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Soliton-antisoliton pair creation in strong external fields

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The features of soliton-antisoliton (fermion-antifermion) pair creation due to instantaneous uniform external electric fields are investigated in the sine-Gordon (massive Thirring) theory. Under the coherent-state approximation of soliton states improved via the generator-coordinate method, it is found that the momentum spectra of solitons and antisolitons have a peak structure, some of whose qualitative properties are quite reminiscent of those of the electron and positron peaks found in recent heavy-ion collision experiments at Gesellschaft für Schwerionenforschung Darmstadt.

In a previous paper[1] we pointed out the possibility that the peak structure in positron[2] and electron[3] energy spectra observed in heavy-ion collision experiments at Gesellschaft für Schwerionenforschung Darmstadt (GSI) is attributable to a nonperturbative mechanism of strong-field QED. In order to analyze such a question we have proposed, in Ref. 1, a framework of nonperturbative QED, the bosonized lowest-partial-wave QED. It is the quantum field theory of $j=\frac{1}{2}$ electrons and $j=0$ electromagnetic fields, first written in a form of two-dimensional fermion theory, and then converted into a boson theory via the bosonization technique. Far away from the external source (which mimics the effect of heavy ions) the theory simply reduces to the sine-Gordon theory. Thus, the electrons are essentially the sine-Gordon solitons in our theory.

In this Rapid Communication we discuss the problem of soliton-antisoliton pair creation due to the rapidly time-varying external source in the sine-Gordon theory. Of course it does not directly address the same question in bosonized QED because of the difference of our theory from the sine-Gordon system, which reflects the original three-dimensional topology of the system. Nevertheless, it is of interest because the developed formalism here is applicable, after suitable modification, to bosonized QED. Furthermore, as we will see later, we obtain important information concerning the nature of soliton-antisoliton pair creation. In particular, we will find a peak structure in momentum spectra of solitons and antisolitons, whose position moves very slowly with the strength Z of the external electric field, i.e., $\propto Z^1$. These features of the soliton-antisoliton pair creation are reminiscent of those observed in GSI experiments.

We are particularly interested in the rapidly changing external source rather than the adiabatic one. This is because the experimental feature of the peak structure in positron and electron energy spectra, if it is the matter of the strong-field QED, definitely favors the hierarchy of the time scales, $\tau_{\text{creation}} \gg \tau_{\text{collision}}$, as discussed in Ref. 1. Therefore, we investigate in this paper the features of electron-positron pair creation in the instantaneous approximation, namely, in the limiting situation where the source has a δ-function-like time dependence. We, however, start with a more general formalism.

We consider the sine-Gordon theory coupled with an external source described by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 + \frac{m^4}{\lambda}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right) - 1\right] + J(x)\phi(x). \tag{1}$$

The particular manner in which the external source couples with the sine-Gordon field in (1), though standard, requires comment. From our original motivation we are interested in the massive Thirring model, the equivalent fermion theory of the sine-Gordon model,[4] in an external electric field. After the bosonization this system reduces to (1), including the coupling with the external source.

We concentrate on evaluating

$$T = {}_{\text{out}}\langle P_1, P_2 \mid S \mid 0\rangle / \langle 0 \mid S \mid 0\rangle, \tag{2}$$

where S denotes the S matrix

$$S=\exp\left[-i\int d^2x\,J(x)\phi(x)\right] \tag{3}$$

of the external-source problem and $|P_1,P_2\rangle_{\rm out}$ represents the asymptotic out state of a soliton-antisoliton pair with momenta P_1 and P_2, respectively. We note that by dropping the time ordering in (3) we merely change the unobservable phase of the S matrix.[5] We will return to the meaning of the denominator in (2) in our later discussions.

Then our whole problem is to determine the asymptotic out state of a soliton-antisoliton pair. In this paper, we rely on the coherent-state approximation of soliton states, improved via the generator-coordinate method.[6]

