Radiative correction to the equivalent-photon spectrum of a relativistic electron and the two-photon process

Martin Landrø, Kjell J. Mork, and Haakon A. Olsen
Institute of Physics, University of Trondheim, N-7055 Dragvoll, Norway
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On the basis of the improved Weizsäcker-Williams equivalent-photon spectrum of a fast charged particle, we calculate the radiative correction to this spectrum. The calculation involves the virtual-photon radiative correction, the soft- and hard-bremsstrahlung contributions, and the vacuum-polarization contribution. The results are used to calculate the radiative correction to two-photon production of neutral bosons and pair production of charged fermions and bosons. General features are that the elastic vertex radiative corrections to these processes are negative and large, up to 10–20 %, while the total radiative corrections, including emission of an extra hard photon, are on the other hand positive and in general considerably smaller, of the order of 1%. The effect of electron tagging on the radiative corrections is briefly discussed.

I. INTRODUCTION

The radiative correction to electron-induced processes at high energies has been studied[1,2] for inelastic electron scattering on nuclei involving nuclear excitation or particle production in the field of the nucleus, and more recently[3,4] for inelastic electron-electron or electron-positron scattering involving creation of leptons or hadrons.

In the present work we calculate the radiative correction to the virtual-photon spectrum and use the result to obtain the radiative correction to electron-induced processes, in particular, to (virtual) 2γ processes. The method of calculation is in principle related to the calculation of Kuo and Yennie[2] for inelastic electron-nucleus scattering and to the method used by Mork and Olsen[5] for calculation of the radiative correction to bremsstrahlung and pair production. Since it appears that the calculation is beset by rather heavy cancellations, a fact also demonstrated by previous work,[3,4] we use the improved Weizsäcker-Williams method[6] which on one hand gives cross sections to a high degree of accuracy as demonstrated previously[6] and, on the other hand, has a fairly simple mathematical structure so that calculations may be performed analytically. Because of the large cancellations mentioned above, it seems to us safe to perform the calculations analytically in order to obtain reliable results for the radiative corrections.

It should be emphasized that as for the cases of radiative corrections to inelastic electron-nucleon scattering[2] and to bremsstrahlung[5] it is the fractional radiative correction Δ in the radiatively corrected cross section $d\sigma = d\sigma_0(1+\Delta)$ which is calculated in the Weizsäcker-Williams approximation while $d\sigma_0$ is in general taken to be the best available numerical or analytical uncorrected cross section, not necessarily calculated in the Weizsäcker-Williams approximation.

The improved Weizsäcker-Williams method is briefly reviewed in Sec. II. The calculation of the virtual-photon radiative correction is given in Sec. III, and the real soft-photon and hard-photon contributions in Secs. IV and V, respectively. The vacuum-polarization contribution is obtained in Sec. VI and the elastic vertex and the total radiative corrections are presented in Secs. VII and VIII. Applications to two-photon processes are discussed in Secs. IX–XII, a general discussion in Sec. IX, followed by specific calculations of radiative corrections to neutral-boson production and charged-fermion and -boson pair production in Secs. X and XI. In Sec. XII a brief discussion of the effect of electron tagging on radiative corrections is given.

II. THE IMPROVED WEIZSÄCKER-WILLIAMS METHOD

Since our calculation of radiative corrections is strongly dependent on the method for improving the Weizsäcker-Williams method in particular in connection with the calculation of the hard-photon radiative correction in Sec. V, we review briefly the calculations of Ref. 6.

The cross section for the process pictured in Fig. 1 where a high-energy equivalent photon of momentum Q together with a high-energy photon of momentum k form a final-state system of momentum P is given by

$$d\sigma(p_1 k \to p_2 P) = \frac{\alpha}{2\pi^2}\int \frac{d^3p_2}{E_1E_2}\frac{1}{q^4}T_0^{\mu\nu}(p_1,Q) \times \tfrac{1}{4}M_\mu^\dagger M_\nu (2\pi)^4 \times \delta^4(p_1+k-P)\,d\Gamma \ , \tag{1}$$

where $q^2=-Q^2$ and where we have taken the vertex $\bar{u}_2\gamma^\mu u_1$ explicitly into account which gives the term

$$T_0^{\mu\nu} = 2p_1^\mu p_1^\nu - \tfrac{1}{2}q^2 g^{\mu\nu} \ . \tag{2}$$

M_μ is the matrix element for the process $Q+k\to P$, for an equivalent-photon polarization μ, and $d\Gamma$ is the invari-

