PHYSICAL REVIEW D VOLUME 36, NUMBER 3 1 AUGUST 1987

Branching, geometrical scaling, and Koba-Nielsen-Olesen scaling

Wei R. Chen and Rudolph C. Hwa
Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403
(Received 12 January 1987)

The phenomenon of Koba-Nielsen-Olesen scaling up to about 100 GeV in c.m. energy in *pp* collision is described in a branching model supplemented by impact-parameter smearing with geometrical scaling as an essential input. A high-accuracy fit of all data points from CERN ISR is achieved by use of only one parameter, which relates the particle productivity to hadron opacity at each impact parameter. It is deduced that the average number of initial clusters is between 4 and 5, depending upon the energy.

I. INTRODUCTION

Since the observation[1] of the violation of Koba-Nielsen-Olesen (KNO) scaling[2] of the multiplicity distribution P_n for c.m. energy between 200 and 900 GeV is accompanied by the observation[3] of low-E_T jets (called minijets) in the same energy range, it is reasonable to regard the two phenomena as intimately related. The central question in soft hadronic processes therefore reverts to the long-standing one: why does P_n exhibit KNO scaling for $\sqrt{s} \leq 100$ GeV? Indeed, without a satisfactory answer to this question, it is not likely that any quantitative investigation of KNO-scaling violation at higher energies can be very reliable. In this paper we show that KNO scaling is a consequence of branching and geometrical scaling.

The study of KNO scaling has a checkered history with many turns, too numerous to be summarized here. In recent years the use of negative-binomial distributions[4] has revitalized considerable interest in the subject.[5] After the discovery by the UA5 group that its data can best be fitted by an s-dependent k parameter (decreasing with s), thus rendering a naive interpretation of k in terms of cells in a stochastic cell model[4] unreasonable on physical grounds, many alternative views have been suggested to accommodate the variation in k and the associated KNO-scaling violation.[6] Concurrently, the interest in studying branching processes grew,[7-9] partly because the branching mechanism offers a possible connection between basic dynamics and KNO scaling; moreover, its solution approaches the asymptotic scaling curve from below, in accord with experimental observation. None of these more recent investigations focuses on the issue of why KNO scaling works so well up to the top of the energies achievable at CERN ISR. In the ISR energy range 20–65 GeV, although the second moment C_2 of the solution G_n^k of the simplest branching equation (which has a modified form of the negative-binomial distribution) can be shown to agree with data for a fixed k (Ref. 10), the higher moments have subsequently been shown to disagree with data.[11] A Poisson smearing of k with a particular s dependence of $\langle k \rangle$ can give a good fit of the data,[12] but no good physical basis for the mathematical procedure has been suggested.

Our viewpoint is that if branching is a sensible dynamical mechanism for multiparticle production at low p_T, then it must apply to hadronic collisions at each impact parameter b. There is no guarantee that after integration over all b the multiplicity distribution would remain as a solution of the branching equation. Indeed, any model that is claimed to have some connection with basic dynamics must face the question at some level of what the effects of impact-parameter smearing (IPS) would do to its predictions. The importance of IPS has been stressed by many authors, a few of whom are listed in Refs. 13–15. It is generally recognized that the broad KNO distribution can be obtained by superimposing narrow distributions at various b (Ref. 14). What we show here is that, while G_n^k not only is wide but for fixed k even broadens at subasymptotic energies, the IPS of G_n^k can give an excellent description of the scaling data. It should be remarked that our concerns are on providing a physical interpretation of the data which are to be examined at a very high level of accuracy. That level was not contemplated earlier,[14,15] but is now forced upon us by the work in Refs. 11 and 12, which showed the inadequacy of the higher moments of the prediction of a simpler branching model[10] without IPS. Nowhere shall we assume a KNO-scaling curve either before or after IPS.

