Chiral hierarchies from slowly running couplings in technicolor theories

Thomas Appelquist and L. C. R. Wijewardhana
Department of Physics, Yale University, New Haven, Connecticut 06511
(Received 4 March 1987)

Chiral-symmetry breaking in asymptotically free theories with slowly running couplings is analyzed. When the confinement scale Λ is much less than the cutoff M beyond which the theory cannot be used in isolation, the dynamical mass $\Sigma(p)$ starts from a value $\simeq\Lambda$ for momenta $p \lesssim \Lambda$ and falls slowly for a significant range $p > \Lambda$. It then takes on the asymptotic form $(\ln p)^a/p^2$ where $a > 1$. This behavior leads to an enhancement of the fermion condensate defined with the ultraviolet cutoff M, while the Goldstone-boson decay constant F remains essentially unaltered. In technicolor theories, with four-fermion interactions of strength $\simeq 1/M^2$ that explicitly break some chiral symmetries, the technifermion condensate determines the values of fermion masses and pseudo-Goldstone-boson masses while F determines W and Z masses. An enhancement of the technifermion condensate will then generate fermion and pseudo-Goldstone-boson masses for higher values of M than naively expected, keeping W and Z masses nearly fixed. This could allow for an adequate suppression of the flavor-changing neutral currents that have plagued technicolor theories. Explicit gauge theories that exhibit slow running are tabulated and the results of a numerical study are reported.

I. INTRODUCTION

Although the $SU(2)\times U(1)$ gauge theory of electroweak interactions has met with considerable experimental success, the symmetry-breaking mechanism that generates masses for the $W^{\pm}$, the Z^0, and the fermions remains yet to be understood. Whether the symmetry breaking is achieved by the condensation of a fundamental scalar field, or a fermion bilinear, or something else, is an important and open question.

The original version of the electroweak theory made use of fundamental scalars to break the gauge symmetry. In fundamental scalar field theories, however, radiative corrections to the scalar mass contain quadratic divergences, necessitating an unnatural fine-tuning of the bare mass in order to keep the physical mass small compared to the cutoff. There is no symmetry in the theory (like chiral symmetry in the case of fermions) to guarantee the smallness of the radiative corrections to the scalar mass. Another problem with fundamental scalars is that the Yukawa couplings to fermions are quite arbitrary. They must be adjusted by hand to fit the various fermion masses.

These and other aesthetic reasons have led to the suggestion that electroweak symmetry breaking might be driven by the condensation of a fermion bilinear due to a strong vector gauge force called technicolor.[1] In these theories, the new fermions coupling to the new gauge force must exhibit a global chiral symmetry at least as large as $SU(2)_L\times SU(2)_R$. Its spontaneous breakdown to $SU(2)_{L+R}$, in the manner of QCD, will produce the three requisite Goldstone bosons to trigger the Higgs mechanism, provided that the confinement scale and also the chiral-symmetry-breaking scale are on the order of several hundred GeV. This mechanism provides an adequate origin for the masses of the $W^{\pm}$ and Z^0 vector bosons.

The problem with technicolor theories is that they do not easily generate the masses of the quarks and leptons. Additional interactions must be introduced to allow the message of spontaneous breakdown in the technisector to be communicated to the quarks and leptons.[2] Whatever the nature of these new interactions is, they play the role of the Yukawa couplings in the conventional, scalar-field Higgs theory.

The scale M associated with these new interactions must be large compared to the technicolor scale. At momenta small compared to M, the new interactions will take the form of effective four-fermion couplings with strength of order $1/M^2$. Fermion masses are then generated through the mechanism of Fig. 1, leading to a result on the order of $\langle \bar{T}T\rangle/M^2$. Here, T represents a technifermion and $\langle \bar{T}T\rangle$ is the appropriately defined technifermion condensate, the vacuum value of the bilinear $\bar{T}T$. The natural expectation, based on experience with QCD, is that $\langle \bar{T}T\rangle$ will be of order Λ^3, the confinement scale of the technicolor theory. The scale M will then have to be much larger than Λ to explain the masses of any of the known fermions.

Whether a realistic theory of fermion masses can be constructed along these lines remains an open question. There is a problem, however, that is likely to plague such a theory. In addition to the four-fermion interactions involving two ordinary fermions and two technifermions, there will be others involving four ordinary fermions and also four technifermions. These too are expected to be of strength $1/M^2$. The problem is that the four-ordinary-fermion interactions typically contain flavor-changing neutral currents. There is no known simple Glashow-Iliopoulos-Maiani (GIM) mechanism.[3] In order to avoid a variety of experimental constraints,[2,4] the scale M must be at least on the order of 300 TeV. This is not only large but probably too large. With M of this order, the typical size of a fermion mass will be no more than 0.1 MeV, much less than many of the quark and lepton masses. Given that no realistic technicolor theory yet exists, the severity of this problem is difficult to assess. Still, it is hard to see how realistic masses of 100 MeV and more could arise under these conditions.

The purpose of this paper is to explore one possible mechanism to alleviate this problem. The method is simply to enhance the value of the condensate $\langle \bar{T}T \rangle$ through a modification of technicolor dynamics.[5,6] The asymptotic freedom of the technicolor theory is maintained but it is assumed that the large number of fermions expected in a realistic technicolor theory will substantially slow down the running of the coupling. A variety of theories exhibiting this behavior will be tabulated. The slow running modifies the ultraviolet behavior of the theory in such a way as to enhance the condensate $\langle \bar{T}T \rangle$ relative to the confinement scale Λ^3. A hierarchy is generated within the technicolor theory that can be as large as two orders of magnitude in some of the models being considered.

Another consequence of condensate enhancement will also be studied in this paper. The full global symmetry of the technifermions will typically be larger than $SU_L(2)\times SU_R(2)$. If so, there will be more Goldstone bosons that can be absorbed by the $W^{\pm}$ and Z^0. Some of these will remain massless until the effects of the four-technifermion interactions are included, then becoming pseudo-Goldstone bosons. These masses can be computed by chiral perturbation theory and they will be proportional to the condensate $\langle \bar{T}T \rangle$. Thus an enhancement of the condensate will also increase the masses of the pseudo-Goldstone bosons. With a two-order-of-magnitude enhancement of the condensate, the pseudo-Goldstone-boson masses can perhaps be pushed out of the range so far excluded by experiment.

In Sec. II the properties of technicolor theories will be reviewed. The new interactions at scale M will be included and the expression for fermion masses and pseudo-Goldstone-boson masses will be derived.

Section III will be devoted to a review of chiral dynamics in asymptotically free gauge theories. The asymptotic form of the dynamical fermion mass $\Sigma(p)$, corresponding to spontaneous rather than explicit chiral-symmetry breaking will be derived. The sideways scale M will play the role of a physical, ultraviolet cutoff in this analysis. An important issue for our purposes is the critical value of the coupling α_c. This is the strength that the running coupling must attain in order to trigger spontaneous chiral-symmetry breaking. The definition of α_c is partially clouded by the fact that the low-momentum components of the theory (on the order of Λ) are involved. These limitations will be stressed.

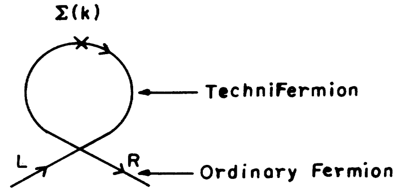

FIG. 1. Fermion mass generation in technicolor theories.

In Sec. IV we will describe the way in which a slowly running coupling modifies the dynamics of spontaneous chiral-symmetry breaking in an asymptotically free gauge theory. It will be shown that if the coupling evolves slowly in the momentum range above Λ, the dynamical mass $\Sigma(p)$ will initially fall more slowly than its ultimate asymptotic behavior. This slower fall will in turn enhance the value of the condensate. In the models we consider, the slower fall of $\Sigma(p)$ will persist out to momenta as large as two orders of magnitude beyond Λ. While this is still less than the cutoff M, it is large enough to enhance the condensate by up to two orders of magnitude.

In Sec. V some of the theories with slowly running couplings and enhanced condensates will be cataloged. The convergence of the loop expansion will be studied and the enhancement effect estimated for each of these theories. For one of the theories, we will summarize the results of a more careful numerical study of the enhancement effect.

Section VI will be devoted to conclusions and to a listing of open questions and possible extensions of the work described in this paper.

II. TECHNICOLOR AND EFFECTIVE FOUR-FERMION INTERACTIONS

In technicolor theories the existence of a new set of fermions interacting by an asymptotically free gauge theory is assumed.[7] If there are N_f technifermions then there is an $SU_L(N_f)\times SU_R(N_f)$-flavor symmetry in this sector. Technicolor dynamics spontaneously breaks this symmetry down to $SU_{L+R}(N)$ by imparting equal vacuum expectation values to $\bar{T}T$ for each technifermion T. This produces $N_F{}^2-1$ Goldstone bosons.

If $N_f=2$, there are 3 Goldstone bosons π^a. Because of the SU(2) transformation properties assigned to the technifermions, the coupling of π^a to the axial-vector currents $J^{\pi}_{5a}=\bar{\psi}\gamma^{\mu}\gamma^5\tau^a/2\psi$ is

$$\langle 0 \,|\, J^{\mu}_{5a} \,|\, \pi_b(k)\rangle = iF\delta_{ab}k^{\mu} \; . \qquad (2.1)$$

Here ψ is the SU(2) doublet of technifermions and τ_a are the Pauli matrices. F is the techni-Goldstone-boson decay constant. Thus $W^{\pm},Z^0$ will acquire the same masses as in a conventional scalar-field Higgs theory by devouring the π^a's, provided the decay constant F is identified with the Higgs vacuum expectation value 250 GeV.