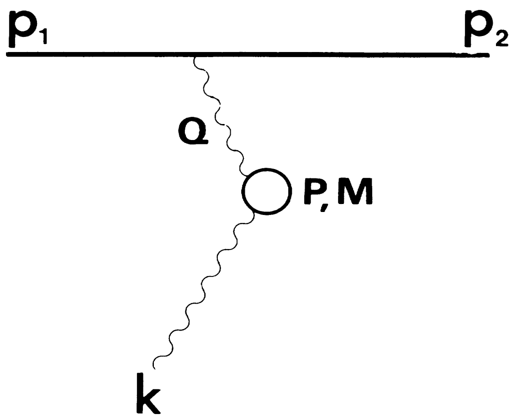

FIG. 1. Creation of a system with momentum P and mass M by one real and one equivalent (virtual) photon.

ant phase-space element for this process. When we neglect longitudinal and scalar equivalent photons, introduce new variables for p_2 so that $d^3p_2=(E_2/2E_1)dq^2dQ_0d\phi$, and perform the azimuthal-angle integration we obtain

$$d\sigma(p_1k\to p_2P)=\int d^2N_0(q^2,Q_0)d\sigma(Qk\to P) , \quad (3)$$

where

$$d^2N_0(q^2,Q_0)=\frac{\alpha\, dq^2dQ_0}{2\pi q^2Q_0}\times\left[\left[1+\frac{E_2{}^2}{E_1{}^2}\right]-\frac{2m^2Q_0}{E_1{}^2q^2}\right]f(q^2) \quad (4)$$

and

$$d\sigma(Qk\to P)=\frac{1}{8Q_0}\sum_{i=1}^{2}|M_i|^2(2\pi)^4\delta^4(p_1+k-P)d\Gamma .$$

The factor $f(q^2)$ in Eq. (4) is a propagator factor and is

$(1+q^2/M^2)^{-2}$ if P is a fermion pair ,

1 if P is a boson .

The equivalent-photon spectrum is obtained by integration over q^2:

$$dN_0(Q_0)=\int_{q_{\min}{}^2}^{q_{\max}{}^2}dq^2d^2N_0(q^2,Q_0)/dq^2 , \quad (5)$$

where the maximum and minimum momentum transfers for fixed equivalent-photon energy Q_0 are given by

$$q_{\max}{}^2=4E_1E_2,\quad q_{\min}{}^2=\frac{Q_0{}^2m^2}{E_1E_2} . \quad (6)$$

For a boson the q^2 integration is cut off at $q^2=M^2$. One finds[6]

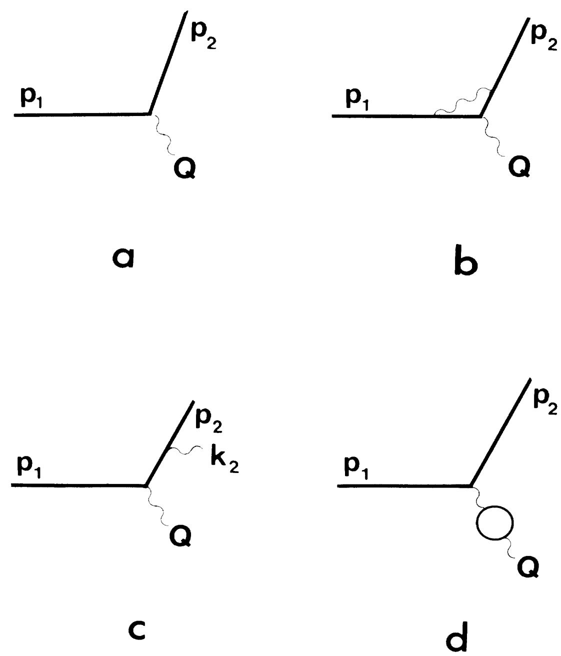

FIG. 2. Equivalent-photon diagrams, including virtual-photon correction diagram b, soft- and hard-real-photon diagram c, and vacuum-polarization diagram d.

$$dN_0(Q_0)=\frac{\alpha\, dQ_0}{2\pi Q_0}\left[\left[1+\frac{E_2{}^2}{E_1{}^2}\right]\left[\ln\frac{M^2}{q_{\min}{}^2}-f\right]-2\frac{E_2}{E_1}\right] , \quad (7)$$

where

$f=1$ if P is a fermion pair,

$f=0$ if P is a boson .

