

**Stochastic quantization of Yang-Mills field theory:
Gauge-fixing parameter dependence and equilibrium limit**

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We calculate, in the framework of stochastic quantization, the one-loop-divergent part of the gluon self-energy and the triple-gluon vertex of pure Yang-Mills field theory, with an arbitrary choice of the stochastic gauge-fixing parameter. This allows us to check that the strong conditions imposed by renormalizability are satisfied up to one-loop order. We compare our results with those coming from the Faddeev-Popov theory and discuss the relationship between both approaches in the equilibrium limit.

I. INTRODUCTION

The method of stochastic quantization¹ (SQ) uses the Langevin or Fokker-Planck equations to describe the evolution of a Euclidean field theory in an artificial time t . The Green's functions of quantum field theory may then be calculated as the equilibrium limit (EL) ($t \rightarrow +\infty$) of the corresponding equal-time stochastic averages. SQ seems particularly useful when applied to non-Abelian gauge field theories that, in the nonperturbative region, suffer from the Gribov ambiguity.² Nevertheless, perturbative calculations are also possible for a pure Yang-Mills gauge field and may be regarded not only as an alternative to the Faddeev-Popov (FP) perturbation theory but as a check of the consistency of SQ and also to be compared with results obtained with nonperturbative methods. In fact, it has been shown³ up to second order in perturbation theory that SQ produces the same result as the FP theory does for a gauge-invariant quantity such as the field-strength propagator.

Although SQ avoids the introduction of a gauge-fixing term, it is convenient, in dealing with perturbative calculation of gauge-noninvariant quantities, to use a stochastic gauge-fixing term⁴ that leaves unchanged expectation values of gauge-invariant objects. It is to be remarked that both gauge fixings are radically different: while in the FP approach one basically averages the gauge field configurations in the neighborhood of the surface $\partial_\mu A^\mu = 0$, the stochastic procedure adds a driving force tangential to the gauge orbits that compels the field configurations to be near the aforementioned surface.

We also verify, up to one-loop order, Zwanziger's formal argument⁵ that gauge fixing should give, in the singular limit $\alpha \rightarrow 0$, the same results as the FP approach in the Landau gauge. Specifically we show this equality for the two- and three-point functions. That of the two-point

function has already been shown in Ref. 6 for gauge theory with fermions and scalar electrodynamics using the continuum regularization scheme.⁷ In general ($\alpha > 0$), the results are expected to be different, as has been explicitly shown in previous works⁸ with the stochastic Feynman gauge ($\alpha = 1$).

We shall present in this paper the results corresponding to choosing an arbitrary value of the stochastic gauge parameter α , which generalizes the previous calculations for $\alpha = 1$ and allows us to check to one-loop order the renormalizability of the theory and also to study the behavior of gauge-noninvariant quantities in the singular limit $\alpha \rightarrow 0$.

The organization of the paper is as follows. In Sec. II we reformulate the Feynman rules in a suitable form for computer calculations and comment on some useful properties as well as the general technique to evaluate a diagram. In Sec. III we present the results of the divergent part of the one-loop correction to the gluon self-energy and triple-gluon vertex. From these calculations it is possible to extract the renormalization constants and to check the Ward identities (Sec. IV).

Finally in Sec. V we discuss the EL and show that the singular limit $\alpha \rightarrow 0$ for the SQ approach coincides with the FP result in the Landau gauge up to one-loop order, while in the $\alpha > 0$ region the differences are important. We also present the correspondence between diagrams in both cases, realizing that reducible diagrams [one-particle irreducible (1PI)] in SQ may contribute, in the EL, to irreducible diagrams (no 1PI) in the FP method.

II. PERTURBATION THEORY

As has been previously shown⁹ the Feynman rules can be obtained from the generating functional

$$Z[J] = \int [dA] \exp \left\{ \int_{-\infty}^{+\infty} dt \int d^Dx \left[J_\mu^a A_\mu^a - \frac{1}{4\gamma} \left(\frac{\partial A_\mu^a}{\partial t} - \gamma \frac{\delta S}{\delta A_\mu^a} + \gamma D_\mu^{ab} V^b \right)^2 \right] \right\}, \tag{1}$$

where S is the Euclidean Yang-Mills action, γ is the diffusion parameter, $D_\mu^{ab} = \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c$ is the covariant derivative, and $V^b = (1/\alpha) \partial_\mu A_\mu^b$ is the stochastic gauge-fixing term.⁴ Notice its different structure from the corresponding one in the FP approach. We have used dimensional regularization ($D = 4 - 2\epsilon$), which has allowed us to disregard a term in the action of (1) which behaves like $\delta^{(D)}(0)$ (Ref. 10) and acts like a counterterm.

