\pi^+p\to\pi^+x)}{2\sigma(K^-p\to\bar{K}^0x)}=1\ , \tag{3.11′}$$

$$\frac{\sigma(K^-p\to K^-x)-\sigma(K^+p\to K^+x)}{\frac{3}{2}\sigma(K^-p\to\bar{K}^0x)-2\sigma(\pi^+p\to\pi^0x)}=1\ , \tag{3.15′}$$

$$\frac{\frac{1}{3}\sigma(K^-p\to K^-x)+\frac{1}{3}\sigma(\pi^-p\to\pi^-x)}{\sigma(K^-p\to\eta x)-\frac{1}{24}\sigma(K^+p\to K^0x)}=1\ , \tag{3.16′}$$

and

$$\frac{\frac{7}{2}\sigma(\pi^+p\to\pi^+x)+\frac{1}{2}\sigma(\pi^-p\to\pi^-x)}{14\sigma(\pi^+p\to\pi^0x)-\frac{5}{2}\sigma(\pi^-p\to\pi^0x)}=1\ . \tag{3.18′}$$

These ratios have also been tested in Table V. A study of the magnitudes of the ratios Eqs. (3.11′) and (3.16′) demonstrates explicitly the expected dominance of the diffractive processes. The apparent deviation of this expectation in Eqs. (3.15′) and (3.18′), however, calls for some extra dynamics of the underlying theory.

B. Symmetry breaking in the model

The typical ratios defined in Eqs. (3.3′)–(3.8′), (3.2′), (3.12′), (3.13′), (3.17′) as well as (3.11′), (3.15′), (3.16′), and (3.18′) deviate away from unity; the pattern of deviation follows Table V. In the present model such features can be incorporated if we assume that the orbital functions $(D^{(\pm)}_{s,a},F^{(\pm)}_{s,a},\bar{D}^{(\pm)}_{s,a},\bar{F}^{(\pm)}_{s,a},d^{(\pm)}_{s,a},f^{(\pm)}_{s,a})$ for the diagonal quark transitions or vacuum exchange $(u\to u,d\to d,s\to s)$, charge exchange $(u\leftrightarrow d)$, hypercharge/strangeness exchange $(u\leftrightarrow s,d\leftrightarrow s)$, and the charm-quark transition $(u,d,s\to c)$ are not identical, but

TABLE V. Ratios of some typical sum rules (theoretical value of the ratios = 1).

Equation number	Experimental values
(3.3)′	7.75×10^{-4}
(3.4)′	3.3×10^{-4}
(3.5)′	2.9×10^{-3}
(3.6)′	1.0×10^{-2}
(3.7)′	3.5×10^{-2}
(3.8)′	4.3×10^{-3}
(3.2)′	0.68
(3.12)′	0.66
(3.13)′	0.57
(3.17)′	0.10
(3.11)′	2.02
(3.15)′	0.05
(3.16)′	3.75
(3.18)′	0.46

correspond to the following four sets, respectively:

$$(D_{s,a}^{(\pm)},F_{s,a}^{(\pm)},\bar{D}_{s,a}^{(\pm)},\bar{F}_{s,a}^{(\pm)},d_{s,a}^{(\pm)},f_{s,a}^{(\pm)})_P\ ,$$

$$(D_{s,a}^{(\pm)},F_{s,a}^{(\pm)},\bar{D}_{s,a}^{(\pm)},\bar{F}_{s,a}^{(\pm)},d_{s,a}^{(\pm)},f_{s,a}^{(\pm)})_Q\ ,$$

$$(D_{s,a}^{(\pm)},F_{s,a}^{(\pm)},\bar{D}_{s,a}^{(\pm)},\bar{F}_{s,a}^{(\pm)},d_{s,a}^{(\pm)},f_{s,a}^{(\pm)})_Y\ ,$$

and

$$(D_{s,a}^{(\pm)},F_{s,a}^{(\pm)},\bar{D}_{s,a}^{(\pm)},\bar{F}_{s,a}^{(\pm)},d_{s,a}^{(\pm)},f_{s,a}^{(\pm)})_C\ .$$

This will yield four separate sets of functions (A_P,B_P,C_P), (A_Q,B_Q,C_Q), (A_Y,B_Y,C_Y), and (A_C,B_C,C_C) in Eq. (2.84) instead of the single set (A_q,B_q,C_q).

The overall pattern of Table V can then be understood if these functions satisfy the condition

$$(A_P,B_P,C_P),(A_Q,B_Q,C_Q),(A_Y,B_Y,C_Y) \gg (A_C,B_C,C_C)\ . \tag{3.19}$$

Furthermore, the sets (A_P,B_P,C_P), (A_Q,B_Q,C_Q), and (A_Y,B_Y,C_Y) should also satisfy the following relationships at $P_{\rm lab}\sim 15.0$ GeV/c and $E_{\rm c.m.}\sim 5.5$ GeV:

$$\frac{A_P}{A_Q}\approx 2.17,\quad \frac{B_P}{B_Q}\approx 1.62,\quad \frac{C_P}{C_Q}\approx 0.24\ , \tag{3.20}$$

$$\frac{A_Q}{A_Y}\approx 2.56,\quad \frac{B_Q}{B_Y}\approx 0.24\ , \tag{3.21}$$

while C_Y does not contribute to the processes under consideration.