We quantize the system (1) by imposing the canonical equal-time commutation relation between $\phi(x)$ and $\pi(x)$, the canonical conjugate of ϕ. Then the one-soliton state with momentum P is taken as (with ϕ_K being the classical kink solution)

$$|P\rangle_s=C(P)\int dx\,e^{iPx}|\phi_K(x)\rangle, \tag{4}$$

which is the superposition of the coherent state

$$|\phi_K(x_1)\rangle=\exp\left[-i\int dx\,\phi_K(x-x_1)\pi(0,x)\right]|0\rangle, \tag{5}$$

localized at $x=x_1$. In (4) $C(P)$ denotes the state normalization constant determined so that $\langle P|P'\rangle=2\pi\delta(P-P')$:

$$|C(P)|^{-2}=\int_{-\infty}^{+\infty}dt\,\exp[itP+F(t)], \tag{6}$$

where $F(t)$ is defined as

$$F(x_1-x_2)=-\tfrac{1}{2}\int dx\,dy[\phi_K(x-x_1)-\phi_K(x-x_2)]\Delta_\pi^{(+)}(x-y)[\phi_K(y-x_1)-\phi_K(y-x_2)]\ , \tag{7}$$

and

$$\Delta_\pi^{(+)}(x-y)=[\pi^{(+)}(t,x),\pi^{(-)}(t,y)]=\frac{1}{4\pi}\int_{-\infty}^{+\infty}dk\,k^0e^{ik(x-y)}\ . \tag{8}$$

We note that the state (5) minimizes the expectation value of the Hamiltonian normal ordered with respect to the annihilation operator:

$$a(k)=\int dx\,e^{-ikx}[k^0\phi(0,x)+i\pi(0,x)]\ . \tag{9}$$

Since (4) gives an approximate eigenstate of the total Hamiltonian, it can be regarded as the asymptotic state. In fact, it can be shown that (4) gives a better approximation than (5) (in the sense of variational method) to the eigenstate of the Hamiltonian as we will see later.

To construct the antisoliton state $|P\rangle_a$ we exploit the charge-conjugation operator of the original fermionic variable. After translation into the Bose variable, it leads to the transformation property $\mathcal{C}\pi(x)\mathcal{C}^{-1}=-\pi(x)$. Therefore we just take $-\phi_K$ instead of ϕ_K in (5) to construct the antisoliton state.

Now our asymptotic out state is constructed as

$$|P_1,P_2\rangle=C(P_1)C(P_2)\int_{-\infty}^{+\infty}dx_1dx_2e^{i(P_1x_1+P_2x_2)}\exp\left[-i\int dx[\phi_K(x-x_1)-\phi_K(x-x_2)]\pi(0,x)\right]|0\rangle\ , \tag{10}$$

which describes an uncorrelated soliton-antisoliton pair with momenta P_1 and P_2, respectively. Given the out state, it is straightforward to calculate the pair-creation amplitude T with the result

$$T=C(P_1)C(P_2)\int dx_1dx_2e^{-i(P_1x_1+P_2x_2)+F(x_1-x_2)}\times\exp\left[\left(\int dt\,dx\,dy[\phi_K(x-x_1)-\phi_K(x-x_2)]J(t,y)[\pi^{(+)}(0,x),\phi^{(-)}(t,y)]\right]\ , \tag{11}$$

where the function $F(t)$ is defined in (7). The superscripts (+) and (−) appearing in (11) imply the positive- and negative-frequency parts, respectively.

We introduce the instantaneous approximation of the source and also assume it uniform based on the physical picture mentioned before. Namely, we take

$$J(t,x)=\delta(t)Z\ . \tag{12}$$

Under the instantaneous approximation we can calculate the commutator appearing in the second exponential in (11) using the fact that

$$[\pi^{(+)}(0,x),\phi^{(-)}(0,y)]=-\frac{i}{2}\delta(x-y)\ .$$

One may suspect that taking the peak position at $t=0$ in (12) is artificial and may ask what happens if we take it to be peaked at $t=t_0$. But it is easy to observe that, from the original expression (2) with the S matrix (3), the change of the peak position merely affects the unobservable phase of the pair-creation amplitude, as expected from physical intuition.

Let us analyze the soliton-antisoliton pair creation described by (11) with (12). First of all, one should note that under the uniform source (12) the produced solitons and the antisolitons have identical momentum distributions. It is obvious from the expression of (11) that we have a δ function $\delta(P_1+P_2)$ under (12) because the rest of the integrand in (11) depends only on x_1-x_2. Therefore, the identity of the soliton and the antisoliton energy spectra is the general consequence of the uniformity of the source, independent of the detail of the production mechanism.

