Critical surfaces and flat directions in a finite theory

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We discuss the phase structure of an $O(N)\times O(N)$-symmetric model with scalar fields in three-dimensional space-time. The effective potential of the theory is calculated in the large-N limit and stability conditions for the model are given. Spontaneous breaking of scale invariance due to the breakdown of the internal symmetry or to nonperturbative mass generation is observed on a critical surface, in connection with the appearance of flat directions in the effective potential. The ground-state energy of the model remains unchanged under the transition from the scale-invariant phase to the various phases, where scale invariance is spontaneously broken. The critical surface allows for arbitrary ratios between the scales associated with breaking of the internal and scale symmetries.

I. MOTIVATION

In a globally scale-invariant theory, spontaneous symmetry breaking may occur[1–4] due to the formation of flat directions in the effective potential. The broken phase is characterized by certain fields gaining expectation values; because of the scale symmetry, the vacuum energy in the broken phase is identical with that obtained for zero expectation value of the fields. In a globally supersymmetric, finite theory, a spontaneous breaking of supersymmetry is thus forbidden, as long as Lorentz invariance is maintained.[4,5] A symmetry can preserve a particular value of the vacuum energy, even if that symmetry is spontaneously broken.

This property will survive, even if other scales arise in the problem. Assume, in particular, that some internal symmetry is broken, but at a much lower scale ϕ_c. It may seem, that the vacuum energy will now be proportional to ϕ_c. By studying a three-dimensional model, in which scale invariance as well as some internal symmetry are broken, we show that the vacuum energy E_0 remains fixed and does not depend on ϕ_c. We also demonstrate that any ratio of the scales of spontaneous symmetry breaking to that of scale-invariance breaking is admissible. Various ratios correspond to different couplings on a critical surface.

Spontaneous breaking of scale invariance due to nonperturbative generation of a mass scale was shown to occur in models with an internal $O(N)$ symmetry in three dimensions.[2] In that model, the internal symmetry remains intact.

By enlarging this theory to an $O(N)\times O(N)$-symmetric model,[6] we obtain a variety of symmetry-breaking patterns, which will produce two scales, whose ratio could adopt all possible values.

In Sec. II we study the stable and unstable regions of the model. We then describe in detail the properties of the critical surface, which separates the stable region from the unstable one, and trace the various symmetry-breaking patterns associated with different parts of the critical surface. In Sec. III the particle content of each phase is analyzed, with emphasis on the Goldstone boson connected to spontaneous breaking of scale invariance—the dilaton.

II. THE MODEL AND ITS EFFECTIVE POTENTIAL

We shall analyze a model described by the following $O(N)\times O(N)$-symmetric action (here written in Euclidean space):

$$S_E=\int d^3\mathbf{x}\left\{\tfrac{1}{2}(\nabla\boldsymbol{\phi}_1)^2+\tfrac{1}{2}(\nabla\boldsymbol{\phi}_2)^2+\tfrac{1}{2}\lambda_1\boldsymbol{\phi}_1{}^2+\tfrac{1}{2}\lambda_2\boldsymbol{\phi}_2{}^2+\frac{1}{4N}[\mu_1(\boldsymbol{\phi}_1{}^2)^2+\mu_2\boldsymbol{\phi}_1{}^2\boldsymbol{\phi}_2{}^2+\mu_3(\boldsymbol{\phi}_2{}^2)^2]\right.$$
$$\left.+\frac{16\pi^2}{6N^2}[h_1(\boldsymbol{\phi}_1{}^2)^3+h_2(\boldsymbol{\phi}_1{}^2)^2\boldsymbol{\phi}_2{}^2+h_3\boldsymbol{\phi}_1{}^2(\boldsymbol{\phi}_2{}^2)^2+h_4(\boldsymbol{\phi}_2{}^2)^3]\right\},$$

