Decoupling of the pion at finite temperature and density

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We use the Nambu–Jona-Lasinio model at finite temperature and density to investigate the static and dynamical properties of the pion. Near the chiral phase transition the pion is found to decouple from matter. The relevance of this result for hadronic matter is discussed.

It is generally believed that with increasing baryon densities and temperature, hadronic matter undergoes a phase transition to a quark-gluon plasma.[1] This exotic form of matter is believed to have been produced in the early stage of the Universe, and hoped to be realized in the laboratory using relativistic heavy-ion colliders.[2] Most of the theoretical insights on this state of matter come from lattice Monte Carlo simulations.[3,4] While all these numerical calculations support the concept of a phase transition both at finite temperature and densities, they still lack the necessary accuracy for a quantitative description of the phenomenon. Aside from systematic errors, the main problems with lattice simulation are related to finite-size effects and the dilemma between chiral symmetry and fermion doubling. At this stage, it is therefore relevant to investigate the temperature and density effects on hadronic matter using alternative methods to Monte Carlo simulation.

Phenomenological attempts to determine the critical temperature and density at which the transition happens already exist.[5] Two types of transition are in general expected: a confinement-deconfinement transition and a chiral-restoring transition. Lattice QCD simulations suggest that these transitions happen at about the same temperature.[4] The purpose of this paper is to investigate the chiral phase transition in the Nambu–Jona-Lasinio model.[6]

At zero temperature and fermion number density, the ground-state wave function can be thought of as a condensate of quark pairs much like Cooper pairs in a superconducting phase. Because of the attractive character of the interaction, it pays for neighboring quarks in the vacuum to pair, causing a spontaneous breakdown of chiral symmetry. In this phase, the pion can be thought of as a coherent isospin excitation in the ground-state condensates. Increasing densities and/or thermal effects will tend to wash out the quark condensation and restore chiral symmetry. When the temperature is increased, the ground state undergoes a phase transition (believed to be second order) from a Goldstone-Nambu phase to a Wigner-Weyl phase. As a result we will show that the pion decouples from matter. The same behavior occurs for increasing densities ρ. When $\rho^{-1/3}$ becomes comparable to the correlation length in the condensed pairs, $q\bar{q}$ constituents behave as free particles causing the pion to decouple again.

To illustrate these effects, consider the Nambu–Jona-Lasinio model in its canonical form:

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/-m)\psi+G[(\bar{\psi}\psi)^2+(\bar{\psi}i\tau\gamma_5\psi)^2] , \tag{1}$$

where m is the current-quark mass and G a dimensionful coupling constant. This model is neither confining nor finite, and one might ask how the confinement-deconfinement phase transition relates to the chiral phase transition in this case. A phenomenological way of enforcing confinement in this model is to use a nonlocal four-fermion interaction. The finite size of the confining potential cuts down the high-momentum components from the spectrum, ensuring at the same time finiteness. In this case the vacuum wave function, the mass gap, and, in general, the low-lying excitations are all dominated by the low-lying components of the spectrum. These effects can also be achieved using a simple momentum cutoff Λ. This cutoff regulates the bad UV behavior in most physical quantities. The cutoff scale is in some way the analog of the size of the confining potential. In either case, a phase transition occurs whenever the temperature scale (T) or density scale (ρ), become comparable to either Λ or G. Throughout, we will assume that G and Λ are temperature and density independent. We will discuss at the end the effect of temperature on both G and Λ.