Obviously there are no functional determinants or ghost fields in (1). In spite of this simplification some other difficulties arise: new vertices [with five and six legs, but with a very simple form (see Ref. 9)] and more complex structures for both propagator and vertices. In fact the main problem, but not the only one, that we have to face is the long but straightforward algebraic manipulation of those structures. Fortunately enough this can be diminished with a computer algebra program. With this purpose we have found it convenient to rewrite the vertices as follows. Let us define

$$U_{\mu\nu\rho}(p) = \left[1 - \frac{1}{\alpha} \right] p_\nu \delta_{\mu\rho} - 2p_\rho \delta_{\mu\nu} + p_\mu \delta_{\nu\rho}, \quad (2)$$

$$V_{\mu\nu}(p, \omega) = (i\omega - \gamma p^2) \delta_{\mu\nu} + \gamma \left[1 - \frac{1}{\alpha} \right] p_\mu p_\nu. \quad (3)$$

Once we have introduced these objects, the triple-gluon vertex (with external momenta p_i , frequencies ω_i , color indices a_i , and Euclidean indices μ_i , where $i = 1, 2, 3$ and we assume $\sum_i p_i = 0$ and $\sum_i \omega_i = 0$) can be written in zero-loop order as

$$\Gamma_{3\mu_1\mu_2\mu_3}^{(0)a_1a_2a_3}(p_1, \omega_1; p_2, \omega_2; p_3, \omega_3) = -i \frac{g}{2} \sum_{\substack{(ijk) \\ i,j,k=1,2,3}} f^{a_i a_j a_k} V_{\mu_i \nu}(p_i, \omega_i) U_{\nu \mu_j \mu_k}(p_j). \quad (4)$$

Analogously, the four-leg vertex (ω independent) takes the form

$$\Gamma_{4\mu_1\mu_2\mu_3\mu_4}^{(0)a_1a_2a_3a_4}(p_1, p_2, p_3, p_4) = \frac{g^2}{2} \sum_{\substack{(ijkl) \\ i,j,k,l=1,2,3,4}} f^{a_i a_j b} f^{a_k a_l b} [\gamma U_{\nu \mu_j \mu_i}(p_j) U_{\nu \mu_l \mu_k}(p_l) + 2V_{\mu_i \mu_k}(p_i, 0) \delta_{\mu_j \mu_l}]. \quad (5)$$

As will be discussed below, it is useful to rewrite the free propagator

$$G_{\mu\nu}^{(0)ab}(p, \omega) = 2\gamma \delta^{ab} \frac{(\omega^2 + \gamma^2 p^4 / \alpha^2) \delta_{\mu\nu} + (1 - 1/\alpha^2) \gamma^2 p^2 p_\mu p_\nu}{(\omega^2 + \gamma^2 p^4 / \alpha^2)(\omega^2 + \gamma^2 p^4)} \quad (6)$$

in the following way:

$$G_{\mu\nu}^{(0)ab}(p, \omega) = 2\gamma \delta^{ab} N_{\mu\rho}(p, \omega) N_{\rho\nu}(p, -\omega), \quad (7)$$

where

$$N_{\mu\nu}(p, \omega) = \frac{(i\omega + \gamma p^2 / \alpha) \delta_{\mu\nu} + (1 - 1/\alpha) \gamma p_\mu p_\nu}{(i\omega + \gamma p^2 / \alpha)(i\omega + \gamma p^2)}. \quad (8)$$

Surprisingly, this tensor satisfies

$$N_{\mu\nu}(p, \omega) V_{\nu\rho}(p, -\omega) = -\delta_{\mu\rho}. \quad (9)$$

Because of Eq. (7) the propagators may be written as the product of two tensors such as Eq. (8) which can, according to (9), cancel eventually with the corresponding piece of the triple-gluon vertex in (4). In this way it is possible to simplify notably the internal-frequency integrals, because each time (9) occurs there is a reduction of the number of poles and, in addition, the numerator gets considerably simpler.