where $\boldsymbol{\phi}_1$, $\boldsymbol{\phi}_2$ are in the vector representation of $O(N)$. The theory can be treated with variational methods[7] or, alternatively, by means of functional integration. We follow here the functional methods methods of Ref. 5 in order to calculate the effective potential in the large-N limit. The result is (we have changed the overall sign of W when passing from Euclidean space to Minkowski space-time)

$$\frac{6}{16\pi^2 N} W(\Phi_1,M_1,\Phi_2,M_2)=M_1{}^3+M_2{}^3+h_1(\Phi_1-M_1)^3+h_2(\Phi_1-M_1)^2(\Phi_2-M_2)+h_3(\Phi_1-M_1)(\Phi_2-M_2)^2+h_4(\Phi_2-M_2)^3 ,$$

where $\Phi_1=(\phi_{1\text{cl}}{}^2)/N$, $\Phi_2=(\phi_{2\text{cl}}{}^2)/N$, $M_1=m_1/(4\pi)$, $M_2=m_2/(4\pi)$. m_1 and m_2 are the masses of the quantum fields ϕ_1 and ϕ_2, and $\phi_{1\text{cl}}$, $\phi_{2\text{cl}}$ are their respective classical expectation values. To simplify the study, the renormalized couplings λ_i^R, μ_i^R were set to zero from the onset. The couplings h_i remain unrenormalized in perturbation theory. An overall constant, which is independent of the variables h_i, is subtracted from the effective potential. The ground-state energy is determined by minimizing W with respect to the non-negative parameters M_1,M_2, $\phi_{1\text{cl}}^k,\phi_{2\text{cl}}^k$ [k being an O(N) index]. W, the effective potential of a truly scale-invariant system, is a homogeneous function of the variables M_1,M_2,Φ_1,Φ_2. Euler's theorem for homogeneous functions ensures that the ground state, if it exists, has zero energy. This simple property of the effective potential is sufficient to prove our statement that, whatever symmetry-breaking pattern will occur, the vacuum (ground-state) energy will not be shifted away from its value in the phase with unbroken scale invariance.

For any number λ, W obeys the relation

$$W(\lambda\Phi_1,\lambda M_1,\lambda\Phi_2,\lambda M_2)=\lambda^3 W(\Phi_1,M_1,\Phi_2,M_2) .$$

In order to obtain a stable ground state, we require, therefore, that

$$W(\Phi_1,M_1,\Phi_2,M_2)\geq 0,\quad \forall\Phi_1,\Phi_2,M_1,M_2\geq 0 . \tag{1}$$

This constraint on the effective potential implies constraints on the couplings h_i. The stability of the effective potential according to (1) is ensured by imposing simultaneously the following six constraints on the coupling constants:

$$(27h_4{}^2h_1+2h_3{}^3-9h_2h_3h_4)^2\geq 4(h_3{}^2-3h_2h_4)^3 , \tag{2a}$$

(2a) with h_4 replaced by $1-h_4,h_2$ by $-h_2$, (2b)

(2a) with h_1 replaced by $1-h_1$,h_3 by $-h_3$, (2c)

(2a) with $h_{1,4}$ replaced by $1-h_{1,4},h_{2,3}$ by $-h_{2,3}$, (2d)

$$0\leq h_1\leq 1 , \tag{2e}$$

$$0\leq h_4\leq 1 . \tag{2f}$$

These constraints divide the four-dimensional space of coupling constants into three subsectors [see Figs. 1(a) and 1(b)].

(i) A bounded region of *stable configurations,* where all six constraints hold simultaneously as exact inequalities. Inside of it, the potential has a unique zero-energy minimum at the point $\Phi_1=\Phi_2=M_1=M_2=0$. In this phase, the theory is O(N)$\times$O(N) symmetric, scale invariant, and contains only massless particles. In Fig. 1(a) this region is contained inside the closed curve carrying indices $a-e$. In Fig. 1(b), it is the region contained inside the closed surface (of which only a two-dimensional cut is shown).

(ii) A region of *instability,* where one of the constraints (2a)—(2f) is violated. Inside of it, the effective potential is unbounded from below. This is the region outside of the closed curve $a-e$ in Fig. 1(a), or outside of the closed surface in Fig. 1(b).