If N_f is greater than 2, there are additional Goldstone bosons in the theory. For example, if one assumes that the technifermions come in a family of 2 techniquarks $\binom{U}{D}$ of 3 colors and 2 technileptons $\binom{E}{N}$, having the same $SU_C(3)\times SU_L(2)\times U_Y(1)$ quantum numbers as the ordinary fermion families [except for the existence of N_R to guarantee $\rho\equiv M_W/M_Z\cos(\theta_W)=1$], then the flavor symmetry is $SU_L(8)\times SU_R(8)$. After spontaneous chiral-symmetry breaking there will be 63 Goldstone bosons.[8] Three of these combine with $W^{\pm}$ and Z^0. There are still 60, apparently massless, scalars to account for. Since all the flavor symmetries are not exact when $SU_C(3)\times SU_L(2)\times U_Y(1)$ interactions of the techniquarks are taken into account, most of these particles are pseudo-Goldstone bosons coming from the breaking of approximate flavor chiral symmetries and acquire nonzero masses. Yet a certain set of Goldstone bosons will remain massless even when strong and electroweak interactions are included. These can receive masses only when the additional interactions, which explicitly break the global symmetries responsible for these Goldstone bosons, are incorporated.

To generate masses for the ordinary quarks and leptons, the technicolor sector must be coupled to the ordinary fermion sector. In the technicolor theory described so far, quarks and leptons are symmetric under the transformation $\psi\to\gamma_5\psi$. This symmetry has to be broken before quarks and leptons can acquire masses. In the ordinary Higgs method of symmetry breaking, Yukawa couplings explicitly break the γ_5 symmetry. The same can be achieved in the technicolor theory by introducing $SU_L(2)\times U_Y(1)\times SU_C(3)\times$ technicolor-invariant interactions which at low enough energies have the form of effective four-fermion couplings. These explicitly break the γ_5 symmetry of the ordinary fermion sector alone. When the technifermions condense, the ordinary fermions will acquire masses through the mechanism depicted in Fig. 1. The fundamental origin of the effective four-fermion interaction is not directly important for the purpose of this paper. It might be the exchange of a very heavy (extended technicolor) gauge boson or perhaps an exchange force in a preonic theory of fermions and technifermions.[9]

The $SU_L(2)\times U_Y(1)$-invariant four-fermion operators connecting ordinary and technifermions can be constructed as follows. First consider the doublet $Q=\binom{U}{D}$ of techniquarks. Ignore color for the moment. Define the 2×2 techni-singlet matrix

$$M_Q=\bar{Q}Q+i\tau\cdot\bar{Q}\gamma_5\tau Q\ . \tag{2.2}$$

Here summation over technicolor indices is assumed. Then under $SU_L(2)$, $M_Q\to UM_Q$ where U is an SU(2) matrix and under U(1), $M_Q\to M_Q e^{i\tau_3\theta/2}$. Therefore the coupling

$$\frac{g_M{}^2}{M^2}(A\bar{q}_L M_Q q_R+B\bar{q}_L M_Q\tau_3 q_R)+\text{H.c.} \tag{2.3}$$

between techni and ordinary quarks is $SU_L(2)\times U_Y(1)$ invariant. Here M is the mass scale associated with the four-fermion interaction and g_M is the dimensionless constant. If the interactions (2.3) arise from the exchange of a heavy gauge boson, then M is its mass and g_M is the gauge coupling constant. The left-handed quark doublet is denoted by q_L, and q_R is the right-handed quark doublet. Possibly family labels on the ordinary quarks have been dropped. After chiral condensation with $\langle\bar{U}U\rangle=\langle\bar{D}D\rangle\neq0$, masses for $q=\binom{u}{d}$ can be generated:

$$\begin{aligned} m_u&=g_M{}^2\frac{\langle\bar{U}U+\bar{D}D\rangle}{M^2}(A+B)\ ,\\ m_d&=g_M{}^2\frac{\langle\bar{U}U+\bar{D}D\rangle}{M^2}(A-B)\ . \end{aligned} \tag{2.4}$$

The first task is to estimate the techniquark condensates $\langle\bar{U}U\rangle$ and $\langle\bar{D}D\rangle$. A naive estimate can be made by simply scaling up from QCD. If the technicolor gauge group is SU(N) and the technifermions are in the fundamental representation of the group, this procedure gives

$$\langle\bar{U}U\rangle=\langle\bar{D}D\rangle=\langle\bar{q}q\rangle\frac{F^3}{f_\pi{}^3}\left[\frac{3}{N}\right]^{1/2}\ , \tag{2.5}$$

where $f_\pi=93$ MeV is the pion decay constant and $\langle\bar{q}q\rangle$ is the QCD quark condensate. Using the PCAC (partial conservation of axial-vector current) relation $f_\pi m_\pi{}^2=(m_u+m_d)\langle\bar{q}q\rangle$ and the conventional current-algebra masses $m_u=5$ MeV and $m_d=10$ MeV, $\langle\bar{U}U\rangle$ and $\langle\bar{D}D\rangle$ can be estimated.

This procedure, however, does not take into account the details of techniguage dynamics. In particular, we will argue that if the technicolor theory is described by a slowly running asymptotically free coupling, and if the condensate is defined by an ultraviolet cutoff on the order of M, the high-momentum contributions to $\langle\bar{U}U\rangle$ and $\langle\bar{D}D\rangle$ will elevate them considerably above the naive estimate (2.5). This will be the business of Secs. IV and V.

This above expression can be generalized to the one-family technicolor model described earlier by defining

$$\begin{aligned} M_Q&=\bar{Q}Q+i\tau\cdot\bar{Q}\gamma_5\tau Q\ ,\\ M_L&=\bar{L}L+i\tau\cdot\bar{L}\gamma_5\tau L\ , \end{aligned} \tag{2.6}$$

where

$$Q_i=\begin{bmatrix}U_i\\D_i\end{bmatrix},\quad L=\begin{bmatrix}E\\N\end{bmatrix},\quad i=1,3$$

are the colors of the techniquarks. Technicolor indices have been suppressed, and summation over both color and technicolor indices is assumed in Eq. (2.6). Then all possible $SU(2)\times U(1)$ invariant four-fermion couplings between and technifermions and a single generation of fermions can be listed as

$$\begin{aligned} &\bar{q}_L M_Q q_R,\quad \bar{l}_L M_Q l_R\ ,\\ &\bar{q}_L M_Q\tau_3 q_R,\quad \bar{l}_L M_Q\tau_3 l_R\ ,\\ &\bar{q}_L M_L q_R,\quad \bar{l}_L M_L l_R\ ,\\ &\bar{q}_L M_L\tau_3 q_R,\quad \bar{l}_L M_L\tau_3 l_R\ . \end{aligned}$$

Here $l_R=\binom{e_R}{\nu_R}$, where ν_R could be eliminated to recover the conventional ordinary fermion mass spectrum. These couplings will lead to mass generation for quarks and leptons after chiral condensation of technifermions. It is quite straightforward to further generalize this to a theory with several generations of ordinary fermions and technifermions.

The standard electroweak model with the GIM mechanism has successfully predicted the rates of flavor-changing neutral-current (FCNC) processes. Technicolor models must do the same to become viable alternatives to it. In the standard model the GIM mechanism ensures that the coupling of neutral currents to the quark mass eigenstates remain flavor diagonal. Furthermore the couplings of the neutral physical Higgs particle to the fermions are proportional to the fermion masses. Therefore the fermion mass matrix and the Higgs couplings are simultaneously diagonalized. Hence the physical Higgs particle does not generate flavor-changing neutral-current processes at the tree level.

For technicolor theories with effective four-fermion couplings the situation is quite different. In general there will be flavor-changing neutral-current interactions. It is easy to understand how these would arise within the context of the underlying extended technicolor (ETC) theory which generates four-fermion couplings. Consider the ETC gauge bosons $A_{q_iQ_j}$ which couple an ordinary fermion q_i to a technifermion Q_j. Successive couplings of $A_{q_iQ_j}$ and $A_{Q_jq_i'}$ can transform q_i to $q_{i'}$. Therefore to close the gauge algebra one needs gauge bosons of the form $A_{q_iq_i'}$ which couple different flavors of ordinary fermions. Unless a GIM-type suppression mechanism can be incorporated into the theory, the scale M (the mass of $A_{q_iq_i'}$ in ETC) must be large enough to adequately suppress FCNC's.

Techicolor theories can also contain a multitude of neutral pseudo-Goldstone bosons, which can mediate flavor-changing neutral-current interactions. This situation here is similar to that of multiple Higgs-doublet models which also contain more than one neutral physical Higgs scalar. In this case the Yukawa couplings of the Higgs particles to quarks and leptons have to be carefully chosen to eliminate the tree-level flavor-changing neutral-current processes. This amounts to letting all the quarks of the same electric charge get their masses from the condensate of the same Higgs doublet.[10] A similar procedure can be applied in technicolor theories to suppress flavor-changing processes mediated by light neutral pseudo-Goldstone-boson masses by letting each quark of the same electric charge acquire mass by coupling to the same technifermion condensate.[11]

Flavor-changing neutral-current effective four-fermion couplings will induce rare decay reactions such as $K\rightarrow\mu e$, $K\rightarrow\pi\mu e$, $K\rightarrow\pi\bar{\nu}\nu$, and $\mu^+\rightarrow e^+e^-e^+$ (Ref. 4). In the absence of a GIM mechanism, the known upper bounds for the branching ratios of such processes give lower bounds for the mass scale M. Another reaction of interest is $\mu\rightarrow e\gamma$, which proceeds through a higher-order process. By taking M/g_M to be at least of order 300 TeV (Refs. 2 and 4), flavor-changing neutral currents can be suppressed below phenomenological bounds. But then the value of ordinary fermion masses given by

$$m_f \simeq g_M{}^2\frac{\langle \bar{T}T\rangle}{M^2}$$

becomes far too small to be consistent with known values. This is a major problem facing technicolor models.