To evaluate the static pion properties at finite temperature and density, we need to construct the pion wave function. For that, we have to understand first, how quarks propagate in the correlated vacuum state. The pairing mechanism in the ground-state wave function can be understood in terms of constituent quarks with a constituent-quark mass different from the current mass m. This can be achieved in terms of a quark self-energy:

$$S^{-1}(k)=\not{k}-\Sigma(k)-m+i\epsilon \ . \tag{2}$$

The mean-field approach to the pairing problem is equivalent to solving the Schwinger-Dyson equation for $\Sigma(k)$ in the Hartree-Fock approximation. From Fig. 1 we have

$$\Sigma=2G\int\frac{d^4k}{(2\pi)^4}\{\mathrm{Tr}[S(k)]-S(k)+i\tau\gamma_5\mathrm{Tr}[i\tau\gamma_5S(k)] -i\tau\gamma_5S(k)i\tau\gamma_5\} \ . \tag{3}$$

Because of the local character of the interaction, Σ turns out to be momentum independent. To analyze the combined effects of temperature and density we will use the imaginary-time formalism. For that, we will make the substitution

$$\int\frac{dk_0}{2\pi}\rightarrow\frac{1}{\beta}\sum_n \ ,$$

where the sum is over all discrete energies $\omega_n=(2n+1)/\beta$, with $\beta=T^{-1}$. Taking into account the symmetry properties of the system, we can use the parametrization

$$\Sigma=\sigma_1-\gamma^0\sigma_0 \tag{4}$$

to reduce (3) into a set of two coupled integral equations of the form

$$\sigma_1=\frac{4G}{\beta}(2n_fn_c+1)(\sigma_1+m)\sum_n\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{{k_n}^2-{\omega_k}^2} \ , \tag{5a}$$

$$\sigma_0=\frac{8G}{\beta}\sum_n\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{k_n}{{k_n}^2-{\omega_k}^2} \ , \tag{5b}$$

where $n_c=3$ and $n_f=2$ are the number of colors and flavors, and for convenience we have defined

$$k_n=\mu+\sigma_0+i\omega_n\,, \quad {\omega_k}^2=\mathbf{k}^2+(\sigma_1+m)^2 \ .$$

In the relation for k_n, μ is the chemical potential. The discrete sums in (5) can be done using contour integrals. The result is

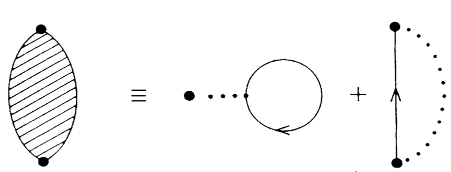

FIG. 1. Hartree-Fock equation for the self-energy kernel.

$$\sigma_1=\frac{G}{2\pi^2}(2n_fn_c+1)(\sigma_1+m)\times\int_0^\Lambda\frac{k^2}{\omega_k}dk\left[\tanh\left[\frac{\beta\omega_k^-}{2}\right]+\tanh\left[\frac{\beta\omega_k^+}{2}\right]\right] \ , \tag{6a}$$

$$\sigma_0=-\frac{2G}{\pi^2}\int_0^\Lambda k^2dk(n_k^+-n_k^-) \ , \tag{6b}$$

where we have defined

$$\omega_k^\pm=\omega_k\pm(\mu+\sigma_0) \ , \tag{7a}$$

$$n_k^\pm=(1+e^{\beta\omega_k^\pm})^{-1} \ , \tag{7b}$$

and used explicitly the short-distance cutoff Λ^{-1}. $n^\pm$ in Eq. (7b) are the quark and antiquark occupation numbers. Notice that the baryon density ρ is related to the chemical potential μ by

$$\rho=-\frac{n_fn_c}{3\pi^2}\int_0^\Lambda k^2dk(n_k^+-n_k^-) \tag{8}$$

so that σ_0 is proportional to ρ.

In the limit of zero temperature and density, Eqs. (6) reduce to the expected gap equation

$$\sigma_1=\frac{G}{2\pi^2}(2n_fn_c+1)(\sigma_1+m)\int_0^\Lambda\frac{k^2}{\omega_k}dk \tag{9}$$

with $\sigma_0=0$. For a given temperature and density, σ_0 and σ_1, through (4), specify entirely the quark propagation in the correlated ground state. In particular, the strength of quark pairs is given by ($q=u,d$)

$$\langle\bar{q}q\rangle=-4n_c\frac{\sigma_1}{\beta}\sum_n\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{({k_n}^2-{\sigma_1}^2)} \ . \tag{10}$$