(iii) A compact *critical surface,* which separates the stable from the unstable region. A one- (two-) dimensional section of the critical surface is shown in Fig. 1(a) [Fig. 1(b)]. Various subsectors of this surface correspond to

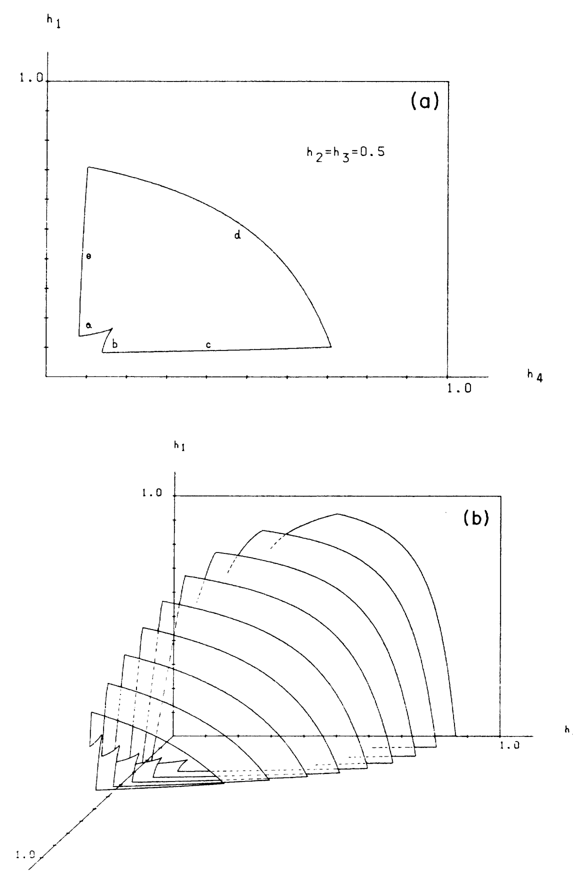

FIG. 1. (a) One-dimensional section of the critical surface in coupling space. (b) Two-dimensional section of the critical surface for $h_3=0.5$.

different symmetry-breaking patterns. They are obtained by enforcing an equality in one of the above constraints, while satisfying all the others. The various phases are symmetric under $O(N-1)\times O(N-1)$ [lines a,b in Fig. 1(a)], $O(N-1)\times O(N)$ (line c), $O(N)\times O(N-1)$ (line e) and $O(N)\times O(N)$ (line d) [the equality taken in (2a)—(2d), respectively]. A remaining $O(N)$ factor implies nonperturbative mass generation for the respective quantum field, such that scale invariance is always broken on the critical surface. For every quadruple of couplings $h_{1,\ldots,4}$ taken from this surface, the effective potential has a flat direction and is stable according to Eq. (1).

The "corner points" of the closed curve in Fig. 1(a) are points where phase transitions occur [except the intersection of lines a and b, which both represent the $O(N-1)\times O(N-1)$-symmetry-breaking pattern]. The effective potential has, at these points, two flat directions and two different degenerate ground states. The "sharp edges" of the closed surface in Fig. 1(b) are lines of phase transitions.

In order that the flat direction will be contained in the shape of non-negative parameters $\Phi_{1,2},M_{1,2}$, additional conditions are necessary. In Table I we write down the symmetry-breaking patterns for particular combinations of signs of $h_{2,3}$, which are not available. Because the flat direction will have only the origin $(M_1=M_2=\Phi_1=\Phi_2=0)$ in common with the region of non-negative parameters Φ_i,M_i, no nonzero scale can be created and all symmetries are preserved.

Therefore, in Fig. 1(a) the lines a and b represent absent symmetry-breaking patterns. The end points of the segment $a\cup b$ *are* points of symmetry breaking (belonging to the phases c and e, respectively), such that the critical surface remains a compact object.