The existence of light pseudo-Goldstone bosons can also be a phenomenological embarassment.[7] To discuss the various mass-generation mechanisms for them it is instructive to enumerate all the Goldstone bosons in the one-family technicolor model mentioned earlier in this section. All the Goldstone bosons π^a in this model couple to axial-vector SU(8) currents $J^\mu_{5a}=\bar{\psi}\gamma^\mu\gamma^5 t^a\psi$. Here $\psi=(U_R,U_G,U_B,D_R,D_G,G_B,E,N)$ is the fermion octet and t^a are the generators of SU(8). The coupling of π^α to the axial-vector currents is

$$\langle 0\,|\,J^\mu_{5a}\,|\,\pi_b(k)\rangle = iF\delta_{ab}k^\mu\ . \tag{2.7}$$

The SU(3)$\times$SU(2)$\times$U(1) interactions break some of these axial symmetries giving masses to some of the Goldstone bosons. Therefore to distinguish between real and pseudo-Goldstone bosons it is advantageous to exhibit explicit strong and electroweak charges of all the Goldstone bosons.

Let Q^i be the colored quark doublet where i is the SU(3)-color index, and L be the lepton doublet. The Goldstone bosons can be enumerated as follows:[7]

$$\begin{aligned}
&\theta_a^\alpha=\bar{Q}\gamma_5\lambda_a\tau^\alpha Q,\quad a=1,\ldots,8,\quad \alpha=1,\ldots,3\ ,\\
&\theta_a=\bar{Q}\gamma_5\lambda_a Q,\quad a=1,\ldots,8\ ,\\
&T_i^\alpha=\bar{Q}^{\,i}\gamma_5\tau^\alpha L,\quad T_i=\bar{Q}^{\,i}\gamma_5 L\ ,\\
&\bar{T}_i^{\,\alpha}=\bar{L}\gamma_5\tau^\alpha Q^i,\quad \bar{T}_i=\bar{L}\gamma_5 Q^i\ ,\\
&\pi^\alpha=\bar{Q}\gamma_5\tau^\alpha Q+\bar{L}\gamma_5\tau^\alpha L\ ,\\
&P^\pm=\bar{Q}\gamma_5\tau^\pm Q-3\bar{L}\gamma_5\tau^\pm L\ ,\\
&P^3=\bar{Q}\gamma_5\tau^3 Q-3\bar{L}\gamma_5\tau^3 L\ ,\\
&P^0=\bar{Q}\gamma_5 Q-3\bar{L}\gamma_5 L\ .
\end{aligned}\tag{2.8}$$

Here λ^a are the Gell-Mann SU(3) matrices and τ^α are the Pauli matrices. The particles θ_a^α and θ_a are color octets. They acquire masses of the order 100 GeV through their color interactions.[7] The same is true for the color-triplet particles T_i^α and T_i. The three π^α are the Goldstone bosons absorbed by $W^\pm$ and Z^0. $P^\pm$ acquire masses of the order 10 GeV through their electroweak interactions and should therefore have been seen in e^+e^- colliders with center-of-mass energy greater than 20 GeV (DESY PETRA). Even more troublesome are the SU(3)$\times$SU(2)$\times$U(1) neutral particles P^0 and P^3 which do not acquire masses at this stage.

To generate nonzero masses for P^0 and P^3 and to raise the mass value of $P^\pm$ one has to make use of the four-fermion interactions among technifermions that explicitly break some of the relevant chiral symmetries of the technisector. Consider the four-Fermi couplings

$$\frac{g_M{}^2}{M^2}(\bar{Q}Q\bar{L}L-\bar{Q}\gamma_5\tau^\alpha Q\bar{L}\gamma^5\tau^\alpha L)\ . \tag{2.9}$$

This interaction is SU(3)$\times$SU(2)$\times$U(1)$\times$technicolor invariant and yet violates separate techniquark and technilepton chiral symmetries. This enables a nonzero mass to be generated for P^0 and P^3 and to raise the masses of $P^\pm$. Masses can be estimated using chiral perturbation theory and Dashen's formula

$$M_{P^a}{}^2=\frac{1}{F^2}\langle 0\,|\,[J_a^0,[J_a^0,H]]\,|\,0\rangle\ , \tag{2.10}$$

where J_a^0 is the time component of the current corresponding to P^a. For example, in the case of P^0,

$$J_0^0=\bar{Q}\gamma^0\gamma_5 Q-3\bar{L}\gamma^0\gamma_5 L\ . \tag{2.11}$$

Evaluating the commutators present in Dashen's formula, we get

$$M_{p^0}{}^2=\frac{g_M{}^2}{F^2M^2}\langle 0\,|\,\bar{Q}Q\bar{L}L\,|\,0\rangle\ . \tag{2.12}$$

Using the result that in $\bar{Q}Q$ there is a summation over three color doublets:[12]

$$\langle \bar{Q}Q\bar{L}L\rangle=3\langle \bar{L}L\bar{L}L\rangle\simeq 3\langle \bar{L}L\rangle^2\ . \tag{2.13}$$

This yields

$$M_P{}^2=\frac{g_M{}^2\langle\bar{\psi}\psi\rangle^2a^2}{F^2M^2}\ , \tag{2.14}$$

where ψ is a typical technidoublet and a is a coefficient of order unity. The enhancement mechanism to be discussed in Sec. IV will raise the value of $\langle\bar{\psi}\psi\rangle$. This enables the use of a higher M/g_M to generate a given fermion mass m_f. Since M_P can also be written as

$$M_P=a\frac{M/g_M}{F}m_f\ , \tag{2.15}$$

this mechanism may also help to raise pseudo-Goldstone-boson masses above current experimental bounds.

III. DYNAMICAL CHIRAL-SYMMETRY BREAKING IN GAUGE THEORIES

If an asymptotically free gauge theory is to drive electroweak symmetry breaking, it must have an exact global chiral symmetry at least as large as $SU(2)_L\times SU(2)_R$. The symmetry could of course be larger, say, $SU(N_f)_L\times SU(N_f)_R$. The additional interactions, which must appear at some higher-energy scale M, can reduce this symmetry back to $SU(2)_L\times SU(2)_R$ and must also break isospin symmetry to give realistic fermion masses.

To begin, we shall neglect these additional interactions and discuss the spontaneous breaking of an exact $SU(N_f)_L\times SU(N_f)_R$ symmetry. This will be done in the presence of the scale M, which will play the role of an ultraviolet cutoff on the technicolor gauge theory dynamics. The new interactions responsible for ordinary fermion mass generation will then be introduced as perturbations in the form of effective four-fermion interactions below the cutoff M. These corrections will induce terms that have the appearance of hard, current-algebra masses below the scale M but which must disappear rapidly as a function of momentum above M in order to ensure that chiral symmetry is not explicitly broken.

If a theory described by a gauge group G is invariant under the global-chiral-symmetry group $SU(N_f)_L\times SU(N_f)_R$, there will be $2(N_f{}^2-1)$ conserved, chiral, gauge-singlet currents. The spontaneous breaking of this symmetry to $SU(N_f)_{L+R}$ will produce a common dynamical mass $\Sigma(p)$ for each of the N_f fermions and leave $N_f{}^2-1$ massless Goldstone bosons in its wake. The dynamics of this process will be governed by the gauge theory whose renormalization-group equation is

$$q\frac{\partial}{\partial q}\alpha(q)=\beta(\alpha)$$
$$=-b\alpha^2(q)-c\alpha^3(q)-d\alpha^4(q)+\cdots\ , \tag{3.1}$$

where we will always take $b>0$ to ensure asymptotic freedom.

This theory can only be used in isolation up to some ultraviolet cutoff M. It is therefore appropriate to analyze the spontaneous breaking of chiral symmetry in the presence of M. Thanks to the asymptotic freedom of the theory, the broad features of the spontaneous breaking will not depend sensitively on M. If they did, the entire approach of initially disregarding the physics at M and beyond would, of course, be inappropriate. It is nevertheless important to keep M to prepare the way for the inclusion of the additional interactions.

If a fermion mass arises spontaneously, the propagator will have the form[13]

$$S^{-1}(p)=\not{p}[1+A(p)]-\Sigma(p)\ . \tag{3.2}$$

It will be assumed that a set of counterterms $Z_i(M)$, appropriate to the massless theory, has been introduced. The wave-function renormalization term $A(p)$ is therefore subtracted and finite in the infinite M limit. It is the behavior of $\Sigma(p)$ that is of central interest. This behavior is governed by the Dyson-Schwinger gap equation which has the general form

$$A(p)\not{p}-\Sigma(p)=\frac{Z_1g^2}{(2\pi)^4}\int d^4k\,D_{\mu\nu}^{ab}(p+k)$$
$$\times\Gamma_\mu^a(p,k)S_f(k)\gamma_\nu\sigma^b-\not{p}(Z_2-1)\ , \tag{3.3}$$

where $D_{\mu\nu}^{ab}$ and Γ_μ^a are the renormalized (subtracted) gauge propagator and vertex. All momenta are Euclidean. Each of the counterterms is defined by using M as a cutoff. Note that no bare mass is present in the equation.