III. GLUON SELF-ENERGY AND TRIPLE-GLUON VERTEX TO THE ONE-LOOP ORDER

It is useful to recognize the tensorial structure of the vertices before computing them. Although some of the properties stated here are also true in higher orders, we shall limit ourselves to one-loop order.

It is easy to show that the divergent part of the self-energy (or two-leg vertex) and the three-gluon vertex have

a polynomial dependence on the frequencies and momenta. On the other hand, the Poincaré invariance implies that the Euclidean indices have to appear only as $p_i^{\mu_i}$ and or $\delta^{\mu\nu}$, with p_i being an external momentum. Further restrictions will be imposed by dimensionality arguments.

The two-leg vertex has to be, in accordance with our previous discussion, a linear combination of $\omega^2 \delta_{\mu\nu}$, $(p^2)^2 \delta_{\mu\nu}$, and $p^2 p_\mu p_\nu$. Notice that in zero-loop order this vertex is just the inverse of the propagator:

$$\Gamma_{2\mu\nu}^{(0)ab}(p, \omega) = \frac{1}{2\gamma} \delta^{ab} \left\{ \frac{\gamma^2}{\alpha^2} p^2 p_\mu p_\nu + \omega^2 \delta_{\mu\nu} + \gamma^2 (p^2 \delta_{\mu\nu} - p_\mu p_\nu) \right\}. \quad (10)$$

In this particular case, the zero-loop approximation depends on all possible objects that satisfy the previous requirements, so that no restrictions are imposed by (10) in order that the theory will be renormalizable. We have three constants in (10), namely, γ , α , and the field strength, which can be adjusted in such a way that one-loop divergences will be absorbed.

For the three-gluon vertex Γ_3 (see Fig. 2) the kind of tensors that we shall obtain is not so evident. With the requirement of dimensionality we can write Γ_3 as a linear combination of 108 objects such as $p_1^{\mu_1} p_1^{\mu_2} p_2^{\mu_3}$, $\omega_1 p_1^{\mu_1} \delta^{\mu_2 \mu_3}$, etc., with coefficients that are independent of the external frequencies and momenta. Fortunately, Γ_3 has to be symmetric under the interchange of the external legs and the

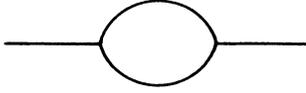


FIG. 1. One-loop diagram for the gluon self-energy.

total frequency and momentum must be conserved. Finally we find that there are only seven linear combinations of the 108 tensors, which are linearly independent and verify all the requirements. A possible choice for them is obtained by summing over all the permutations of the legs the product of the structure constant $f^{a_i a_j a_k}$ by the tensors:

$$\begin{aligned} & p_i^{\mu_j} p_i^{\mu_k} p_k^{\mu_i} ; p_i^{\mu_i} p_i^{\mu_j} p_j^{\mu_k} ; \\ & i\omega_i p_j^{\mu_j} \delta^{\mu_i \mu_k} ; i\omega_i p_j^{\mu_k} \delta^{\mu_i \mu_j} ; \\ & p_i^2 p_j^{\mu_j} \delta^{\mu_i \mu_k} ; p_i^2 p_j^{\mu_i} \delta^{\mu_j \mu_k} ; p_i^2 p_j^{\mu_k} \delta^{\mu_i \mu_j} \quad (i, j, k = 1, 2, 3) . \end{aligned} \quad (11)$$

The resulting expressions will be referred to as E_1, E_2, \dots, E_7 , respectively. $\Gamma_3^{(0)}$ in terms of these tensors reads

$$\Gamma_3^{(0)} = -i \frac{g}{2} \left[\frac{\gamma}{\alpha^2} E_2 - \frac{1}{\alpha} E_3 + \gamma (E_6 + 2E_7 - E_2) \right], \quad (12)$$

where we have omitted, for notational simplicity, the obvious dependence on p_i, ω_i, a_i , and μ_i ($i = 1, 2, 3$).