Minimizing the effective potential W with respect to the components $\phi^k_{1\mathrm{cl}}$, $\phi^k_{2\mathrm{cl}}$ leads to the gap equations

$$M_1{}^2\phi^k_{1\mathrm{cl}}=0,\quad k=1,\ldots,N\ ,$$

$$M_2{}^2\phi^k_{2\mathrm{cl}}=0,\quad k=1,\ldots,N\ .$$

The four sets of parameters satisfying these equations are

$$M_1=0,\quad M_2=0,\quad M_2=0,\quad \phi^k_{1\mathrm{cl}},\phi^k_{2\mathrm{cl}}\ \text{arbitrary}\ ,$$

$$\phi^k_{1\mathrm{cl}}=0,\quad \phi^k_{2\mathrm{cl}}=0,\quad M_1,M_2\ \text{arbitrary}\ ,$$

$$M_1=0,\quad \phi^k_{2\mathrm{cl}}=0,\quad \phi^k_{1\mathrm{cl}},M_2\ \text{arbitrary}\ ,$$

$$\phi^k_{1\mathrm{cl}}=0,\quad M_2=0,\quad M_1,\phi^k_{2\mathrm{cl}}\ \text{arbitrary}\ .$$

In the stable region, all four variables are zero at the minimum of the effective potential. The critical surface is divided into the subsections mentioned above; on each such subsection, the potential minima are situated along a flat direction. In each subsection, however, a definite relation holds between some pair of the effective potential's variables, while the others vanish.

Let us take, as an example, the subspace defined by $M_1=M_2=0$; the remaining equations

$$\partial W/\partial M_1=0,\quad \partial W/\partial M_2=0$$

read

$$3h_1\Phi_1{}^2+h_3\Phi_2{}^2+2h_2\Phi_1\Phi_2=0\ ,$$

$$h_2\Phi_1{}^2+3h_4\Phi_2{}^2+2h_3\Phi_1\Phi_2=0\ .$$

The solution is nontrivial, provided that

$$B^2=4AC\ , \tag{3}$$

where $B=9h_1h_4-h_2h_3$, $A=h_3{}^2-3h_2h_4$, $C=h_2{}^2-3h_2h_1$. In this case, $\boldsymbol{\phi}_{1\mathrm{cl}}$ and $\boldsymbol{\phi}_{2\mathrm{cl}}$ are related by

$$\boldsymbol{\phi}_{1\mathrm{cl}}{}^2/\boldsymbol{\phi}_{2\mathrm{cl}}{}^2=(A/C)^{1/2}\ .$$

This is a phase, in which the $O(N)\times O(N)$ symmetry is broken down to $O(N-1)\times O(N-1)$. The expression $(A/C)^{1/2}$ can be shown to adopt arbitrary positive values with varying couplings h_i. By choosing the couplings properly, we might thus create any arbitrary separation between the two scales Φ_1 and Φ_2.

TABLE I. Absent symmetry-breaking patterns. The symmetry-breaking pattern marked in a certain cell is unavailable for this particular combination of signs of the couplings $h_{2,3}$.

h_3 h_2	+	−
+	$O(N-1)\times O(N-1)$	$O(N-1)\times O(N)$
−	$O(N)\times O(N-1)$	$O(N)\times O(N)$

III. THE PARTICLE CONTENT OF THE VARIOUS BROKEN-SYMMETRY PHASES

A. The phase with $O(N)\times O(N)$ broken down to $O(N-1)\times O(N-1)$

We use the following Lagrangian in Minkowski space-time

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\boldsymbol{\phi}_1)^2+\tfrac{1}{2}(\partial_\mu\boldsymbol{\phi}_2)^2-V(\boldsymbol{\phi}_1{}^2,\boldsymbol{\phi}_2{}^2)\ , \tag{4a}$$

$$V(\boldsymbol{\phi}_1{}^2,\boldsymbol{\phi}_2{}^2)=(\eta/6N^2)(a^2\boldsymbol{\phi}_1{}^2+\boldsymbol{\phi}_2{}^2)(b^2\boldsymbol{\phi}_1{}^2-\boldsymbol{\phi}_2{}^2)^2\ . \tag{4b}$$