A dynamical mass $\Sigma(p)$ is expected to have some nonzero value at $p\simeq 0$ and then fall rapidly as $p\to\infty$. The determination of $\Sigma(0)$ in terms of, say, the confinement scale, is a nonperturbative problem which we set aside for the moment. To determine the large momentum behavior of $\Sigma(p)$, a simplified version of Eq. (3.3) can be used. First of all the equation can be linearized in $\Sigma(p)$. Next a gauge can be chosen such that the renormalization-group running coupling appears explicitly in the equation.

To see how this goes, consider the graphical expansion of the right-hand side of Eq. (3.3). The one-loop corrections in Figs. 2(b)–2(e) can all be identified as contribution to the running coupling. In the Landau gauge, however, Fig. 2(c) [and also 2(d)] will not contribute. Equivalently, Z_2 is finite through one loop in this gauge. The subgraph corrections in Figs. 2(b) and 2(e) can be seen to be functions of the quantity $(k-p)^2$. Thus neglecting Fig. 2(f) for the moment, the Landau gauge linearized gap equation for the high-momentum components of $\Sigma(p)$ is

$$\Sigma(p)=\frac{3C_2(R)}{4\pi^3}\int^M d^4k\frac{\bar{\alpha}((k-p)^2)}{(k-p)^2}\frac{\Sigma(k)}{k^2}\ , \tag{3.4}$$

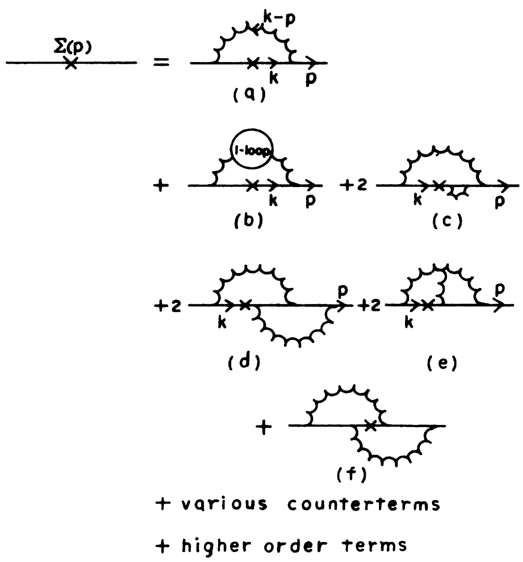

FIG. 2. The graphical expansion of the linearized gap equation for $\Sigma(p)$. Each cross indicates the insertion of $\Sigma(k)$.

where $\bar{\alpha}((k-p)^2)$ is the running coupling developed so far through one loop. To get the leading-logarithmic behavior of $\Sigma(p)$, the approximation

$$\bar{\alpha}((k-p)^2)=\bar{\alpha}(k)\theta(k^2-p^2)+\bar{\alpha}(p)\theta(p^2-k^2) \quad (3.5)$$

can be used. Then doing the angular integrations, Eq. (3.4) becomes

$$\Sigma(p)=\frac{3C_2(R)}{2\pi}\left[\int^{p}\frac{k\,dk}{p^2}\bar{\alpha}(p)\Sigma(k)+\int_p^M\frac{dk}{k}\bar{\alpha}(k)\Sigma(k)\right] . \quad (3.6)$$

Before looking at the solutions to this equation, a few remarks about its derivation are in order.

(1) The particular form of Eqs. (3.4) and (3.5) only emerges in a gauge in which Z_2 is finite. This condition defines the Landau gauge at one loop and the generalized Landau gauge in higher orders. This will be the gauge of choice throughout this paper. It is important to stress that while $\Sigma(p)$ and the equation it satisfies are gauge dependent, the final physical conclusions of the paper will be expressed in terms of gauge-independent quantities.

(2) Figure 2(f) is a higher-order correction to the linearized gap equation. At lower-momentum scales corresponding to strong coupling it will not in general be suppressed. For SU(N) theories at large N, however, it will be suppressed in a $1/N$ expansion. There are other, still higher-order contributions to the gap equation that are not suppressed in a $1/N$ expansion. Whether they are numerically small in the theories of interest here remains an open question.

The solution to Eq. (3.6) satisfies the differential equation[14]

$$\frac{4\pi}{3C_2(R)}\frac{d}{dp^2}\frac{1}{d/dp^2[\bar{\alpha}(p)/p^2]}\frac{d\Sigma(p)}{dp^2}=\Sigma(p) \quad (3.7)$$

subject to the boundary condition

$$p^2\frac{d\Sigma}{dp^2}+[1-4\pi\, d\bar{\alpha}(p)/d\ln p^2]\Sigma(p)=0 . \quad (3.8)$$

Note that this condition follows from the absence of a bare mass term $m_0(M)$ in Eq. (3.6). For momenta large compared to the confinement scale the well-known solution to Eq. (3.6) is

$$\Sigma(p)=C_1\Sigma_1(p)+C_2\Sigma_2(p) , \quad (3.9)$$

where

$$\Sigma_1(p)\sim\frac{1}{p^2}(\ln p/\Lambda)^{A/2-1} \quad (3.10)$$

and

$$\Sigma_2(p)\sim(\ln p/\Lambda)^{-A/2} \quad (3.11)$$

with $A=3C_2(r)/\pi b$. The boundary condition (3.8) then leads to the relation

$$\frac{C_2}{C_1}=-\frac{3C_2(R)}{\pi b}\frac{1}{M^2}\left[\ln\frac{M}{\Lambda}\right]^{A-2} . \quad (3.12)$$

It is clear from this relation that in the infinite cutoff limit $M\to\infty$, only $\Sigma_1(p)$ survives. In the physical case of interest here, in which the cutoff is kept finite but large ($M\gg\Lambda$), the solution $\Sigma_1(p)$ will dominate for $p\ll M$.

The absence of a bare mass term $m_0(M)$ means that the N_f axial-vector currents $J^a_{5\mu}(x)$ of the theory are conserved. Some of these symmetries will be broken by the higher-dimension four-fermion operators which must yet be added, and these will also change the structure of the gap equation. For the moment, these perturbations will continue to be neglected.

In the above discussion of the asymptotic behavior, it was assumed that (1) a nonzero solution $\Sigma(p)$ to the full gap equation actually exists and is energetically preferred to the solution $\Sigma(p)=0$, that is, that spontaneous chiral-symmetry breaking $SU(N_f)_L\times SU(N_f)_R$-$SU(N_f)_{L+R}$ does take place, and (2) the magnitude of $\Sigma(0)$ is small compared to the cutoff M. If it is on the order of the confinement scale Λ, then the use of the linearized gap equation can be justified at about the same momentum at which the perturbative running coupling can be used.

To justify these assumptions and to estimate $\Sigma(0)/\Lambda$ one has to go beyond the realm of perturbation theory. No obvious, tractable truncation of the gap equation (3.3) is known, and indeed the gap equation itself may not be the best starting point for the analysis. It is probably not unreasonable, however, to make use of the gap equation to get some qualitative information about these questions. For our purposes, it is only order of magnitude information about $\Sigma(0)/\Lambda$ that we need. It will turn out, even in the theories of interest with slowly running couplings, that $\Sigma(0)/\Lambda\sim 1$.

The other important quantity for our purposes is the critical value α_c of the coupling strength. This is the value that the coupling must exceed in order to trigger spontaneous chiral-symmetry breaking. The determination of α_c is another task that may well take one beyond any simple truncation of the gap equation. Its value will however play a central role in our analysis and it is important to fix it as precisely as possible.

A variety of analyses which will be briefly describe lead to the following value for the critical coupling:

$$\frac{3C_2(R)\alpha_c}{\pi}=1\ . \tag{3.13}$$

This value emerges most directly from an analysis of the gap equation (3.3), neglecting the running of the coupling constant.[15–17] This is a limit that will be of considerable interest for our slowly running theories, and it will be discussed in more detail in the next section. Here we simply summarize the main result. The one-loop nonlinear equation for $\Sigma(p)$ in this case is

$$\Sigma(p)=\frac{3C_2\alpha}{2\pi}\left[\int_0^p k^3\frac{dk}{p^2}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}+\int_p^M k\,dk\frac{\Sigma(k)}{k^2+\Sigma^2(k)}\right]. \tag{3.14}$$

The analysis of this equation[15,16] shows that solutions corresponding to spontaneous chiral-symmetry breaking only occur for $\alpha\geq\alpha_c$. This can also be demonstrated by examining the effective potential in the linearized approximation.[17] In the approximation of Eq. (3.14), the only dimensionful parameter is the ultraviolet cutoff M. It is then found that unlike the asymptotically free case with a confinement scale Λ, $\Sigma(0)\propto M$.

The extension of the above analysis to the case of a running coupling has been considered by several authors.[14,18,19] Higashijima studies Eq. (3.14) but with α replaced by the perturbative running coupling of QCD. This is just a nonlinear version of Eq. (3.6). Even though it is only the linearized version of this equation [relevant for $p>>\Sigma(0)$] that can be derived from QCD, this is perhaps not an unreasonable thing to do. The infrared growth of the running coupling is cut off at some momentum $\hat{p}$ and is it found that spontaneous chiral-symmetry breaking can only take place if $\hat{p}$ is small enough so that $\alpha(\hat{p})\geq\alpha_c$ (Ref. 18). This analysis is extended to a study of the effective potential by Castorina and Pi.[19] Within the limitations described above, it is found that the chirally symmetric vacuum becomes unstable for $\alpha(\hat{p})\geq\alpha_c$. These results have been verified in our numerical studies, one of which will be reported in Sec. V.