To investigate the chiral phase transition we will consider first the case $m=0$. In solving Eqs. (6), Λ and G are chosen so as to reproduce as well as possible the pion decay constant f_π and the fermion condensate $\langle\bar{q}q\rangle^{1/3}$ at $T=0$ and $\mu=0$ (Ref. 7). Specifically, $\Lambda=700$ MeV and $G\Lambda^2=2.00$ with $f_\pi=100$ MeV and $-\langle\bar{q}q\rangle^{1/3}=270$ MeV. This parameter set will be referred to as (I). To probe the sensitivity of our results to Λ and G, we will also consider a second set of parameters (II) with $\Lambda=925$ MeV and $G\Lambda^2=2.00$ which yields $f_\pi=134$ MeV and $-\langle\bar{q}q\rangle^{1/3}=357$ MeV. The behavior of $\langle\bar{q}q\rangle$ with T and ρ is shown in Figs. 2(a) and 2(b), respectively. As expected, increasing thermal fluctuations cause the fermion condensate to vanish at the critical temperature T_c for $m=0$. Similarly, increasing baryon densities liberate the quark pairs by percolation. Notice that near the critical density ρ_c, the typical correlation length in the pair $\langle\bar{q}q\rangle^{-1/3}$ becomes comparable to the characteristic baryon scale $\rho^{-1/3}\sim1$ fm. The dashed curves show the result for a finite current-quark mass $m=5$ MeV. Because of the explicit chiral-symmetry-breaking mass term, the transition region is smoothed out. However, one can still define a transition temperature and density at somewhat higher values. Changes in the parameters Λ and G do not affect qualitatively the above results. Finally, we show in Fig. 2(c) the phase diagram in the $T\rho$ plane for $m=0$. At sufficiently low temperature, the critical density is an increasing function of temperature. At higher temperatures, however, the critical density decreases. A similar behavior has been obtained in the σ model by Wakamatsu and Hayashi.[8] Throughout the critical line, the difference in the energy density is positive definite, indicating that the phase transition is second order. The set of parameters (II) leads to values of T_c and ρ_c, respectively, of 275 MeV and $5\rho_0$, where $\rho_0=0.17$ fm^{-3} is the nuclear matter density, that are consistent with lattice QCD calculations. The value of $5\rho_0$ corresponds to a chemical potential of 450 MeV. Values of T_c and ρ_c for set (I) can be obtained from (II) by scaling T_c by Λ and ρ_c by Λ^3. Notice that, in general, T_c is less sensitive than ρ_c to variations in Λ and G. Indeed, the range of values obtained for T_c by comparing different calculations is much smaller than the one for ρ_c.

As already pointed out, the pion can be thought of as a long-range isospin excitation in the quark-pair condensates. Since the latter are substantially modified by temperature and baryon density, we expect similar modifications in the pion properties. To illustrate this, we will have to define a pion wave function. This can be achieved using the Bethe-Salpeter equation for a massive pion at rest.[9] If we denote by $G^a(m_\pi)$ the pion vertex for a zero-momentum pion, then the Bethe-Salpeter equation shown in Fig. 3(a) reads

$$G^a(m_\pi)=2G\int\frac{d^4k}{(2\pi)^4}\{\mathrm{Tr}[S(k)G^a(m_\pi)S(k)]+i\tau\gamma_5\mathrm{Tr}[S(k)G^a(m_\pi)S(k)i\tau\gamma_5] -S(k)G^a(m_\pi)S(k)-i\tau\gamma_5S(k)G^a(m_\pi)S(k)i\tau\gamma_5\}\ . \tag{11a}$$