Condition (2a) or, equivalently, (3) are automatically satisfied for this choice of couplings, regardless of the values of η,a,b. We define shifted fields σ_1,σ_2 by

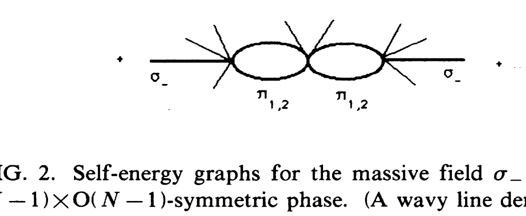

FIG. 2. Self-energy graphs for the massive field σ_- in the $O(N-1)\times O(N-1)$-symmetric phase. (A wavy line denotes a factor of f.)

$$\phi_1^2=\pi_1^2+(\sigma_1+f)^2 , \quad (5a)$$

$$\phi_2^2=\pi_2^2+(\sigma_2+bf)^2 , \quad (5b)$$

where $b=(b^2)^{1/2}>0$.

We obtain, for the potential V in the large-N limit (keeping η finite as $N\to\infty$),

$$V=\frac{\eta(a^2+b^2)}{6N^2}\times[8b^2f^4\sigma_-^2+4\sqrt{2}bf^3\sigma_-(b^2\pi_1^2-\pi_2^2)+f^2(b^2\pi_1^2-\pi_2^2)] , \quad (6a)$$

$$\sigma_-=(b\sigma_1-\sigma_2)/\sqrt{2} . \quad (6b)$$

From the expressions (6), we deduce the particle content of this phase. There are $2(N-1)$ massless Goldstone bosons (the fields π_1^k,π_2^k). The field

$$\sigma_+=\sigma_1+b\sigma_2 \quad (6c)$$

(which is the coordinate along the flat direction of the effective potential) does not appear in V. Its self-energy is, therefore, zero in leading order of N. σ_+ is interpreted as the massless dilaton, the product of spontaneous breakdown of scale invariance due to the generation of the scales Φ_1,Φ_2. Unlike in the model for a single O(N) vector field,[2] the dilaton is in our case a O($N-1$) singlet and not a O(N) singlet. The field σ_- (the coordinate orthogonal to the flat direction) is the only massive particle in this phase. Its self-energy, in the leading-N approximation, is given by the graphs in Fig. 2. The induced two-, three-, and four-point couplings can be read from Eq. (6a). We find, for the inverse propagator of the σ_- particle,

$$\mathrm{Im}[iD^{-1}(q^2)]=\tfrac{2}{9}\eta^2(a^2+b^2)^2b^2f_0^2\frac{[b^2(b^2+1)(q^2)^{1/2}+(\eta^2/144)(a^2+b^2)^2(b^2-1)(b^4+1)^2f_0^4]}{q^2+(\eta^2/144)(a^2+b^2)^2(b^4+1)^2f_0^4} , \quad (7a)$$

$$\mathrm{Re}[iD^{-1}(q^2)]=\frac{q^2\{q^2-(8\eta/3)(a^2+b^2)f_0^4[b^2-(\eta/384)(a^2+b^2)(b^4+1)^2]\}}{q^2+(\eta^2/144)(a^2+b^2)^2(b^4+1)^2f_0^4} , \quad (7b)$$

where we have kept $f_0=f/N^{1/2}$ finite as $N\to\infty$. The inverse propagator has a zero at $q^2=0$. For

$$\eta<\eta_c\equiv384b^2/[(b^4+1)^2(a^2+b^2)]$$

an additional resonance bound state appears.