II. THE GEOMETRICAL BRANCHING MODEL

In describing the geometrical properties of hadronic collisions, we adopt the formalism that incorporates the empirical property of geometrical scaling.[16] Thus the eikonal function $\Omega(s,b)$ is a function of the dimensionless scaling variable $R = b/b_0(s)$, where $b_0(s)$ is the size parameter that describes the s dependence of σ_{el}, σ_T, and σ_{in}. The inelastic cross section

$$\sigma_{\text{in}}(s) = \pi \int_0^\infty db^2 (1 - e^{-2\Omega(s,b)}) \tag{1}$$

becomes simply $\pi b_0{}^2(s)$, with the inelasticity function $g(s,b)$ becoming $g(R) = 1 - \exp[-2\Omega(R)]$, which satisfies the normalization constraint

$$\int_0^\infty dR^2 g(R) = 1 \ . \tag{2}$$

If the multiplicity distribution at s and b is $G_n(s,b)$, then the observed distribution is

$$P_n(\bar{n}) = \int_0^\infty dR^2 g(R) G_n(s,b) , \quad (3)$$

where $\bar{n}(s)$ is the average multiplicity at s, i.e., $\bar{n} = \sum nP_n$. We write $G_n(s,b)$ as $G_n(N)$, where $N(s,R)$ is the average multiplicity at s and R: $N(s,R) = \sum nG_n(N)$. Averages over R^2 as in Eq. (3) shall hereafter be abbreviated by the notation $\langle \ \rangle$, e.g., $P_n(\bar{n}) = \langle G_n(N) \rangle$, and $\bar{n} = \langle N \rangle$.

We assume that the multiplicity distribution at every R satisfies the branching equation for the Furry process[17]

$$\frac{d}{dt} Q_n^k = (n-1) Q_{n-1}^k - nQ_n^k , \quad (4)$$

where we have added a superscript k to Q_n to specify the initial multiplicity: $Q_n^k(t=0) = \delta_{n,k}$. With $N = \sum_n nQ_n^k$, we have $t = \ln(N/k)$, a relation which can be verified by summing Eq. (4) over n weighted by n. Let w be the evolution parameter

$$w = N(s,R)/k(s,R) \quad (5)$$

in terms of which the solution of Eq. (4) is[7,8,17]

$$Q_n^k(w) = \frac{\Gamma(n)}{\Gamma(k)\Gamma(n-k+1)} \left[\frac{1}{w} \right]^k \left[1 - \frac{1}{w} \right]^{n-k} . \quad (6)$$

This can be rewritten in the form of a negative-binomial distribution, and has been used to compare with data for $w = \bar{n}/k$, resulting in some difficulty with high moments.[11,12] Our view in this paper is that Eq. (6) is for a fixed R and is specified by Eq. (5) at that R; upon substitution of Eq. (6) for $G_n(N)$, it is Eq. (3) after IPS that should be compared to data.

Since, after IPS of the branching solution, $P_n(\bar{n})$ does not, in general, satisfy the branching equation, the processes of branching and IPS are not independent. Nevertheless, in searching for a simple description of the geometrical properties of the multiparticle production processes, we assume that $N(s,R)$ and $k(s,R)$ are factorizable, and that they have the same R dependence: i.e.,

$$N(s,R) = \bar{n}(s) h(R) , \quad (7)$$

$$k(s,R) = \bar{k}(s) h(R) . \quad (8)$$

Since by definition

$$\langle N \rangle = \bar{n}, \quad \langle k \rangle = \bar{k} , \quad (9)$$

we require $h(R)$ to satisfy

$$\langle h \rangle = 1 . \quad (10)$$

The property that branching and IPS are not independent implies that $\bar{n}$ and $\bar{k}$ are not related directly by branching. Yet it is convenient and appropriate to refer to $\bar{k}$ as the average number of initial clusters. Indeed, it follows from Eqs. (5), (7), and (8) that $w = \bar{n}(s)/\bar{k}(s)$, a function of s only, but, we repeat, $P_n(\bar{n})$ is not $Q_n^{\bar{k}}(w)$.