Before turning to the pecularities of theories with slowly running couplings, it will be useful to record expressions for the condensate $\langle\bar{T}T\rangle$ and the Goldstone-boson decay constant F, in terms of the dynamical fermion mass $\Sigma(p)$. The high-momentum components of the condensate can be computed perturbatively to give

$$\langle\bar{T}T\rangle_M\simeq\frac{N}{2\pi^2}\int^M p\,dp\,\Sigma(p)\ , \tag{3.15}$$

where M is the ultraviolet cutoff and where N is the dimensionality of the fermion representation. It is clear from the form of the solution (3.9) that an ultraviolet cutoff, or a subtraction procedure, is necessary to define $\langle\bar{T}T\rangle$. We have chosen to cut the integral off at M, assuming that beyond this scale the effective four-fermion vertex giving rise to $\langle\bar{T}T\rangle$ in the computation of fermion and pseudo-Goldstone-boson masses, will fall rapidly. The more conventional definition, subtracting $\langle\bar{T}T\rangle$ at a momentum on the order of the confinement scale, is not as natural for our purposes.

A similar perturbative expression for the high-momentum component of F is[20]

$$F^2\simeq\frac{N}{2\pi^2}\int^\infty\frac{dp}{p}\left[\Sigma^2(p)-\frac{1}{4}p\Sigma(p)\frac{d\Sigma(p)}{dp}\right]. \tag{3.16}$$

In this case, the ultraviolet cutoff M has been removed since even a logarithmic fall of $\Sigma(p)$ with increasing p is enough to converge the integral.

The condensate $\langle\bar{T}T\rangle_M$ is clearly much more sensitive to the high-momentum behavior of $\Sigma(p)$ than is the decay constant F. The latter determines the W- and Z-boson masses and the former determines fermion masses and pseudo-Goldstone-boson masses in a technicolor theory. Thus a slowing of the asymptotic fall of $\Sigma(p)$ at least over a range of momenta, can enhance these masses while not substantially affecting the W and Z masses. It will be shown that in theories with a slowly running coupling, $\Sigma(p)$ will indeed drop more slowly over a range of momenta than indicated by its ultimate asymptotic form $\Sigma_1(p)$ [Eq. (3.10)]. The corresponding enhancement of $\langle\bar{T}T\rangle_M$ can be quite substantial.

IV. THE SLOWLY RUNNING COUPLING

A qualitative conclusion of the studies referred to in the previous section is that chiral-symmetry breaking sets in at some momentum scale μ at which $\alpha(\mu)\gtrsim\alpha_c$. Both μ and $\Sigma(0)$ will be on the order of the confinement scale Λ. The numerical results to be reported in the next section also lead to this result.

In this section, a qualitative description of the consequences of a slowly running coupling will be developed. To begin, we suppose that $\alpha_\mu\equiv\alpha(\mu)$ is small enough so that in the theories of interest, the expansion of the β function [Eq. (3.1)] converges. For momenta between μ and M, the running coupling takes its traditional form

$$\alpha(p)=(1/b)\ln p/\Lambda'+\text{higher-order terms}\ , \tag{4.1}$$

where Λ' is a renormalization scale. Specifying Λ' is equivalent to, say, specifying the strength of the coupling $\alpha(M)$ at the ultraviolet cutoff.

For a range of momenta above μ, $\alpha(p)$ can be written in the form

$$\alpha(p)\simeq\alpha_\mu[1-(b\alpha_\mu+c\alpha_\mu{}^2+\cdots)\ln p/\mu+\cdots]\ , \tag{4.2}$$

where $\alpha_\mu=(1/b)\ln\mu/\Lambda'$ and where the second-order term has been explicitly included. Suppose now that

$$b\alpha_\mu+c\alpha_\mu{}^2+\cdots<<1 \tag{4.3}$$

and that the series on the left-hand side is reasonably convergent. Under these circumstances, the variation in the coupling can be neglected to first approximation, in a momentum range below

$$\mu \exp[1/(b\alpha_\mu + c\alpha_\mu^2)] , \tag{4.4}$$

where the expansion has here been truncated at second order. A new, large scale has entered the problem, which for several of the theories to be listed in the next section, can be up to two orders of magnitude larger than $\mu \simeq \Sigma(0) \simeq F$. Recalling that in technicolor theories, these scales are on the order of a few hundred GeV and that the ultraviolet cutoff is at least on the order of a few hundred TeV, the new scale will be somewhat less than the ultraviolet cutoff in the theories of interest.

For a range of momenta below $\mu \exp[1/(b\alpha_\mu + c\alpha_\mu^2)]$, the approximate behavior of $\Sigma(p)$ can be determined by replacing the running coupling constant by α_μ in Eq. (3.6). The corresponding differential equation can be written in the form

$$\Box \Sigma(p) = -\frac{\alpha_\mu}{\alpha_c} \frac{1}{p^2} \Sigma(p) , \tag{4.5}$$

where

$$\Box \equiv \left[\frac{d}{dp} \right]^2 + \frac{3}{p} \frac{d}{dp} . \tag{4.6}$$

The solutions are

$$\Sigma(p) \sim p^{-1+(1-\alpha_\mu/\alpha_c)^{1/2}}, \quad \frac{\alpha_\mu}{\alpha_c} < 1 \tag{4.7}$$

$$\Sigma(p) \sim \frac{1}{p} \left[\cos \left[\frac{\alpha_\mu}{\alpha_c} - 1 \right]^{1/2} \ln p \right], \quad \frac{\alpha_\mu}{\alpha_c} > 1 . \tag{4.8}$$

It is quite clear from these expressions that if $\alpha_\mu \simeq \alpha_c$, $\Sigma(p)$ will fall approximately like $1/p$ in this range of momenta beyond μ. It is this fall, much slower than the final asymptotic behavior $\Sigma_1(p)$ [Eq. (3.10)], that is responsible for enhancing the condensate $\langle \bar{T}T \rangle$ [Eq. (3.15)].

The numerical study to be summarized in Sec. V will confirm this qualitative picture. It sill be seen that with $\mu \equiv 2\Sigma_0$, $\alpha_\mu \simeq \alpha_c$ and $\Sigma(p) \sim 1/p$ for a range of momenta $p > \mu$. Although a steeper drop will begin before p reaches $\mu \exp[1/(b\alpha_\mu + c\alpha_\mu^2)]$, a crude estimate of the enhancement effect can be made by evaluating Eq. (3.15) with $\Sigma(p) \simeq \Sigma_0^2/p$, and with M replaced by this new scale, the result is

$$\langle \bar{T}T \rangle \simeq \frac{N}{2\pi^2} \mu^3 \exp[1/(b\alpha_\mu + c\alpha_\mu^2)] . \tag{4.9}$$

This rough estimate is to be compared with what might be called the "normal" size of $\langle \bar{T}T \rangle$. If the asymptotic behavior (3.10) were to set in immediately beyond $p \simeq \mu$, as it would if the coupling were to run at a normal rate, the estimate for $\langle \bar{T}T \rangle$ would become

$$\langle \bar{T}T \rangle_{\text{normal}} \simeq \frac{N\mu^3}{2\pi^2} \int^M \frac{dp}{p} (\ln p/\mu)^{A/2-1} \lesssim 5 \frac{N\mu^3}{2\pi} . \tag{4.10}$$

The final estimate comes from assuming that $A/2$ is a number of order unity and that $M/\mu \leq 10^3$. A number of order unity for $A/2$ is reasonable if b is not especially small, as expected in the "normal running" case. In some of the slow running examples discussed in the next section, the exponential factor in Eq. (4.9) can be of order 100, nearly two orders of magnitude larger than the "normal" size.

It is worth emphasizing that the slow running of $\alpha(p)$ leading to the slow fall of $\Sigma(p)$ with p and to the enhanced condensate is also found in theories with the existence of an ultraviolet fixed point is assumed.[21,22] The problem there, of course, is that no realistic theory of this type has even been exhibited. Asymptotically free theories, even those with slowly running couplings, do exist and offer a natural explanation of why the spontaneous breaking of chiral symmetry takes place at a scale μ, much smaller than the ultraviolet cutoff M.

The analysis of the gap equation in the slowly running limit naturally leads one to the question of scale invariance. To the extent that running can be completely neglected, the gap equation contains no intrinsic scale other than the ultraviolet cutoff. This suggests that a nonzero solution $\Sigma(k)$ to the gap equation could correspond to a spontaneous breaking of scale invariance as well as chiral symmetry. However, as discussed in Sec. III, the solution to the scale-invariant, one-loop gap equation (3.14) in the presence of an ultraviolet cutoff M does not obviously exhibit spontaneous breaking of scale invariance. The ultraviolet cutoff M sets the scale, giving $\Sigma(0) \propto M$ (Refs. 15 and 16). Some speculation exists[23] that $\Sigma(0)$ could remain finite as $M \to \infty$, reinstating spontaneous breaking of scale invariance, but we know of no convincing argument that this is the case.[24]

In the slow-running case of interest here, it is even more clear that no spontaneous breaking of scale invariance exists. The value of $\bar{\alpha}(p)$ for $p \simeq M$ will be less than α_c. The scale μ at which it approaches α_c will be much less than M and, as discussed above, will be on the order of the physical confinement scale Λ. It is these intrinsic quantities that then determined $\Sigma(0)$. There is no sense in which the underlying theory, leading to spontaneous chiral-symmetry breaking, is scale invariant.