Contrary to the $\Gamma_2^{(0)}$ case, $\Gamma_3^{(0)}$ in (12) is a linear combination of three tensors, namely, E_2, E_3 , and $E_6 + 2E_7 - E_2$, while there was the possibility of seven different terms. This simple structure is imposed by the renormalizability of the theory, since we have only three adjustable parameters. This structure must be conserved up to every order in perturbation theory. These parameters could be in conflict with those coming from the renormalization of Γ_2 . We shall come back to this point later.

For brevity, we shall avoid a detailed exposition of the calculations involved in one-loop order. Nevertheless, we think it is interesting to comment on some general ideas. The computation of the divergent part of the contribution of a given diagram consists in the multiplication of propagators and vertices, followed by integrations in the internal frequencies and momenta. In order to carry out an efficient utilization of a computer algebra program, it is very interesting to work with only polynomials. In our case the integrand is always a rational function with a denominator factorized in terms such as $(\omega + iaq^2)$ where a is a constant. So it is possible to perform in a systematic way the ω integrations and also to use the Feynman pa-

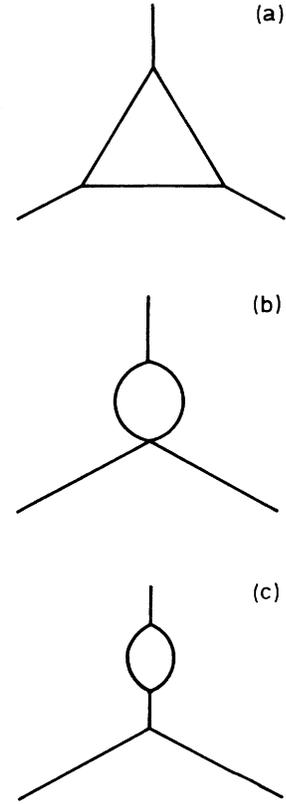


FIG. 2. One-loop diagrams for the triple-gluon vertex.

rametrization to obtain a denominator such as $[(q+b)^2 + c^2]^n$ with b and c constants. After the q integration, the result is just a polynomial in the Feynman parameters which are trivially integrated out. Although the procedure schematically presented above may seem straightforward, we must mention two sources of difficulties. The first one is related with the appearance of factors in the denominator which are not quadratic in the internal momentum. Fortunately this problem can be adequately treated with several methods. In general, the difficulties coming from the denominator increase slowly enough to allow the computation of complex diagrams. Surprisingly, in spite of being the integration process almost independent of the form of the numerator, its mere calculation is more involved. For this reason, the extensive use of (8) plays a fundamental role.

There is only one diagram (see Fig. 1) that contributes to the gluon self-energy to one-loop order. Its divergent part is

$$[\Gamma_2^{(1)}]_{\text{div}} = C \frac{\delta^{ab}}{2\gamma} \left[\frac{(6-\alpha)\gamma^2}{\alpha^2} p^2 p_\mu p_\nu + \frac{\alpha(5-\alpha)}{1+\alpha} \omega^2 \delta_{\mu\nu} + \frac{52+67\alpha-3\alpha^2}{3(1+\alpha)} \gamma^2 p^2 (p^2 \delta_{\mu\nu} - p_\mu p_\nu) \right], \quad (13)$$

where

$$C = \frac{N}{(4\pi)^2} \left[\frac{g}{2} \right]^2 \frac{1}{-\epsilon}$$

[for an $SU(N)$ gauge group].

There are two topologically different diagrams [Figs. 2(a) and 2(b)] that contribute to the three-gluon vertex to one-loop order. The result we found is

$$[\Gamma_3^{(1)}]_{\text{div}} = -i \frac{g}{2} C \left[\frac{3(-2-3\alpha+\alpha^2)}{2\alpha^2(1+\alpha)} \gamma E_2 + \frac{3(3-\alpha)}{2(1+\alpha)} E_3 + \frac{-86-131\alpha+9\alpha^2}{6(1+\alpha)} \gamma(E_6+2E_7-E_2) \right]. \quad (14)$$

As was to be expected, the structure of $[\Gamma_3^{(1)}]_{\text{div}}$ is similar to that of $\Gamma_3^{(0)}$, a fact that can be interpreted as a check of the renormalizability of the theory. Remarkably, this structure is only recuperated when the contributions of Figs. 2(a) and 2(b) are summed.