It can be established from Eq. (6) by negative-binomial expansion that

$$\sum_{n=k}^{\infty} \frac{\Gamma(n+m)}{\Gamma(n)} Q_n^k(w) = \frac{\Gamma(k+m)}{\Gamma(k)} w^m . \quad (11)$$

Performing the IPS of Eq. (11) and solving for the moments

$$C_m = \overline{n^m}/\bar{n}^m = \left\langle \sum_n n^m G_n^k \right\rangle \Big/ \bar{n}^m , \quad (12)$$

we obtain, using Eq. (10) and $w = \bar{n}/\bar{k}$,

$$C_2 = \langle h^2 \rangle + (w-1)/\bar{n} , \quad (13a)$$

$$C_3 = \langle h^3 \rangle + 3(w\langle h^2 \rangle - C_2)/\bar{n} + 2(w^2-1)/\bar{n}^2 , \quad (13b)$$

$$C_4 = \langle h^4 \rangle + 6(w\langle h^3 \rangle - C_3)/\bar{n} + 11(w^2\langle h^2 \rangle - C_2)/\bar{n}^2 + 6(w^3-1)/\bar{n}^3 , \quad (13c)$$

$$C_5 = \langle h^5 \rangle + 10(w\langle h^4 \rangle - C_4)/\bar{n} + 35(w^2\langle h^3 \rangle - C_3)/\bar{n}^2 + 50(w^3\langle h^2 \rangle - C_2)/\bar{n}^3 + 24(w^4-1)/\bar{n}^4 . \quad (13d)$$

The experimental values of these moments are independent of energy, at least through the ISR range, a reflection of the KNO scaling. On the other hand, $\bar{n}$ and w are both energy dependent. Thus the challenge is to discover a simple way to achieve energy independence for all the moments as well as to obtain the right values for those moments. The burden is clearly on finding the proper form of $h(R)$ that can yield the appropriate moments $\langle h^m \rangle$ after IPS.

III. PHENOMENOLOGY

Let us now adopt the Gaussian form for $\Omega(R)$ in order to be specific; the exact form is not important as we shall discuss later. Thus we write

$$\Omega(R) = \Omega_0 e^{-\beta R^2} . \quad (14)$$

The value of Ω_0 is not free to adjust because it is determined by the position of the first minimum of the elastic diffraction peak.[13] We shall use the value $\Omega_0 = 1.4$. The value of β is also not adjustable on account of Eq. (1), which yields $\beta = 1.624$. Since the number of particles produced at any fixed R should be related to the opacity of the colliding hadrons at that R, we assume the form

$$h(R) = h_0 \Omega^\gamma(R) , \quad (15)$$

where γ is the only parameter in this problem, h_0 being determined by Eq. (10). Thus we have, for all $m \geq 0$,

$$\langle h^m \rangle = h_0^m \beta^{-1} \int_0^{\Omega_0} d\Omega (1 - e^{-2\Omega}) \Omega^{m\gamma - 1} . \quad (16)$$

We vary γ to fit C_5, since it is most sensitive to the precise value of γ. The energy dependence (or, more precisely, the $\bar{n}$ dependence) of w is specified by Eq. (13a). For $\gamma = 0.25$, we have $\langle h^2 \rangle = 1.055$; thus for $C_2 = 1.2$ we get $a = 0.145$ in

$$w = 1 + a\bar{n}, \quad a = C_2 - \langle h^2 \rangle . \quad (17)$$

If we choose $\gamma = 0.35$, then $\langle h^2 \rangle = 1.10$ and the value of a would be 0.1. What is surprising is that for these values of γ, or any in between, (a) all of the calculated moments C_3 to C_5 agree well with the ISR data, as shown in Fig. 1, and (b) those moments are essentially independent of energy (or $\bar{n}$). While the $\bar{n}$ independence of C_2 is guaranteed by Eq. (17), the virtual independence on $\bar{n}$ of