Assuming that $G^a(m_\pi)$ is a smooth function of m_π, the general solution to (11a) reads

$$G^a(m_\pi)=\tau^a\gamma_5\sigma_1 f_\pi^{-1}+O(m_\pi^2)\ . \tag{11b}$$

The normalization in (11b) is fixed by the chiral Ward identity at $m=m_\pi=0$. The pion decay constant f_π can be deduced from the transition amplitude of the axial-vector current between the pion state (11b) and the vacuum:

$$if_\pi m_\pi=\frac{\sigma_1+m}{2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[S\left[k+\frac{m_\pi}{2}\right]\tau_3\gamma_5 S\left[k-\frac{m_\pi}{2}\right]\gamma_0\gamma_5\tau_3\right] \tag{12}$$

as illustrated in Fig. 3(b). At finite density ρ and zero temperature, the pion decay constant is

$$f_\pi{}^2=\frac{3}{2\pi^2}(\sigma_1+m)\int_0^\Lambda dk\frac{k^2}{\omega_k{}^3}\theta(\omega_k{}^2-(\mu+\sigma_0)) \tag{13a}$$

and at finite temperature T and zero density it is

$$f_\pi{}^2=\frac{3}{2\pi^2}(\sigma_1+m)^2\int_0^{1/2}du\int k^2dk\left\{\frac{2}{a^3}\left[\tanh\left[a\frac{\beta}{2}\right]\right]-\frac{\beta}{a^2}\left[\frac{1}{\cosh^2\left[\frac{\beta}{2}a\right]}\right]\right\}, \tag{13b}$$

where

$$a=[k^2+(\sigma_1+m)^2+(u^2-\tfrac{1}{4})m_\pi{}^2]^{1/2}\ .$$

In general, f_π follows from (12) using the general propagator at finite T and μ and discrete external energies. The result is then analytically continued to $m_\pi+i\epsilon$.

The behavior of f_π vs T/T_c and ρ/ρ_c is shown in Figs. 4(a) and 4(b), respectively, for the set of parameters (I). The full curve corresponds to the case $m=0$. The pion decay constant vanishes at T_c and ρ_c, and consequently the pion decouples from matter. The same although less sharp transition than in $\langle\bar{q}q\rangle$ is observed for f_π when one breaks the symmetry explicitly using a small mass term. Equation (11b) enables us to define the quark-pion coupling constant in the Nambu-Goldstone phase as

$$g_{\pi qq}=\frac{\sigma_1}{f_\pi}\ . \tag{14}$$

At the quark level the Goldberger-Treiman relation remains valid as one increases the temperature and density up to T_c and ρ_c, respectively. This is, of course, expected since chiral symmetry is still spontaneously broken. This same behavior has also been noted in Ref. 10. $g_{\pi qq}$ is rather stable up to the transition region where it decreases near ρ_c and increases near T_c by about 10%. Above T_c and ρ_c, Eq. (14) becomes meaningless since the pion state crosses the $q\bar{q}$ continuum. In this case, the pion becomes unstable against $q\bar{q}$ decays. This was first observed by Kunihiro and Hatsuda,[11] and confirmed in our calculation. There is an increase in the pion mass by about 40% near T_c and 50% near ρ_c.