B. The phase with unbroken O(N)×O(N) symmetry and broken scale invariance

We expect, in this phase, the spontaneous generation of masses m_1 and m_2 for the quantum fields ϕ_1,ϕ_2, respectively. There will be $2N$ massive fields ϕ_1^k,ϕ_2^k, $k=1,\dots,N$, and a massless O(N) scalar: the dilaton. The calculations in this paragraph are performed in Euclidean space. We use the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(\nabla\phi_1)^2+\tfrac{1}{2}(\nabla\phi_2)^2+(16\pi^2/6N^2)[h_1(\phi_1^2)^3+h_2(\phi_1^2)^2\phi_2^2+h_3\phi_1^2(\phi_2^2)^2+h_4(\phi_2^2)^3] . \quad (8)$$

Nonzero masses can be created, provided that the couplings satisfy the relation

$$[9(1-h_4)(1-h_1)-h_2h_3]^2=4[h_3^2+3h_2(1-h_4)][h_2^2+3h_3(1-h_1)] .$$

The parameters m_1 and m_2 will be related by

$$[9(1-h_4)(1-h_1)-h_2h_3]m_1=2[h_3^2+3h_2(1-h_4)]m_2 .$$

We look for the dilaton as a zero pole in the four-point amplitude shown in Fig. 3. We find for the Euclidean O(N)-singlet amplitude in the limit of low external momenta:

$$\Gamma^{(4)}(q^2)\to\frac{192\pi}{Nq^2}\frac{m_1^3m_2[h_3m_1-3(1-h_4)m_2]}{(h_2+h_3)m_1m_2-3(1-h_1)m_1^2-3(1-h_4)m_2^2} \quad \text{as } q^2\to0 .$$

The dilaton appears as a massless, O(N)-singlet bound state of the ϕ_1^k fields, as it did in the single-O(N) case.[2]

C. The phase with O(N)×O(N) broken down to O($N-1$)×O(N)

In this phase, the field ϕ_1 adopts a nonzero expectation value

$$\langle\phi_1\rangle=\phi_{1\mathrm{cl}}$$

and a nonzero mass m_2 is generated for the field ϕ_2. We consider the Lagrangian in Minkowski space-time,

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\phi_1)^2+\tfrac{1}{2}(\partial_\mu\phi_2)^2-(16\pi^2/6N^2)[h_1(\phi_1^2)^3+h_2(\phi_1^2)^2\phi_2^2+h_3\phi_1^2(\phi_2^2)^2+h_4(\phi_2^2)^3] ,$$

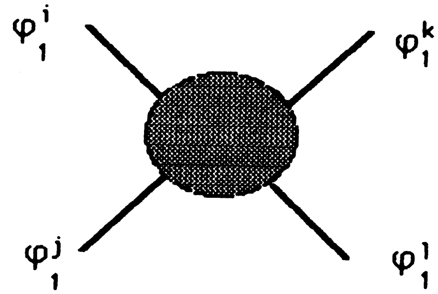

FIG. 3. Four-point amplitude for the dilaton pole in the $O(N)\times O(N)$-symmetric phase with broken scale invariance.

and shift the field ϕ_1,

$$\phi_1{}^2=\pi_1{}^2+(\sigma_1+f)^2\ , \tag{9}$$

with $f=\sqrt{N}f_0$, where f_0 is kept finite as $N\to\infty$.

The two scales $f_0{}^2$ and m_2 are related by

$$(m_2/4\pi)[h_2h_3+9h_1(1-h_4)]=2(h_2{}^2-3h_1h_3)f_0{}^2\ .$$

This holds, provided that the couplings h_i satisfy the relation

$$[9h_1(1-h_4)+h_2h_3]^2=4(h_2{}^2-3h_1h_3)[h_3{}^2+3h_2(1-h_4)]\ .$$

We have the following particle content in this phase: There are N massive fields ϕ_2^k, $k=1,\dots,N$, and $N-1$ massless Goldstone bosons π_1^k, $k=1,\dots,N-1$. In addition, we find a massive particle σ_1 [see Eq. (9)] and a massless $O(N)$-singlet state: the dilaton. For details of the calculation, see the Appendix.