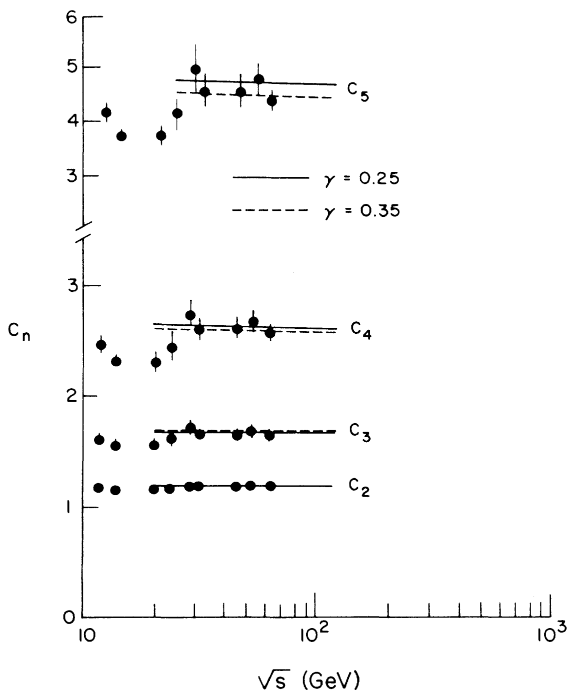

FIG. 1. Moments of multiplicity distribution vs energy. The solid and dashed lines are calculated results for two different values of γ. The data points are from Ref. 1.

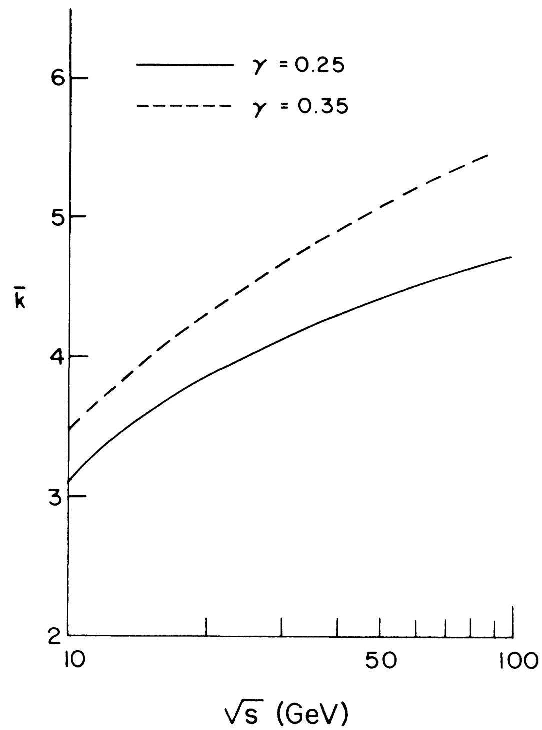

FIG. 2. Average number of initial clusters.

C_3 to C_5 is a consequence of the structure of the moment equations (13) resulting from IPS of the branching solution. We therefore regard our dynamical (branching) and geometrical (IPS) description of particle production linked by Eq. (15) with $\gamma=0.3\pm0.05$ as a satisfactory answer at this stage to the puzzling question about the origin of KNO scaling.

We have checked the dependence of our result on the exact input for $\Omega(R)$. We varied Ω_0 by 20% and found that the moments C_m change by less than 2%. We also tried some other form for $\Omega(R)$, e.g., $\Omega_0 \exp(-\beta_1 R^4)$, and found no significant effects on the fits of the moments. Thus the value $\gamma=0.3\pm0.05$ is quite stable against variations on the details about $\Omega(R)$.

IV. DISCUSSION

We can learn from our result some properties of branching as applied to hadronic collisions. First, the evolution parameter w that specifies the extent of branching increases with $\bar{n}$, and hence s, without bound, as evidenced by Eq. (17). This is, of course, eminently reasonable. Second, and less obviously, the average number of initial clusters $\bar{k}$ also increases with s, but is bounded. This follows from Eqs. (5), (9) (or directly $w=\bar{n}/\bar{k}$), and (17):

$$\bar{k}=(a+1/\bar{n})^{-1} . \tag{18}$$

Using the known experimental relationship between $\bar{n}$ and s, we have plotted $\bar{k}$ against $\sqrt{s}$ in Fig. 2. Evidently, $\bar{k}$ can vary between 3 and 5, as $\sqrt{s}$ is increased up to 100 GeV. This increase is physically more sensible than a naive interpretation of the decrease of k in the phenomenological fit of the data[1] by UA5 based on negative binomials. The decrease of k is necessary there to accommodate a broadening distribution. But this rule is not applicable in a branching model; indeed, $\bar{k}$ must increase at nearly the same rate as $\bar{n}$ to maintain KNO scaling in the subasymptotic regime. The result suggests that as s increases, more and more of the hadronic stuff (in the language of the droplet model[13]) becomes effective in producing particles until it is exhausted, whence $\bar{k}$ reaches its asymptotic value. We may also associate $\bar{k}$ with the average number of chains in the dual-parton model[18] or with the average number of clusters in the fireball model[19] (except that the latter prefers three fireballs in a minimal description). To discover the interconnections among all these approaches would be very interesting and worthwhile.