We conclude this section by recording the expressions for the first three terms in the expansion of the β function and briefly discussing the question of convergence. For a general gauge group G with fermions transforming according to representation R,

$$[R^a, R^b] = i f^{abc} R^c, \quad f^{acd} f^{bcd} = C_2(G) \delta^{ab} , \qquad R^a R^a = C_2(R) I, \quad \mathrm{Tr} R^a R^b = T(R) \delta^{ab} . \tag{4.11}$$

Thus

$$d(R) C_2(R) = r T(R) , \tag{4.12}$$

where $d(R)$ is the dimension of representation R and r is the rank of the group. For an SU(N) theory with the fermions in the fundamental representation,

$$C_2(G) = N, \quad C_2(R) = \frac{N^2-1}{2N}, \quad T(R) = \tfrac{1}{2} . \tag{4.13}$$

The first two terms in the β function [Eq. (3.1)] are[25]

$$b=[\tfrac{11}{3}C_2(G)-\tfrac{4}{3}T(R)n_f]\frac{1}{2\pi} \quad (4.14)$$

and

$$c=[\tfrac{34}{3}C_2{}^2(G)-\tfrac{20}{3}C_2(G)T(R)n_f-4C_2(R)T(R)n_f]\frac{1}{8\pi^2}\ , \quad (4.15)$$

where n_f is the number of fermion multiplets in representation R. These coefficients are both gauge independent and renormalization-convention independent. The three-loop contribution has been computed[26] using the minimal-subtraction (MS) scheme, with the result

$$d=[\tfrac{2857}{54}C_2{}^3(G)-\tfrac{1415}{27}C_2{}^2(G)T(R)n_f+\tfrac{158}{27}C_2(G)T^2(R)n_f{}^2-\tfrac{205}{9}C_2(G)C_2(R)T(R)n_f+\tfrac{44}{9}C_2(R)T^2(R)n_f{}^2+2C_2{}^2(R)T(R)n_f]\frac{1}{32\pi^3}\ . \quad (4.16)$$

This coefficient, and in fact the entire β function, has been shown to be gauge independent within the MS scheme.[25] For the SU(N) theory with fermions in the fundamental representation,

$$\beta(\alpha)=-\frac{1}{2\pi}(\tfrac{11}{3}N-\tfrac{2}{3}n_f)\alpha^2-\frac{1}{8\pi^2}\left[\frac{34}{3}N^2-\frac{10}{3}Nn_f-\frac{N^2-1}{N}n_f\right]\alpha^3-\frac{1}{32\pi^3}\left[\frac{2857}{54}N^3-\frac{1415}{54}N^2n_f+\frac{79}{54}Nn_f{}^2-\frac{205}{18}N\frac{N^2-1}{2N}n_f+\frac{11}{9}\frac{N^2-1}{2N}n_f{}^2+\left[\frac{N^2-1}{2N}\right]^2n_f\right]\alpha^4+\cdots\ . \quad (4.17)$$

Using the above expressions, some general remarks about slow running and spontaneous chiral-symmetry breaking can be made. The key question is whether it is possible to implement the program we have in mind in the context of some systematic expansion in a small parameter. The criticality condition (3.13) suggests that a $1/N$ expansion could be useful. For an SU(N) technicolor theory, the large-N limit of this condition is

$$\frac{3}{2\pi}N\alpha_c=1\ , \quad (4.18)$$

meaning that $\alpha_c=O(1/N)$.

The problem is that a simple $1/N$ expansion, with its inclusion of all planar graphs and its neglect of fermion loops, does not especially simplify the computation of the β function and certainly does not lead to slow running in the neighborhood of α_c. The fermions are crucial for slow running. While this suggests that something like a combined large-N, large-n_f expansion might be of use, we have so far not found a convergent expansion scheme of this sort. If one could be found, leading both to convergence and to a small value for $\beta(\alpha_c)/\alpha_c$, the slow-running enhancement mechanism could be put on much sounder footing.

V. SLOWLY RUNNING THEORIES AND NUMERICAL RESULTS

Without a systematic expansion scheme, we have simply proceeded numerically. By surveying all gauge theories of rank $\lesssim 8$ with an even number of fermions, a list of theories can be assembled which share the following properties: (1) They are asymptotically free ($b>0$); (2) the β function exhibits reasonable convergence through low orders for α as large as α_c; (3) the quantity $\beta(\alpha_c)/\alpha_c$, governing the rate of running of the coupling for α in the neighborhood of α_c, is small.

The results of this survey are summarized in Table I. With these results in hand, several important observations can be made.

(1) Many theories exist that satisfy the above criteria of asymptotic freedom, reasonable convergence of $\beta(\alpha_c)$, and $\beta(\alpha_c)/\alpha_c \ll 1$.

(2) While these properties seem easier to achieve with the fermions in higher representations of the gauge group, there are several examples with fermions in the fundamental representation.

(3) Examples have been included in which the two-loop term and the three-loop term are small compared with the one-loop term, but in which the three-loop term is larger than the two-loop term. Without higher-order estimates, it is not completely clear whether this is reasonable.

(4) The general question of convergence remains open. Even if apparent convergence through three loops is achieved, there is no guarantee that this will persist in higher orders.

We next provide a more precise, numerical estimate of fermion and pseudo-Goldstone-boson masses in an explicit technicolor model with a slowly running coupling. We choose an SU(4) gauge theory with 14 fermion flavors as our example, and truncate the expansion at second order. Then $\alpha_c=0.56$, $b=0.85$, $c=-0.73$, and

$$\beta(\alpha_c)/\alpha_c \simeq b\alpha_c+c\alpha_c{}^2=0.47-0.22=0.25\ , \quad (5.1)$$

which is a reasonable number to induce slow running of the coupling.

To be more explicit, the 14 technifermions can be composed of a colored weak doublet (U,D), a color-singlet weak doublet (E,N), and 2 colored weak singlets (P,Q). This selection of fermion representations leads to the cancellation of all axial anomalies in the electroweak sector. Below the technicolor scale μ, the b parameter of the QCD β function becomes $b_{\rm QCD}=1/2\pi(11-\frac{2}{3}b)=2/2\pi$, while above μ, $b_{\rm QCD}=-11/6\pi$. Thus asymptotic freedom is lost above the technicolor scale and $\alpha(q)$ will start to grow. Yet it will still be well below its 1-GeV value when the cutoff scale M is reached, provided $M\lesssim 1000$ TeV. Above this scale QCD could be embedded in a larger gauge group restoring asymptotic freedom.

To estimate fermion and pseudo-Goldstone-boson masses we must evaluate $\langle \overline{T}T\rangle_M$. The high-momentum contribution to $\langle \overline{T}T\rangle_M$ is given by Eq. (3.15). To estimate its overall size we note that

$$F=250\ \text{GeV}/2=125\ \text{GeV}\ , \tag{5.2}$$

since there are four SU(2) doublets. Using this result, Σ_0 can be estimated by scaling up from QCD. This scaling-up procedure is only approximately correct because of the slow running of the coupling in the technicolor theory. The estimate should be fairly accurate however, since from Eq. (3.6) it can be seen that Σ_0 is not very sensitive to the high-momentum components. Thus by simple scaling,[27]

$$\Sigma_0=(\Sigma_0)_{\rm QCD}\frac{F}{f_\pi}\left[\frac{3}{4}\right]^{1/2}$$

$$\simeq 300\ \text{MeV}\frac{125\ \text{GeV}}{93\ \text{MeV}}\left[\frac{3}{4}\right]^{1/2}\simeq 350\ \text{GeV}\ . \tag{5.3}$$

Therefore

$$\langle \overline{T}T\rangle_M\simeq\frac{4}{2\pi^2}(350\ \text{GeV})^3\int^{M/\Sigma_0}\chi^d\chi^\sigma(\chi)\ , \tag{5.4}$$

where $\sigma=\Sigma(p)/\Sigma_0$ and $\chi=p/\Sigma_0$. The cutoff must be at least 300 TeV$\times g_M$ to adequately suppress flavor-changing neutral processes.[4]

A rough measure of the integral can be obtained by noting that $\sigma(\chi)\simeq 1/\chi$ for χ up to on the order of $\exp[1/(b\alpha_c+c\alpha_c^2)]$. Thus a chiral hierarchy

$$\langle \overline{T}T\rangle_M/(2\Sigma_0^3/\pi^2)\simeq\exp[1/(b\alpha_c+c\alpha_c^2)] \tag{5.5}$$

can be expected. For the model under discussion this gives a value of about 50 for the integral and using the result $m_f=g_M^2/M^2\langle \overline{T}T\rangle$, one estimates the fermion mass to be 5 MeV.

To be more accurate, this model, along with many others, has been studied numerically. The gap equation (3.6), cutoff in the ultraviolet at M, is solved by a self-consistent iterative procedure. Here we assume that $g_M^2/4\pi^2\simeq 1$, so that $M=300$ TeV$\times g_M\simeq 1200$ TeV. A chiral-symmetry-breaking scale Σ_0 is assumed to exist and below the threshold $\mu=2\Sigma_0$, the running coupling will evolve faster, due to the absence of the retarding effects of the decoupled fermions. The existence of a physical confinement scale Λ is also assumed at which $\alpha(q)$ reaches some value $\alpha_\Lambda\gtrsim\alpha_c$. Below Λ we have substituted a consistent value for $\alpha(q)$ in the gap equation. The maximum value of Σ,Σ_0 is determined in terms of Λ. In the case when $\Lambda<\mu$ the running of the coupling in the region $\Lambda<q<\mu$ is taken to be

$$\alpha(q)=\frac{1}{1+\alpha(\mu)b\ln(q/\mu)}\ . \tag{5.6}$$

The parameter b is of order unity since the fermions do not contribute to the running coupling in this region.