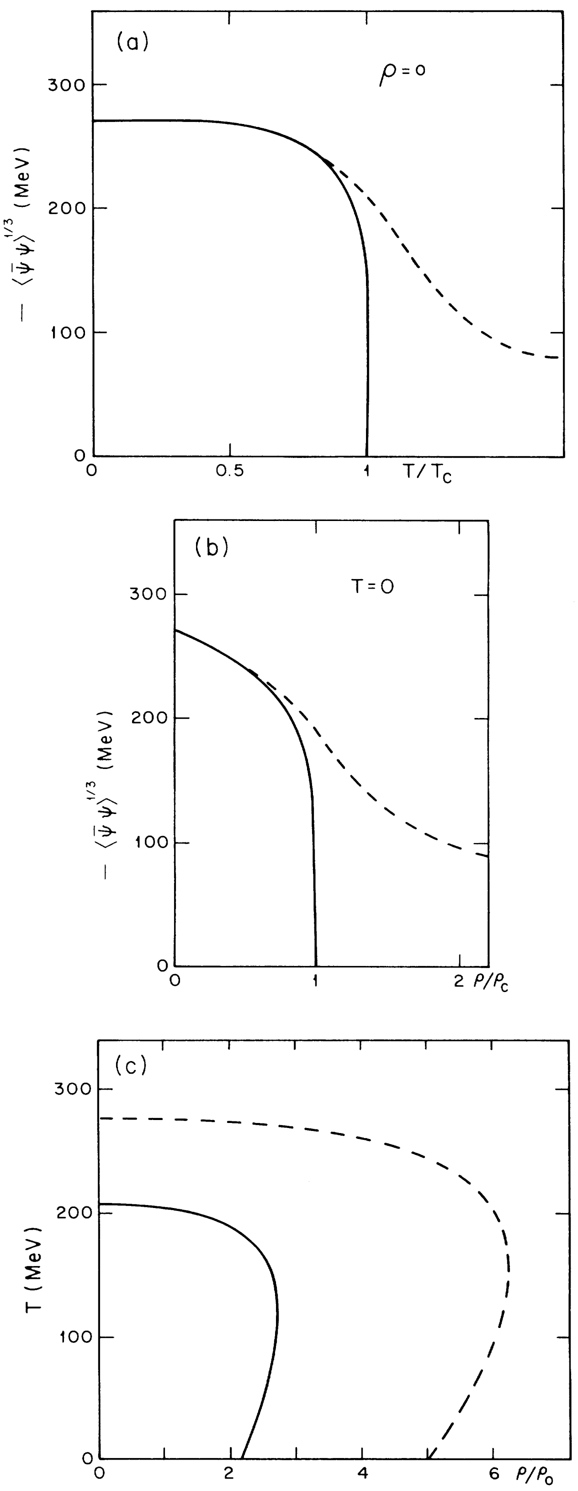

FIG. 2. (a) Behavior of the fermion condensate with T/T_c. T_c is the critical temperature defined for $m=0$. The solid (dashed) curve corresponds to $m=0$ ($m=5$ MeV) and $\Lambda=700$ MeV. (b) Behavior of the fermion condensate with ρ/ρ_c. ρ_c is the critical density defined for $m=0$. The solid (dashed) curve corresponds to $m=0$ ($m=5$ MeV) and $\Lambda=700$ MeV. (c) Phase diagram in the $T\rho$ plane. The solid (dashed) curve corresponds to a value of Λ of 700 MeV (925 MeV) and $G\Lambda^2=2$.

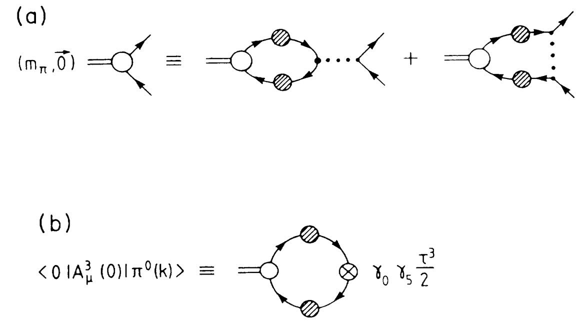

FIG. 3. (a) Bethe-Salpeter equation for the pion state in its rest frame. (b) Transition amplitude for the axial-vector current between the pion state defined in (a) and the vacuum, to leading order.