Finally, we show how IPS of the branching solution results in the KNO curve. If we break up the continuous R^2 integration in Eq. (3) into a sum of four sections centered at $R^2=0.15$, 0.5, 1.0, and 2.2, we get the four dashed curves shown in Fig. 3 for $g(R^2)\bar{n}Q_n^k(w)$, the solid curve being the integrated result. The case for $\gamma=0.25$ is plotted, and the normalization has been changed to 2 in order to compare with the data on charged particles produced at ISR (Ref. 20). Note that the individual contributions have wide distributions themselves, a characteristic of the branching solution. The region with $n>\bar{n}$ is dom-

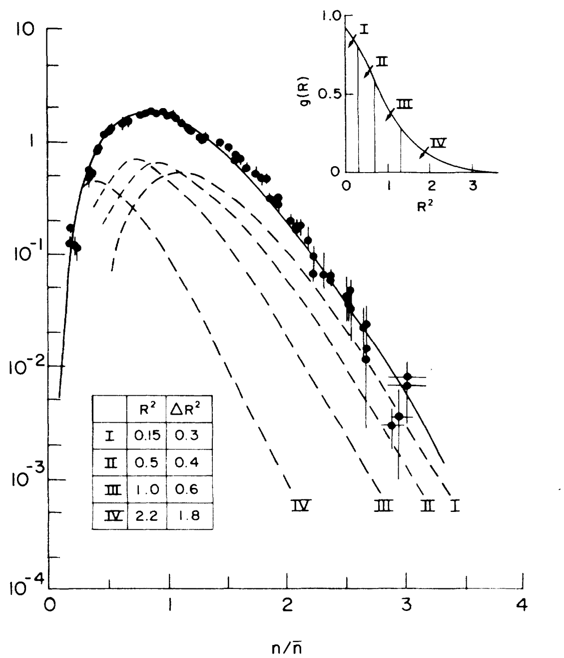

FIG. 3. KNO plot of multiplicity distribution. The data points are from ISR, as presented in Ref. 1. Solid line is the integrated overall distribution. Dashed lines are contributions from various impact parameters, as indicated by the four regions in R^2 in the insets.

inated by the small-R contribution, as is reasonable. The curves for small R^2 turn over at small n because $k(s,R)$ is relatively large ($\simeq 5.1$ for $R^2=0.15$) so the probability of emitting a small number of particles ($n \geq k$) is suppressed. At large R, the region of n, where such a suppression mechanism operates, is pushed to the extreme low end in n, since $k(s,R)$ is itself small ($\simeq 2.2$ for $R^2=2.2$). The large-n behavior for all R^2 is similar for all branching solutions with $n \gg k$ being basically exponential in character. The most striking feature of Fig. 3 is that the overall KNO curve is not a superposition of narrow distributions of the type appropriate for describing multiplicity distribution in e^+e^- annihilation.[14,15] Here we have come to a rather basic point: after the geometrical complication is disentangled, is the production mechanism at a given R universal in the sense of having no distinction among pp, e^+e^-, and ep collisions, or is the soft production process in pp collision even at fixed R basically different from the hard processes in e^+e^- and ep collisions? Our answer is clearly affirmative in the latter option.

V. SUMMARY

In summary we have amalgamated branching and geometrical scaling in a way that gives rise to KNO scaling. There is one free parameter γ which relates the efficiency of particle production to the opacity of hadron scattering. For $\gamma=0.3\pm0.05$ we have found a nearly perfect description of the multiplicity distributions throughout the ISR energy region. While the dynamical origin for the value of γ remains to be understood at a deeper level, the stage is set for an investigation of the violation of KNO scaling by minijets at CERN S$\bar{p}p$S, which takes into account the geometrical aspect of the hadrons.