It should be stressed that the gap equation (3.6) is basically perturbative and cannot be expected to completely govern the dynamics that lead to chiral-symmetry breaking. In particular it is almost surely unreliable in detail in the region $q\lesssim\mu$. Our expectation, however, is that a reasonable estimate of the ratio Σ_0/Λ can be obtained by our procedure. For the gauge theory parameters dis-

TABLE I. Theories with slowly running couplings. The conditions are (1) asymptotic freedom $(b>0)$ and (2) reasonable convergence $(c\alpha_c^2, d\alpha_c^3, <\frac{1}{2}b\alpha_c)$. The exponential of the inverse of the number of the final column is a measure of the condensate enhancement for each theory.

Group	$d(R)$	α_c	n_f	$b\alpha_c$	$c\alpha_c^2$	$d\alpha_c^3$	$\beta(\alpha_c)/\alpha_c$
SU(4)	4	0.56	12	0.59	−0.09	−0.25	0.24
SU(4)	6	0.42	6	0.44	−0.99	−0.12	0.23
SU(4)	10	0.23	2	0.25	−0.06	−0.03	0.16
SU(5)	15	0.19	2	0.27	−0.05	−0.03	0.19
SU(6)	6	0.36	18	0.57	−0.09	−0.23	0.26
SU(6)	21	0.19	2	0.28	−0.04	−0.02	0.22
SU(7)	7	0.31	22	0.53	−0.13	−0.25	0.15
SU(7)	21	0.18	4	0.36	−0.06	−0.055	0.24
SU(8)	8	0.27	24	0.56	−0.09	−0.22	0.25
SU(8)	28	0.16	4	0.33	−0.07	−0.06	0.19
SU(8)	36	0.12	2	0.30	−0.02	−0.02	0.26
SU(9)	9	0.24	28	0.54	−0.12	−0.24	0.17
SU(9)	36	0.13	4	0.30	−0.08	−0.05	0.17
SU(9)	45	0.11	2	0.31	−0.03	−0.02	0.26
SO(7)	7	0.70	8	0.43	−0.12	−0.16	0.15
SO(11)	11	0.42	14	0.48	−0.11	−0.17	0.19
SO(13)	13	0.35	16	0.53	−0.07	−0.16	0.295
SO(13)	64	0.21	2	0.32	−0.06	−0.04	0.21
SO(15)	15	0.30	20	0.50	−0.10	−0.19	0.20
SO(17)	17	0.26	22	0.53	−0.07	−0.17	0.28
Sp(6)	6	0.60	12	0.63	−0.09	−0.30	0.23
Sp(6)	14	0.28	14	0.35	−0.02	−0.02	0.30
Sp(10)	44	0.21	2	0.37	−0.04	−0.04	0.29
Sp(12)	65	0.17	2	0.35	−0.06	−0.05	0.23
Sp(14)	14	0.28	24	0.59	−0.09	−0.25	0.24
Sp(14)	90	0.15	2	0.31	−0.07	−0.05	0.19
Sp(16)	16	0.24	28	0.56	−0.12	−0.27	0.16
Sp(16)	119	0.13	2	0.30	−0.08	−0.05	0.16
SO(8)	8	0.60	10	0.41	−0.14	−0.18	0.08
SO(10)	10	0.47	12	0.49	−0.08	−0.16	0.24
SO(10)	16	0.37	6	0.39	−0.08	−0.09	0.22
SO(12)	12	0.38	16	0.46	−0.13	−0.19	0.13
SO(12)	32	0.25	4	0.30	−0.09	−0.06	0.14
SO(14)	14	0.32	18	0.51	−0.09	−0.17	0.24
SO(14)	64	0.18	2	0.33	−0.04	−0.03	0.25
G_2	7	1.04	6	0.55	−0.09	−0.21	0.24
E_6	27	0.72	6	0.38	−0.07	−0.08	0.21

cussed here it turns out that $\mu \simeq \Lambda$ so that there is essentially no intermediate region $\Lambda < q < \mu$. For other parameters (say larger values of α_Λ) we get $\Lambda < \mu$. It is always the case, however, that $\mu/\Lambda \sim 1$. This can be understood from the fact that for $q < \mu$ there is no small parameter that enters the dynamics of the system. We will only make use of this order-of-magnitude equivalence of μ and Λ in addressing the suppression of flavor-changing neutral currents. The fundamental result of this paper, the enhancement of $\langle \bar{T}T \rangle_M$ comes from the momentum range $p > \mu$, where the gap equation is presumably more reliable.

The variation of $\ln\Sigma(k)$ with $\ln(k)$ is depicted in Fig. 3. We notice that the variation of $\Sigma(k)$ with k shows the $1/k$ behavior for a significant range of k beyond the chiral-symmetry-breaking scale $\mu = 2\Sigma_0$. Eventually a behavior closer to $1/k^2$ is approached in the asymptotic region.

The gap equation for this model has been solved for a range of α_Λ between 0.6 and 0.9. The integral

$$I = \int^{M/\Sigma_0} \chi \, d\chi \, \sigma(\chi)$$

turns out to be nearly 200 for all values of α_Λ in the above range. Therefore for $M/g_M \simeq 300$ TeV, and $\Sigma_0 \simeq 350$ GeV corresponding to the SU(4) model with the fermion content discussed earlier:

$$m_f \simeq 20 \text{ MeV} . \tag{5.7}$$

This is to be compared to the naive estimate of not much more than 0.1 MeV.

We next estimate the pseudo-Goldstone-boson masses. This theory with 14 fermion flavors will have 195 Goldstone bosons. Of these, three will combine with $W^\pm$ and Z^0. Out of the leftover 192, some will acquire masses due to their electroweak and color interactions. There will be some neutral pseudo-Goldstone bosons which will remain exactly massless at this level. Appropriate effective four-fermion couplings among techifermions can lift their masses up from zero.[28] In technicolor models without enhanced condensates, these masses remain somewhat less than 1 GeV. There are also charged pseudo-Goldstone bosons in these models which get masses of order 10 GeV from electroweak interactions. They too get contributions from four-fermion interactions which are less than 1 GeV without enhanced condensates. The enhancement mechanism described in this paper can lift all these masses to the 50-GeV range.

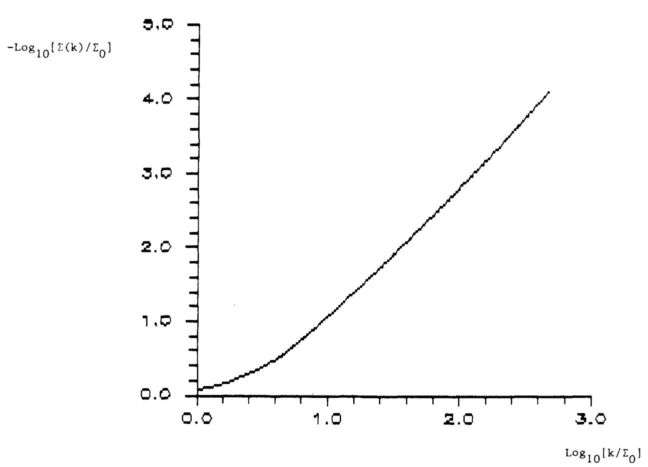

FIG. 3. $-\log_{10}[\Sigma(k)/\Sigma_0]$ is plotted against $\log_{10}(k/\Sigma_0)$ for the case of the SU(4) gauge theory with $N_F = 14$ and $\alpha_\Lambda = 0.9$. The slope of the curve is seen to be approximately one up to $k/\Sigma_0 \gtrsim 10$ and then to increase slowly for larger k/Σ_0.

Using the expression for M_P given in Eq. (2.15), namely,

$$M_P = \frac{aM/g_M m_f}{F} ,$$

we estimate the pseudo-Goldstone-boson masses to be

$$M_P \simeq 50a \text{ GeV} , \tag{5.8}$$

where a is a parameter of order unity. Thus the enhancement mechanism discussed here raises all the pseudo-Goldstone-boson masses to the 50-GeV range. Pseudo-Goldstone bosons of this mass range could not have been produced by any accelerator experiments so far completed.[29] They might, of course, be produced at one of the colliders coming on line in the near future.

VI. CONCLUSIONS

In this paper we have analyzed dynamical mass generation in asymptotically free gauge theories with slowly running couplings. Slowness is achieved by including a sufficient number of fermion flavors. We have solved the Dyson-Schwinger gap equation in such theories in the presence of a cutoff M. The perturbative corrections to the propagators and vertices were taken into account by employing the appropriate form of running coupling in the gap equation. The generalized Landau gauge was used in the analysis.

Analytical and numerical studies of the gap equation have indicated that there is a critical coupling $\alpha_c = \pi/[3C_2(R)]$ that the running coupling $\alpha(q)$ must exceed before chiral condensation can set in. The scale q_c at which $\alpha(q) = \alpha_c$ turns out to have the same order of magnitude as the physical confinement scale Λ. Furthermore, the maximum value Σ_0 of the dynamical mass $\Sigma(k)$, is also of order Λ. This can be understood from the fact that below $\mu = 2\Sigma_0$ the condensed fermions decouple from the running of the coupling. Therefore, for momenta $\lesssim \mu$, the β function governing the dynamics gets contributions only from the gauge fields, and is of order unity. Thus there is no small parameter to set a hierarchy between μ and Λ.