IV. CONCLUSIONS

We have studied a scale-invariant theory, which is exactly solvable in the large-N limit. Spontaneous breakdown of scale invariance occurs on a compact critical surface. It manifests itself in various phases, in some of which it is a consequence of internal-symmetry breaking. The ground-state energy E_0 is fixed, regardless of the special pattern of symmetry breaking or of symmetry breaking occurring at all. This is an example of a theory where symmetries are used to determine the cosmological constant; its value is preserved, whether the symmetry is preserved or spontaneously broken. The fact that this can even occur when there are large ratios of scales may be relevant to combined solutions of the hierarchy problem and the cosmological constant problem.

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APPENDIX

In the phase where $O(N)\times O(N)$ is broken down to $O(N-1)\times O(N)$, we look for the dilaton in the four-point amplitude for the fields ϕ_2^k, which is analogous to Fig. 3. The Euclidean $O(N)$-singlet amplitude in the limit of low external momenta is

$$\Gamma^{(4)}(q)=\frac{48g_c{}^2M_2{}^3}{Nq}\,\frac{4q(3h_4M_2-h_3f_0{}^2)+g_cM_2(3h_1f_0{}^2-h_2M_2)}{4(3h_4M_2-h_2f_0{}^2)q^2+g_cM_2(3h_1f_0{}^2-h_2M_2)q+48g_cM_2{}^2[h_3f_0{}^2++3(1-h_4)M_2]}$$

with $g_c=16\pi^2$, $M_2=m_2/4\pi$, $q=(q^2)^{1/2}$.

The dilaton pole of the form $1/q^2$ is mixed with the branch cut due to the Goldstone fields, and is exposed only, when we take the limit of a single $O(N)$ vector field by setting $h_1=h_2=h_3=0$, $h_4=1$.

We would also like to calculate the propagator of the massive σ_1 particle in Minkowski space-time. The induced coupings relevant in the large-N limit are indicated in Fig. 4. For the sake of simplicity, we specify the theory to the case

$$h_1=h_2=H,\ \ h_3=h_4-1=-H,\ \ f_0{}^2=m_2/4\pi=M_2\ .$$

The stability conditions (2) impose, furthermore, the constraint

$$0\le H\le 0.75\ .$$

We obtain the following results for the inverse propagator of the σ_1 particle.

Case 1. The external momentum q^2 satisfies the relation $\alpha^2\equiv q^2/m_2{}^2\le 4$. The inverse propagator is then