ACKNOWLEDGMENTS

One of us (R.C.H.) has discussed various aspects of the general problem with C.S. Lam over the years and most recently at the Aspen Center for Physics during a visit; to him we are very grateful. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG06-85ER40224-A001.

[1]G. J. Alner *et al.*, Phys. Lett. **160B**, 193 (1985); **167B**, 476 (1986).
[2]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. **B40**, 317 (1972).
[3]G. Arnison *et al.* (unpublished); G. Piano Mortari, in *Proceedings of the Oregon Meeting,* proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, Eugene, 1985, edited by R. C. Hwa (World Scientific, Singapore, 1986), p. 615.
[4]A. Giovannini, Nuovo Cimento **A15**, 543 (1973); P. Carruthers and C. C. Shih, Phys. Lett. **127B**, 242 (1983).
[5]For a brief overview see P. Carruthers, in *Strong Interactions and Gauge Theories,* proceedings of the XXIst Rencontre de Moriond, Les Arcs, 1986, edited by J. Tran Thanh Van (Editions Frontiéres, Gif-sur-Yvette, 1986), p. 229.
[6]A. Giovannini and L. Van Hove, Z. Phys. C **30**, 391 (1986); G. N. Fowler, E. M. Friedlander, R. M. Weiner, and G. Wilk, Phys. Rev. Lett. **56**, 14 (1986); C. C. Shih, Phys. Rev. D **34**, 2710 (1986).
[7]B. Durand and S. D. Ellis, in *Design and Utilization of the SSC, Snowmass 1984,* proceedings of the 1984 Division of Particles on Fields Summer Study, Snowmass, Colorado, 1984 edited by R. Donaldson and J. G. Morfin (Fermilab, Batavia, Illinois, 1985), p. 234.
[8]C. S. Lam and M. A. Walton, Phys. Lett. **140B**, 246 (1984); D. C. Hinz and C. S. Lam, Phys. Rev. D **33**, 3256 (1986); R. C. Hwa and C. S. Lam, Mod. Phys. Lett. **A1**, 303 (1986).
[9]B. Durand and I. Sarcevic, Phys. Lett. **172B**, 104 (1986).
[10]R. C. Hwa and C. S. Lam, Phys. Lett. **173B**, 346 (1986).
[11]C. J. Maxwell, Phys. Lett. **185B**, 192 (1987).
[12]M. Biyajima, T. Kawabe, and N. Suzuki (unpublished).
[13]T. T. Chou and C. N. Yang, Phys. Rev. **170**, 1591 (1968); A. W. Chao, and C. N. Yang, Phys. Rev. D **8**, 2063 (1973); T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. Lett. **54**, 510 (1985).
[14]S. Barshay, Phys. Lett. **51B**, 376 (1974); S. Barshay and Y. Yamaguchi, *ibid.* **51B**, 376 (1974); S. Barshay, *ibid.* **116B**, 193 (1982).
[15]C. S. Lam and P. S. Yeung, Phys. Lett. **119B**, 445 (1982).
[16]J. Dias de Deus, Nucl. Phys. **B59**, 231 (1973); A. J. Buras and J. Dias de Deus, *ibid.* **B71**, 481 (1974).
[17]S. K. Srinivasan, *Stochastic Theory and Cascade Processes* (El-

sevier, New York, 1969), p. 7.
[18]A. Capella, U. Sukhatme, C. I. Tan, and J. Tran Thanh Van, Phys. Lett. **81B**, 68 (1979); A. Capella, U. Sukhatme, and J. Tran Thanh Van, Z. Phys. C **3**, 329 (1980).
[19]Liu L.-S. and Meng T.-C., Phys. Rev. D **27**, 2640 (1983); Chou K.-C., Liu L.-S., and Meng T.-C., *ibid.* **28**, 1080 (1983).
[20]The ISR data are collectively presented by Alner *et al.* (Ref. 1).