Let us now enter into the question of the shape of the momentum spectra. To gain qualitative insight we first work with approximations

$$\phi_K(x)=\frac{2\pi m}{\sqrt{\lambda}}\theta(x)\ , \tag{13a}$$

$$\Delta_\pi^{(+)}(x)=\frac{m}{2}\delta(x)\ , \tag{13b}$$

where $\theta(x)$ denotes the step function. Equation (13b) implies the replacement of $\sqrt{k^2+m^2}$ by m in (8). These approximations are only meant to be an attempt to grasp the gross features of the soliton-antisoliton pair creation. We will return to the problem of the accuracy of these approximations in later discussions.

Under the approximations (13) we obtain an analytic expression of T:

$$T=2\pi\delta(P_1+P_2)\frac{\left(\frac{\pi^2m^3}{\lambda}\right)^2+P_1{}^2}{\left[\left(\frac{\pi^2m^3}{\lambda}\right)^2+\left(P_1-\frac{\pi m}{\sqrt{\lambda}}Z\right)^2\right]^2}\ . \tag{14}$$

That is, we have a peak structure in momentum spectra of the soliton and the antisoliton, which are identical to each other. The peak position moves with the strength Z of the external electric field as $\propto Z^1$. The width of the peak is not, however, particularly narrow. It is of the order of the soliton mass, in accord with our physical intuition. (Recall that the classical soliton mass formula gives $m_{\rm cl}=8m^3/\lambda$.)

We now turn to a more accurate numerical analysis of (11), without recourse to the approximations (13). In fact, a close examination of (11) indicates the necessity of an improved form of the soliton profile because the integral in (7), without using (13b), has an ultraviolet divergence, which is the artifact of the infinite slope of the soliton profile in (13a).

To overcome such a problem, we improve the soliton profile (13a) by adding the correction term

$$\phi_K(x)=\frac{2\pi m}{\sqrt{\lambda}}[\theta(x)-\tfrac{1}{2}\epsilon(x)e^{-(2m/\pi)|x|}]\ , \tag{15}$$

where $\epsilon(x)=\pm1$ for $x\gtrless0$. The parameters of the second term are determined so that the first derivative of the kink solution at $x=0$ coincides with that of the exact solution, $\phi_K'(0)=2m^2/\sqrt{\lambda}$. We employ the approximate form (14) rather than the exact solution itself because the latter does not have a manageable Fourier transform.

Under (14) the function $F(t)$ defined in (7) allows a simple form of momentum representation

$$F(t)=-\frac{\pi m^2}{\lambda}\left(\frac{2m}{\pi}\right)^4\int_{-\infty}^{+\infty}dq(1-\cos qt)\frac{q_0}{q^2\left[q^2+\left(\frac{2m}{\pi}\right)^2\right]^2}\ , \tag{16}$$

which is usable in numerical computation.

In our numerical analysis we concentrate on the case of the strong coupling, $\sqrt{\lambda}/m=2\sqrt{\pi}$, which corresponds to the bosonized QED.[1] In Fig. 1 we plot the momentum dependence of the pair creation amplitude at $Z=2.5m_{\rm cl}$, whose square is proportional to the soliton production cross section. The result (14) with the approximation (13) is also plotted.

Instead of presenting similar plots for different value of Z, we show in Fig. 2 the Z dependence of the peak position and the width in units of the classical soliton mass. Here the peak width refers to the full width at half maximum of the amplitude squared. We can observe from these figures that Z^1 dependence of the peak position is well reproduced already at relatively small Z, but with more gentle slope than the asymptotic value predicted by the approximate result (14). On the other hand, the approximate result (14) gives rather poor estimations for the height and the width of the peak. Roughly speaking, however, the latter is of the order of the soliton mass.

The simultaneous presence of the peak structure in the soliton and the antisoliton momentum spectra and the weak Z dependence of the peak positions are welcome to our interpretation of the peak structure in GSI experiments as the effect of strong-field QED. The peak width expected from our calculation, however, is not particularly narrow, and clearly is insufficient to explain the "line" structure observed experimentally.[2,3]

Does T squared really imply the production probability of soliton and antisoliton? The answer is no, because it is $\langle P_1P_2|S|0\rangle$ squared not T squared that measures the probability. For our choice of the uniform external field, however, the denominator in (2) has a form $\exp(-Z^2L/4m)$, with L being the size of the one-dimensional world, and becoming vanishingly small as $L\to\infty$. Therefore the exclusive probability for the creation of one soliton pair vanishes in our case. This should not come as a surprise, because the unitarity dictates that each exclusive probability goes to zero if the average number of the pair increases as the size of our world grows. Despite vanishing absolute probability, our investigation of the properties of the soliton-antisoliton pair creation is meaningful since it reveals interesting features of the system, once they are created, due to the nonperturbative production mechanism.