IV. RENORMALIZATION CONSTANTS AND WARD IDENTITIES

As we have shown in Eqs. (13) and (14), the one-loop-order contributions of the gluon self-energy and three-leg vertex diverge when $D \rightarrow 4$. As expected, the divergent parts have the same tensorial structure as in the tree approximation. Then it is possible to introduce the bare parameters in such a way that we end up with finite (renormalized) Green's functions. In the case of the Γ_3 vertex there are four renormalization constants to be introduced, $Z_A, Z_g, Z_\gamma, Z_\alpha$, while there are only three terms [see (14)] which diverge. On the other hand, we can introduce three (new) renormalization constants $Z'_A, Z'_\gamma, Z'_\alpha$ for the Γ_2 vertex (inverse propagator) that absorb the three divergent terms in $\Gamma_2^{(1)}$ [Eq. (13)]. It is the gauge invariance of the theory that makes equal, through the Ward identities, the renormalization constants needed for γ, A and α in Γ_2 and Γ_3 , respectively.

To be more specific, we have to determine the values of a set of constants Z_A, Z_g, Z_γ , and Z_α such that

$$Z_A^3 \left[-i \frac{g}{2} \right] Z_g \left[\frac{Z_\gamma}{Z_\alpha^2} \frac{\gamma}{\alpha^2} E_2 - \frac{1}{Z_\alpha \alpha} E_3 + Z_\gamma \gamma (E_6 - 2E_7 - E_2) \right] + [\Gamma_3^{(1)}]_{\text{div}} = [\Gamma_3^{(0)}]_{\text{bare}}, \quad (15)$$

$$Z_A^2 2\gamma Z_\gamma \delta^{ab} \left[\frac{Z_\gamma}{Z_\alpha^2} \frac{\gamma^2}{\alpha^2} p^2 p_\mu p_\nu + \omega^2 \delta_{\mu\nu} + Z_\gamma^2 \gamma^2 (p^2 \delta_{\mu\nu} - p_\mu p_\nu) \right] + [\Gamma_{2\mu\nu}^{(1)ab}]_{\text{div}} = [\Gamma_{2\mu\nu}^{(0)ab}]_{\text{bare}}. \quad (16)$$

The resultant system of equations is, of course, compatible and its solution reads

$$\begin{aligned} Z_A &= 1 + C \left[\frac{26 + 41\alpha - 3\alpha^2}{6(1+\alpha)} \right], \\ Z_g &= 1 + C \left(-\frac{22}{3} \right), \\ Z_\gamma &= 1 + C \left(\frac{26}{3} \right), \\ Z_\alpha &= 1 + C \left[\frac{17 + 26\alpha}{3(1+\alpha)} \right]. \end{aligned} \quad (17)$$

It is to be remarked that the gauge independence of Z_g and Z_γ , and consequently of the universal functions associated with these constants, was proven previously in Ref. 11.

A different but equivalent approach was used in Ref. 12. It consists in the addition of eight counterterms in the Lagrangian with their corresponding renormalization constants. They can be reduced to four independent quantities by using four Ward identities.

V. THE EQUILIBRIUM LIMIT

As has already been stated in the Introduction, the equal-time correlation functions or stochastic averages give in the EL the Euclidean Green's functions of the quantum field theory involved. However, it is possible to obtain the EL even for finite t (Ref. 13), if the initial condition for the stochastic equations is taken at $t \rightarrow -\infty$, as it has been assumed in deriving (1). So we simply have to make them equal time in order to get the corresponding EL. This amounts in the Fourier ω - p space to integrate out all of the external frequencies as can be easily shown.