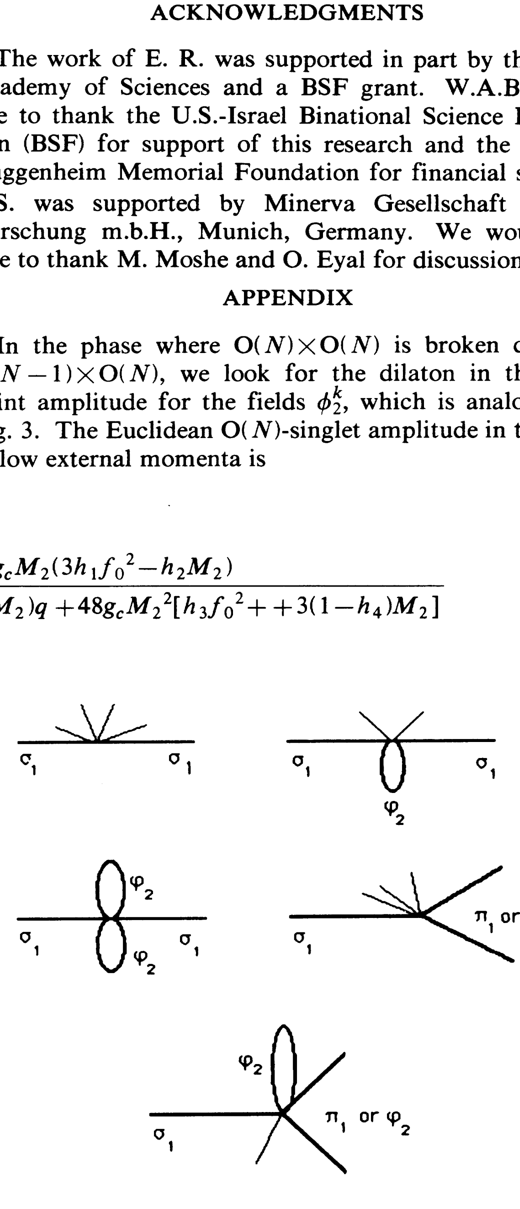

FIG. 4. Induced couplings for the massive σ_1 particle in the $O(N-1)\times O(N)$-symmetric phase.

$$\mathrm{Re}[iD^{-1}(\alpha^2)]=m_2{}^2\left[\alpha^2-\tfrac{1}{3}H\frac{(21\omega-4HF)(3\omega+4HF)\alpha^2-\frac{1}{4}g_cH^2\omega^2}{(3\omega+4HF)^2\alpha^2+\frac{1}{4}g_cH^2\omega^2}\right], \tag{A1}$$

$$\mathrm{Im}[iD^{-1}(\alpha^2)]=\frac{16\pi m_2{}^2H^2\omega^2\alpha}{(3\omega+4HF)^2\alpha^2+\frac{1}{4}g_cH^2\omega^2}, \tag{A2}$$

with $g_c=16\pi^2$, $\omega=\omega(\alpha)=1-2F(\alpha)$, $F(\alpha)=(1/\alpha)\mathrm{arcsinh}[\alpha(4-\alpha^2)^{-1/2}]$.

The asymptotic behavior is

$$\mathrm{Re}[iD^{-1}(\alpha^2)]\to\tfrac{1}{3}Hm_2{}^2 \text{ as } \alpha^2\to 0\ . \tag{A3}$$

Case 2. $\alpha^2\equiv q^2/m_2{}^2\geq 4$. The inverse propagator is

$$\mathrm{Re}[iD^{-1}(\alpha^2)]=m_2{}^2\left[\alpha^2-H\frac{[(21\omega-4HG)\alpha^2-g_cH/8]A-\frac{1}{16}g_c\alpha^2(2H\omega+2H+21)B}{3(A^2+\frac{1}{16}g_c\alpha^2B^2)}\right], \tag{A4a}$$

$$\mathrm{Im}[iD^{-1}(\alpha^2)]=m_2{}^2\frac{[16H(\omega^2+\omega+2G)\alpha^2+g_cH]\pi\alpha H}{A^2+\frac{1}{16}g_c\alpha^2B^2}, \tag{A4b}$$

with

$$g_c=16\pi^2,\quad A=(3\omega+4HG)\alpha^2+\tfrac{1}{8}g_cH\ ,$$

$$B=2H\omega+2H-3,\quad \omega=1-2G\ ,$$

$$G=G(\alpha)=(1/\alpha)\mathrm{arccosh}[\alpha(\alpha^2-4)^{-1/2}]\ .$$

The asymptotic behavior is

$$\mathrm{Re}[\Sigma(\alpha^2)]\to\tfrac{7}{3}Hm_2{}^2 \text{ as } \alpha^2\to 0$$

with the tree mass being exactly $\frac{7}{3}Hm_2{}^2$. A numerical investigation of these expressions gives the following results. For the allowed values of H, only expression (A1) [not (A4a)] has zeros. For $H=0$, the inverse propagator is obviously

$$iD^{-1}(\alpha^2)=m_2{}^2\alpha^2=q^2$$

and we have, in the zero-coupling limit, a zero of the inverse propagator only at $q^2=0$ (a free, massless particle). For growing H, we find two different zeros for every value of H, corresponding to resonance bound states. These two poles of the propagator merge to form a single one again, when H reaches a critical value $H_c=0.0417\ldots$. For $H>H_c$, there are no poles in the propagator. The two-point function of the σ_1 particle shows, thus, resonance bound states in a limited domain of the coupling H.

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