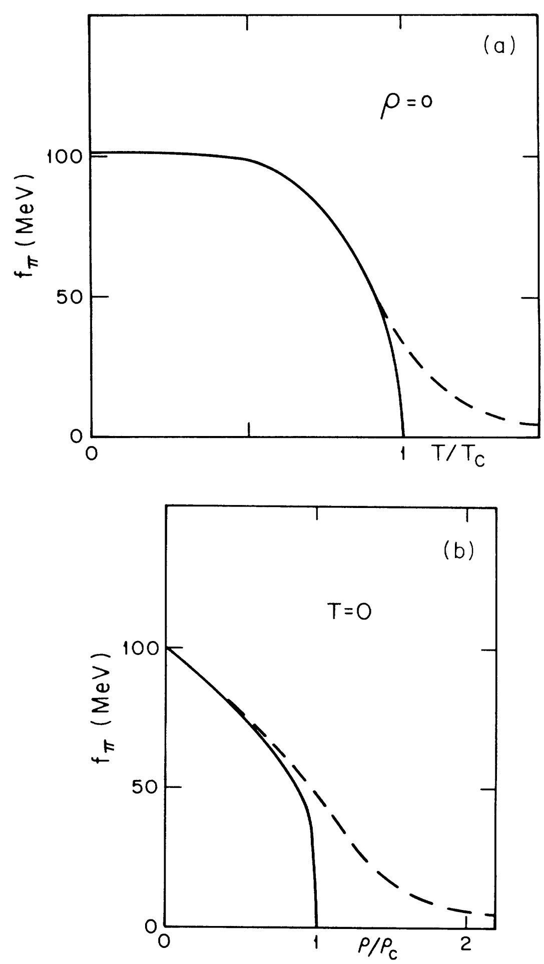

FIG. 4. (a) Behavior of f_π vs T/T_c. The solid (dashed) curve corresponds to $m=0$ ($m=5$ MeV). (b) Behavior of f_π vs ρ/ρ_c. The solid (dashed) curve corresponds to $m=0$ ($m=5$ MeV).

So far, we have assumed that the coupling constant G is temperature independent. Lattice simulations, however, suggest that at high temperature and/or density the string tension vanishes and the quarks deconfine. There are numerical indications that the deconfinement temperature is about equal to the critical temperature for restoring chiral symmetry. Assuming that this is indeed the case, we can use the parametrization of Alvarez and Pisarski[12] for a bosonic string in $d \to \infty$, to write

$$\frac{G}{G_0} = \left[1 - \left[\frac{T}{T_0} \right]^2 \right]^{1/2} , \tag{15}$$

where T_0 is some critical temperature presumably related to T_c. To arrive at (15), one only uses arguments based on dimensionality and scaling.[13] Figure 5 shows the effects of (15) on the $T\rho$-phase diagram. The qualitative behavior of all discussed quantities remains unchanged. Interestingly, the constituent-quark mass σ_1 scales like the string tension with temperature, an indication in favor of the lattice scaling.[14] This behavior was also discussed in Ref. 15 in the context of potential models in Coulomb gauge, where it was observed that the chiral-restoring transition coincides with the deconfinement transition. The effects of density on the string tension are unfortunately more subtle for a simple parametrization. Finally, one might argue at this stage that by changing μ and T the cutoff scale Λ changes as well. While this is not completely excluded, we do not expect these changes to affect qualitatively the character of our results.

We have investigated the dual effects of temperature and density on the static properties of the pion in the context of the Nambu–Jona-Lasinio model. We have shown in particular, that near the chiral phase transition the pion decay constant f_π vanishes and its mass enters the $q\bar{q}$ continuum. In other words, the pion decouples from matter. We have also argued that this behavior is enhanced by deconfinement. It would be interesting at this stage to see how the isoscalar bound state behaves in this model. Intuitive arguments suggest that like the pion, the σ meson will also decouple. This decoupling will have important consequences on the relativistic many-body problem. This issue will be brought up next.

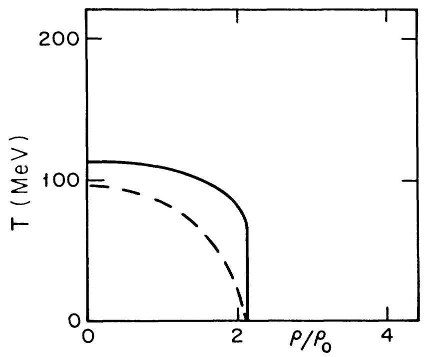

FIG. 5. $T\rho$-phase diagrams for a temperature-dependent coupling constant as defined in (15). The solid (dashed) curve corresponds to $T_0 = 0.3\Lambda$ ($T_0 = 0.22\Lambda$).

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