For the propagator we have

$$[G_{\mu\nu}^{(1)ab}(p)]^{\text{eq}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} G_{\mu\rho}^{(0)ac}(p, \omega) \times \Gamma_{2\rho\sigma}^{(1)cd}(p, \omega) G_{\sigma\nu}^{(0)db}(p, \omega). \quad (18)$$

That is, we are not able to compare directly amputated Green's functions. This phenomenon has already been observed in Ref. 6 for a gauge theory with fermions. Nevertheless the equilibrium result may be factorized again as

$$[G_{\mu\nu}^{(1)ab}(p)]^{\text{eq}} = [G_{\mu\rho}^{(0)ac}]^{\text{eq}} [\Gamma_{2\rho\sigma}^{(1)cd}]^{\text{eq}} [G_{\sigma\nu}^{(0)db}]^{\text{eq}}. \quad (19)$$

The EL of the free propagator coincides with the FP result. Hence the EL of the divergent part contribution to the self-energy up to one-loop order may easily be obtained from (19) for $\alpha > 0$. However, in the singular limit $\alpha \rightarrow 0$, the equilibrium-free propagator is not invertible, and then it is better to directly compare complete Green's functions. After a straightforward application of (18) using Eqs. (6) and (13), one obtains

$$[G_{\mu\nu}^{(1)ab}(p)]_{\text{div}}^{\text{eq}} = C \frac{\delta^{ab}}{p^4} \left[\frac{26 + 41\alpha - 3\alpha^2}{3(1+\alpha)} (p^2 \delta_{\mu\nu} - p_\mu p_\nu) + (3\alpha + 5\alpha^2 - \alpha^3) p_\mu p_\nu \right] \quad (20a)$$

or

$$[G_{\mu\nu}^{(1)ab}(p)]_{\text{div}}^{\text{eq}} = C \frac{\delta^{ab}}{p^4} \left[\left(\frac{26}{3} + 5\alpha \right) (p^2 \delta_{\mu\nu} - p_\mu p_\nu) + 3\alpha p_\mu p_\nu + O(\alpha^2) \right]. \quad (20b)$$

If $\alpha=0$ this coincides with that found within the FP approach in the Landau gauge.¹⁴ We must point out that the stochastic gauge-fixing procedure is by no means equivalent to the usual one.¹⁵ Even for a small but not zero α [see Eq. (20b)], the differences are not a global variation of the α dependence, also a nontransversal part appears.

The next step is the calculation of the EL of the three-gluon vertex by adding the external lines and integrating over the two independent frequencies. One could naively expect, according to what we have learned in the propagator case, that this result would be equal to the FP one in the $\alpha \rightarrow 0$ and Landau gauges, respectively. In fact we are not allowed to compare the EL of 1PI diagrams in SQ with 1PI in FP. Instead what must be compared are the whole three-leg Green's function including no 1PI diagrams such as the one shown in Fig. 2(c).

The expression that we obtain for the complete one-loop-order divergent contribution to the amputated three-leg vertex is, in the $\alpha \rightarrow 0$ limit,

$$\lim_{\alpha \rightarrow 0} [\hat{\Gamma}_3^{(1)}]_{\text{div}}^{\text{eq}} = -i \frac{g}{2} C f^{a_1 a_2 a_3} [\delta^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + \delta^{\mu_2 \mu_3} (p_2 - p_3)^{\mu_1} + \delta^{\mu_3 \mu_1} (p_3 - p_1)^{\mu_2}] \left(-\frac{122}{3} \right). \quad (21)$$

Remarkably this result coincides with the corresponding one calculated in the FP approach in the Landau gauge. Because of the problems associated with the noninvertibility of $G^{(0)}$, $[\hat{\Gamma}_3]_{\text{div}}^{\text{eq}}$ is not uniquely determined from the nonamputated vertex in $\alpha=0$, but the expression in (21) is the only one that verifies the symmetry properties and is linear in the momenta. We do not write $[\hat{\Gamma}_3^{(1)}]_{\text{div}}^{\text{eq}}$ for an arbitrary α due to its complexity.

In conclusion, we realize, with one-loop calculations, that the relationship between both approaches, FP and SQ, is not trivial at all. From the existence in SQ of new vertices and the absence of ghost fields, it is clear that a direct comparison between individual graphs is not to be expected. Moreover there is not even such a correspondence between sets of 1PI diagrams but complete Green's

functions. This means that the way of quantizing and, in particular, the gauge-fixing procedure is very different.

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