The solution to the gap equation for constant $\alpha > \alpha_c$ has a $1/p$ power behavior multiplied by an oscillatory function. At $\alpha = \alpha_c$ the solution has an exact $1/p$ power behavior. The running coupling $\alpha(q)$ reaches the value α_c around the scale μ. Thus if the theory has a slowly running coupling, $\alpha(q)$ will remain close to α_c for a range of momenta above μ. In this range $\Sigma(p)$ will have an approximate $1/p$ behavior. This relatively slow fall will allow a higher value of the cutoff M than naively expected, for a given value of the fermion mass. Yet the $W^\pm$ and Z^0 masses remain essentially unaltered. This raising of the cutoff leads to the suppression of flavor-changing neutral currents. The same enhancement mechanism can also raise pseudo-Goldstone-boson masses above current accelerator bounds. Fermion masses on the order of 20 MeV and pseudo-Goldstone-boson masses on the order of 50 GeV can be obtained for a cutoff of order 300 TeV. Some gauge theories that exhibit slow running and enhanced fermion condensates have been tabulated. As large as the enhancement effect is, it is not yet clear whether it is adequate to produce realistic masses in a realistic model.

To conclude we discuss some of the open questions that remain to be addressed. Our discussion of chiral-symmetry breaking using the gap equation was restricted to the Landau-type gauges. It remains to be seen whether the concept of critical coupling, which was an essential ingredient in the discussion, persists in a general gauge. A careful analysis of gauge invariance ought to be carried out.

The convergence properties of the β function for α as large as α_c should also be studied more generally. In this paper, a set of theories was listed in which the coupling runs slowly and in which the β function converges reasonably well through three loops. These are simply numerical results however. Since the expansion is not in terms of an obvious small parameter, there is no guarantee that convergence will persist in higher orders. If some convergent expansion scheme could be found, the enhancement mechanism discussed in this paper could be studied more systematically. Whether a realistic technicolor theory comes with such a convergent expansion is another question.

Asymptotically free gauge theories with slowly running couplings can have special properties besides elevated chiral condensates. For example, they can change the convergence properties of integrals used in instanton computations and render them ultraviolet cutoff dependent[30,31] (as opposed to infrared cutoff dependent in theories with normally running couplings). This behavior has been employed to raise the value of the ordinary axion mass.[30]

There has been some discussion in the literature about the possibility of spontaneous breaking of scale invariance in gauge theories.[22,23] While these studies remain inconclusive, it is not unreasonable that the phenomenon might appear if spontaneous breaking of chiral symmetry takes place in a theory which is at least approximately scale invariant. In the theories we have examined, a physical confinement scale Λ as well as an ultraviolet cutoff M are present to explicitly break the scale invariance. Even with a slowly running coupling, $\Sigma(0)\simeq\Lambda\ll M$. The confinement scale sets the scale of chiral-symmetry breaking. There does not appear to be an approximate scale invariance or an approximate dilaton in the theories of interest here.

A natural origin for slowly running couplings might be found in supersymmetric theories. For example, $N=4$ supersymmetric Yang-Mills theory has a vanishing β function and therefore a nonrunning coupling. Dynamical fermion condensation in such theories should be examined to see if they can lead to slow running and to the enhancement mechanism discussed in this paper.

Electroweak symmetry breaking and fermion mass generation remain the most important unsolved problems in particle physics. Some hints towards understanding the underlying physics could come in the next generation of accelerator experiments. Therefore it is all the more important to continue to develop various symmetry-breaking schemes and to explore their consequences. Some work along these lines, incorporating the field-theoretical ideas discussed in this paper, has already appeared.[32]

ACKNOWLEDGMENTS

We are very grateful to Kenneth Lane for a critical reading of this manuscript, including the correction of a numerical mistake in our original mass estimates. Helpful conversations with A. DeRujula, M. Dine, H. Georgi, B. Holdom, R. Jaffe, and M. Voloshin are also acknowledged. We thank D. Karabali who participated in the early stages of this work, and D. Carrier for producing the computerized theory survey leading to Table I. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER0 3075.

[1]S. Weinberg, Phys. Rev. D **13**, 974 (1976); **19**, 1277 (1979); L. Susskind, *ibid.* **20**, 2619 (1979).

[2]S. Dimopoulos and L. Suskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).

[3]M. A. B. Beg, Phys. Lett. **124B**, 403 (1983); **129B**, 113 (1983); S. Dimopoulos, H. Georgi, and S. Raby, *ibid.* **127B**, 101 (1983); S. C. Chao and K. Lane, *ibid.* **159B**, 135 (1985).

[4]S. Dimopoulous and J. Ellis, Nucl. Phys. **B182**, 505 (1981); D. Wyler, in *Proceedings of the XXIII International Conference on High Energy Physics,* Berkeley, California, 1986, edited by S. Loken (World Scientific, Singapore, 1987).

[5]T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[6]T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987).

[7]Some recent revews of technicolor theories are E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981); K. Lane and M. Peskin, in *Electroweak Interactions and Unified Theories,* edited by J. Tran Thanh Van (Editions Frontieres, Derux, France, 1980); R. Kaul, Rev. Mod. Phys. **55**, 449 (1983).

[8]The axial-U(1) symmetry is explicitly broken by the anomaly eliminating the 64th Goldstone boson.

[9]I. Bars, Nucl. Phys. **B208**, 77 (1982); J. Preskill, in *Particles and Fields—1981: Testing the Standard Model,* proceedings of the APS/DPF Meeting, Santa Cruz, California, 1981, edited by C. A. Heusch and W. T. Kirk (AIP Conf. Proc. No. 81) (AIP, New York, 1982), pp. 512–589.

[10]S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977); E. A. Paschos, *ibid.* **15**, 1966 (1977).

[11]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and P. S. Sikivie,

Nucl. Phys. B182, 529 (1981).

[12]The factorization of the matrix element $\langle 0 | \bar{\psi}\psi\bar{\psi}\psi | 0 \rangle$ is assumed here. This can be established for the high-momentum components of interest using the asymptotic freedom of the model.

[13]All momenta will be Euclidean throughout the paper.

[14]P. I. Fomin, V. P. Gusynin, V. A. Miransky, and Yu. A. Sitenko, Rev. Nuovo Cimento **6**, 1 (1983).

[15]T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1976).

[16]R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1976).

[17]M. Peskin, in *Recent Advances in Field Theory and Statistical Mechanics*, 1982 Les Houches Lectures, edited by J. B. Zuber and R. Stora (North-Holland, Amsterdam, 1984).

[18]K. Higashijima, Phys. Rev. D **29**, 1228 (1984).

[19]P. Castorina and S. Y. Pi, Phys. Rev. D **31**, 411 (1985); R. Casalbuoni, S. De Curtis, D. Dominici, and R. Gatto, Phys. Lett. **150B**, 295 (1985).

[20]R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973); H. Pagels and S. Stokar, *ibid.* **20**, 2947 (1979); R. Casalbuoni, *Proceedings of the 1985 International Symposium on Composite Models for Quarks and Leptons,* Tokyo, 1985, edited by H. Terezawa and M. Yasué (Institute for Nuclear Study, University of Tokyo, Tokyo, Japan, 1985).

[21]B. Holdom, Phys. Rev. D **24**, 1441 (1981).

[22]K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); Phys. Lett. **178B**, 308 (1986).

[23]V. A. Miransky, Kiev Report No. ITP-84-152E (unpublished); V. A. Miransky and P. I. Fomin, Fiz. Elem. Chastits At. Yadra **16**, 469 (1985) [Sov. J. Part. Nucl. **16**, 203 (1985)].

[24]For further analysis, see W. A. Bardeen, C. N. Leung, and S. T. Love, Phys. Rev. Lett. **56**, 1230 (1986); Nucl. Phys. **B273**, 649 (1986), and also Ref. 22.

[25]For a review, see S. Narison, Phys. Rep. **84**, 263 (1982).

[26]O. V. Tarasov, A. A. Vladimirov, and A. Yu. Zharkov, Phys. Lett. **93B**, 429 (1980).

[27]We have simply taken $(\Sigma_0)_{\rm QCD}$ to be the constituent mass of the u or d quark $\simeq \frac{1}{3} M_{\rm proton}$.

[28]P. Binetruy, A. Chada, and P. Sikivie, Phys. Lett. **107B**, 425 (1981) have shown that in generic ETC models the neutral pseudo-Goldstone bosons P^0 and P^3 remain exactly massless even after all the gauge interactions including ETC are taken into account. Binetruy *et al.* have observed that the vertical unification of techniquarks and technileptons into a Pati-Salam-type single multiplet is sufficient to break explicitly the global axial symmetries responsible for the masslessness of P^0 and P^3. We assume here that appropriate four-Fermi couplings that break these axial symmetries are included in the theory.

[29]E. Eichten, I. Hinchliffe, K. D. Lane, and C. Quigg, Phys. Rev. D **34**, 1547 (1986).

[30]M. Peskin and B. Holdom, Nucl. Phys. **B208**, 397 (1982). In this paper the QCD coupling is assumed to lose asymptotic freedom above the electroweak scale.

[31]M. Dine and N. Seiberg, Report No. CCNY-HEP-86/2, 1986 (unpublished).

[32]S. King, Boston University Report No. BUHEP-86-7, 1986 (unpublished); K. Lane (private communication).