In this paper we have discussed the property of soliton-antisoliton (which mimics e^+e^-) pair creation due to the rapidly changing external electric fields. We have demonstrated that the mechanism exists which is responsible for fermion-antifermion pair creation with qualitative features reminiscent of the peak structure in GSI experiments. Of course we have dealt with a related but different model from bosonized QED, and the obtained peak width is not sufficiently narrow. But we believe that our result opens the possibility of explaining the anomalous peak structure in e^+e^- spectra, as the effects of non-

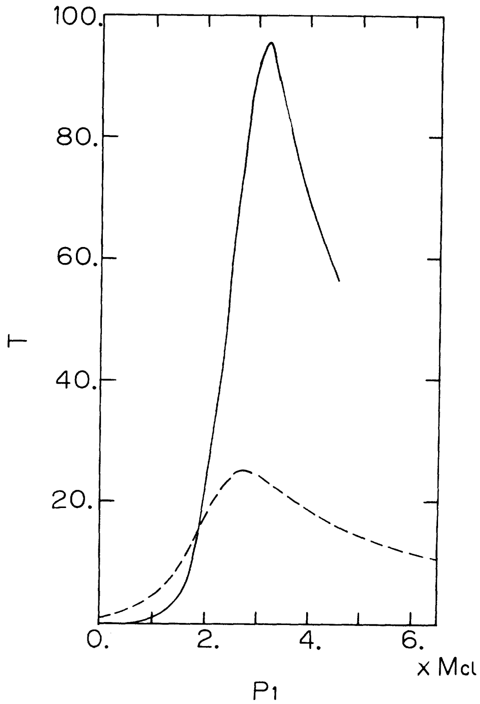

FIG. 1. The momentum dependence of the soliton (or antisoliton) production amplitude at $Z = 2.5m_{cl}$ is plotted (solid line). The momentum is measured in units of m_{cl}, the classical soliton mass. The same quantity multiplied by 5 computed by the approximation (13) is also plotted (dashed line). The amplitudes, not amplitudes squared, are presented for ease of drawing the figure.

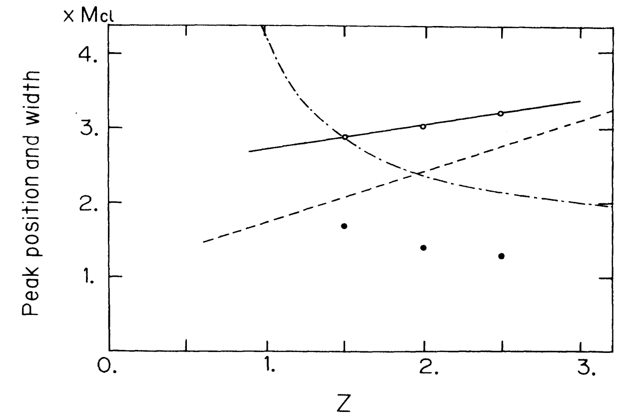

FIG. 2. The numerically calculated results of source dependences of the peak position (○) and the width of the peak (●) are plotted in units of m_{cl}, the classical soliton mass. The Z dependence of the peak position is shown to be well approximated by the straight line $0.36Z+2.31$. The same quantities obtained by using the approximation (13) are also plotted; the dashed line for peak position and the dash-dotted line for the width. All the dimensionful quantities are measured in units of m_{cl}.

perturbative QED in the strong field, in supporting the suggestion given in Ref. 1.

Finally some comments are in order.

(1) What is the essential difference between our approach and the old QED approach[7] to the spontaneous positron creation in the strong field? It is simply the fact that they are working with the different hierarchy of the time scales. Namely, while the old approach was dealing with the situation $\tau_{\text{creation}} \ll \tau_{\text{collision}}$ where the adiabatic approximation holds, we have examined the case with opposite hierarchy.

(2) We have not addressed the question of how good the coherent-state approximation is improved via the generator-coordinate method. Concerning the soliton mass, this method with the Gaussian approximation[6] and (13b) gives the correction to the classical mass formula, $m_s = m_{cl} - \frac{1}{4} m$, which should be compared with $-(1/\pi)m$, the result of exact one-loop computation.[8] While this result is encouraging, we certainly need better ways of testing our approximations used in this paper.

(3) To deal with bosonized QED we have to take into account other factors which come from the original three-dimensional topology of the system. Work toward this direction is in progress.

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