C. Possible dynamics of symmetry breaking

The underlying theory is thus to satisfy the phenomenological condition (3.19). One plausible way to do this is to assume that the functions A_q,B_q,C_q of Eq. (2.84) are Regge behaved. In the triple-Regge limit $(s \gg M^2 \gg 1)$ inclusive cross sections behave as[19] $\sim (s/M)^{2\alpha(t)-1}$, where $\alpha(t)$ is the linear Regge trajectory for the exchange process under consideration. It is then possible to understand qualitatively the relative suppression of charm-production cross sections over others due to the conjectured negative intercept of the charmed trajectory.[39] Though strictly valid for x near 1, such a feature provides a reasonable rationale in the range of x even beyond $x\approx 1$. Furthermore, the relation (3.20) shows that the leading functions (A_P,B_P) of the diffractive transitions satisfy the inequalities

$$A_P > A_Q \quad\text{and}\quad B_P > B_Q\ . \tag{3.22}$$

Within Regge behavior, relation (3.22) can be understood to be the consequence of the displacement between the Pomeron trajectory and the charge exchange ρ trajectory $[\alpha_{\rm P}(0)=1,\ \alpha_\rho(0)=0.58]$. Similarly, the inequality

$$A_Q > A_Y \tag{3.23}$$

of Eq. (3.21) is consistent with the displacement between the ρ and the K^* trajectory with $\alpha_{K^*}(0)=0.3$. The phenomenological conditions

$$C_P < C_Q,\quad B_Q < B_Y \tag{3.24}$$

of Eqs. (3.20) and (3.21), on the other hand, ensure the experimental observations of the type

$$\sigma(K^\pm p \to K^\pm x) < \sigma(\pi^\pm p \to \pi^0 x)$$

and

$$\sigma(K^- p \to \bar{K}^0 x) < \sigma(K^- p \to \pi^0 x)$$

within the present model.

Thus the overall qualitative feature of the symmetry breaking indicates that the underlying theory should have suitable mechanisms to generate linear Regge trajectories. It is then possible that the residue functions, which control the dynamics of the theory of the present model have expected Regge behavior consistent with Eqs. (3.19), (3.22), and (3.23). Besides such features, these functions should also satisfy certain approximate dynamical relations such as (3.20) and (3.21).

To calculate a Regge trajectory or such relationships is beyond the scope of the present work. However, educated guesses as to how the dynamics may work to generate such trajectories are not impossible,[40] viz., Regge trajectories with $\alpha(0)\le\frac{1}{2}$ are identified with quark exchange, while the Pomeron with $\alpha(0)=1$ is identified with gluon exchange. It will be worthwhile to incorporate such features quantitatively in the model in future.

Let us also comment on the possibility of interpreting the above results in terms of quark models specifically pursued by other authors. In the $^3P_0$ model of Le Yaouanc *et al.*,[41] the reason that charm-changing amplitudes are smaller is due to much lower probability of creating a $c\bar{c}$ pair out of vacuum than the noncharmed ones. Hence most of the symmetry breaking [Eq. (3.19) of the text] can be understood in the model through such differences in the pair-creation amplitudes, while the remaining discrepancy can be attributed to the differences in wave functions containing a charm quark.

D. Effects of gluons

Let us now discuss how Eq. (2.89) affects our results. As it is expressed in terms of three effective parameters $(A_q\cos^2\phi + A_g\sin^2\phi)$, $(B_q\cos^2\phi+B_g\sin^2\phi)$, and $(C_q\cos^2\phi + C_g\sin^2\phi)$, all the sum rules Eqs. (3.1)–(3.18) remain unaltered by this modification even though the individual cross sections change in structure.

From Eq. (2.89) however one infers that such modification is, in general, flavor dependent as the ratio of the cross sections Eq. (2.89) and Eq. (2.84) (defined as σ_{q+g} and σ_q, respectively) is expressible as

$$\frac{\sigma_{q+g}}{\sigma_q} = \cos^2\phi + R_g(u,v,w)\sin^2\phi \tag{3.25}$$

with

$$R_g(u,v,w) = \frac{uA_g + vB_g + wC_g}{uA_q + vB_q + wC_q} \tag{3.26}$$

for any process with definite flavor factors (u,v,w) defined in Eq. (2.84). Further work on this aspect of the model is in progress.

IV. CONCLUSIONS

In this paper we have addressed ourselves to a study of the inclusive reactions $A+B \rightarrow C+X$ within a constituent-quark model proposed by Mitra[20] and pursued by us recently.[24-26] Besides charm and color degrees of freedom, an attempt is being made to incorporate gluon effect as well in the nucleon wave function. Our analysis indicates that symmetries alone are not adequate, rather the underlying theory should contain suitable mechanisms to generate Regge-type behavior in the inclusive reactions. It also suggests a flavor-dependent effect due to gluons. Further studies at high-momentum transfer will hopefully establish whether or not such gluonic effect is indeed of perturbative origin.

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