M is the invariant mass of the produced system $P^2=M^2$. The main contribution to the equivalent-photon spectrum occurs for equivalent-photon energies Q_0 which are considerably smaller than the electron energy. We shall accordingly make the approximation $Q_0\ll E_1$ throughout the paper.

III. THE VIRTUAL-PHOTON RADIATIVE CORRECTION

The radiative correction due to the virtual-photon diagram in Fig. 2(b) is obtained in the same way as N_0. The virtual-photon vertex is given by[7]

$$\bar u_2\gamma_\mu u_1F_1(q^2)+\frac{i}{2m}\bar u_2\sigma_{\mu\nu}u_1Q^\nu F_2(q^2) . \quad (8)$$

Here the fermion charge form factor is

$$F_1(q^2)=\frac{\alpha}{2\pi}\left[2\left[1+\frac{1+\xi^2}{1-\xi^2}\ln\xi\right]\ln\frac{m}{\lambda}-2-\frac{3(1+\xi^2)+2\xi}{2(1-\xi^2)}\ln\xi+\frac{1+\xi^2}{1-\xi^2}\left[\frac{\pi^2}{6}-\frac{1}{2}\ln^2\xi+2\ln(1+\xi)\ln\xi+2L_2(-\xi)\right]\right] , \quad (9)$$

where λ is the virtual-photon infrared cutoff and ξ is related to q^2 by

$$q^2/m^2=(1-\xi)^2/\xi \ , \tag{10}$$

and $L_2(z)$ is the Euler dilogarithm

$$L_2(z)=-\int_0^z \frac{\ln(1-x)}{x}dx \ . \tag{11}$$

The anomalous-magnetic-moment form factor is given by

$$F_2(q^2)=-\frac{\alpha\xi}{\pi(1-\xi^2)}\ln\xi \ . \tag{12}$$

As shown in Appendix A the contribution from the anomalous-magnetic-moment form factor is negligible and the virtual-photon radiative correction is obtained directly as

$$d^2N_{\rm vir}(q^2,Q_0)=2\,d^2N_0(q^2,Q_0)F_1(q^2) \tag{13}$$

which is the correction to the spectrum Eq. (4). The correction to the virtual-photon spectrum Eq. (7) is obtained by integrating Eq. (13),

$$dN_{\rm vir}(Q_0)=\int_{q_{\rm min}^2}^{q_{\rm max}^2} dq^2 d^2N_{\rm vir}(q^2,Q_0)/dq^2$$

with the result given in Appendix B, which becomes, for $Q_0=E_1-E_2\ll E_1$,

$$dN_{\rm vir}(Q_0)=\frac{\alpha^2}{2\pi^2}\frac{dQ_0}{Q_0}\left[\left[-8\ln^2(M/m)+16\ln(M/m)-\frac{2\pi^2}{3}-12\right]\ln\frac{m}{\lambda}-\frac{8}{3}\ln^3(M/m)+10\ln^2(M/m)\right.$$
$$\left.-14\ln(M/m)+8L_3(-1)+\frac{\pi^2}{6}+12\right] \tag{14}$$

with $L_3(z)$ the Euler trilogarithm, $L_3(-1)=0.9015$. It should be noted that this spectrum has the simple $1/Q_0$ dependence, which will be a general feature of the radiative correction spectra.

IV. THE SOFT-PHOTON RADIATIVE CORRECTION

The radiative correction due to the emission of a real soft bremsstrahlung photon Fig. 2(c) may be written down, from the work of Mork and Olsen,[5] as

$$d^2N_{\rm soft}(q^2,Q_0)=d^2N_0(q^2,Q_0)I(q^2,\Delta\omega_2) \ , \tag{15}$$

where $I(q^2,\Delta\omega_2)$ is the soft-photon bremsstrahlung contribution:

$$I(q^2,\Delta\omega_2)=\frac{\alpha}{4\pi^2}\int_{\omega_2<\Delta\omega_2}\frac{d^3k_2}{\omega_2}\left[\frac{p_1}{p_1k_2}-\frac{p_2}{p_2k_2}\right]^2$$
$$=-\frac{\alpha}{\pi}\left\{2\left[1+\frac{1+\xi^2}{1-\xi^2}\ln\xi\right]\ln\frac{\Delta\omega_2 m}{\lambda E_1}\right.$$
$$+\frac{1}{2}\frac{(1+\xi^2)}{(1-\xi^2)}\left[2\ln\xi\ln(1-\xi^2)-\ln^2\xi-\ln\left[\frac{E_2}{E_1}\xi\right]\ln\left[1-\frac{E_2}{E_1}\xi\right]\right.$$
$$+\ln\frac{E_2}{E_1\xi}\ln\left[\frac{E_2}{E_1}-\xi\right]+\ln\xi\ln\frac{E_2}{E_1}+L_2\left[\frac{E_1-E_2\xi}{E_1(1-\xi^2)}\right]$$
$$\left.\left.+L_2\left[\frac{E_2-E_1\xi}{E_2(1-\xi^2)}\right]-L_2\left[\frac{\xi(E_2-E_1\xi)}{E_1(1-\xi^2)}\right]-L_2\left[\frac{\xi(E_1-E_2\xi)}{E_2(1-\xi^2)}\right]\right]\right\} \tag{16}$$

as shown in Ref. 5. The soft-photon radiative correction to the equivalent-photon spectrum is obtained in Appendix B for $Q_0\ll E_1$:

$$dN_{\rm soft}(Q_0)=-\frac{\alpha^2}{2\pi^2}\frac{dQ_0}{Q_0}\left[\left[-8\ln^2(M/m)+16\ln(M/m)-\frac{2\pi^2}{3}-12\right]\ln\frac{\Delta\omega_2 m}{\lambda E_1}-\frac{8}{3}\ln^3(M/m)\right.$$
$$\left.+4\ln^2(M/m)+8L_3(-1)-\frac{\pi^2}{3}\right] \ . \tag{17}$$

It is apparent from Eqs. (14) and (17) that the infrared cutoff λ drops out when the virtual and soft photons are added together, as it should.

V. THE HARD-PHOTON RADIATIVE CORRECTION

The calculation of the hard-bremsstrahlung contribution Fig. 2(c) to the radiative correction is the most complicated part of the calculation. The procedure is as in Sec. II. The cross section is again given by an expression of the form Eq. (1):

$$d\sigma(p_1k\to p_2P)=\frac{\alpha}{2\pi^2}\int\frac{d^3p_2}{E_1E_2}\frac{d^3k_2}{q^4}T^{\mu\nu}(p_1,Q,k_2)\tfrac{1}{4}M_\mu^\dagger M_\nu(2\pi)^4\delta^4(p_1+k-P)d\Gamma\ . \tag{18}$$

The tensor $T^{\mu\nu}$ is now, however, the complicated Compton-type term

$$T^{\mu\nu}(p_1,Q,k_2)=\frac{e^2}{(2\pi)^3}\left[\frac{1}{(p_1k_2)(p_2k_2)}[p_1p_2T_0^{\mu\nu}(p_1,Q)-m^2k_2^\mu k_2^\nu]+\frac{1}{p_2k_2}(2p_1^\mu p_1^\nu-p_1k_2g^{\mu\nu})\right.$$
$$\left.-\frac{m^2}{(p_2k_2)^2}T_0^{\mu\nu}(p_1,Q)+(p_1\leftrightarrow -p_2)\right]\ ,$$

where $T_0^{\mu\nu}$ is given by Eq. (2). It is nice to notice that in the soft-energy limit of k_2 we obtain the correct low-photon-energy bremsstrahlung limit,

$$T^{\mu\nu}(p_1,Q,k_2)M_\mu^\dagger M_\nu=\frac{-e^2}{(2\pi)^3}\left[\frac{p_1}{p_1k_2}-\frac{p_2}{p_2k_2}\right]^2T_0^{\mu\nu}(p_1,Q)M_\mu^\dagger M_\nu\ ,$$

since in the limit $k_2=0$, $T_0^{\mu\nu}(p_1,Q)M_\mu^\dagger M_\nu=T_0^{\mu\nu}(p_2,Q)M_\mu^\dagger M_\nu$ because of gauge invariance, $Q^\mu M_\mu=0$.

As in Sec. II we retain only transverse equivalent photons, introduce new variables q^2 and Q_0 in Eq. (18), and perform the azimuthal-angle integrations with the result

$$\int d\phi\, T^{ij}(p_1,Q,k_2)M_i^\dagger M_j=\pi\frac{e^2}{(2\pi)^3}\left[\left[\frac{p_1p_2}{(p_1k_2)(p_2k_2)}-\frac{m^2}{(p_2k_2)^2}\right](2\mathbf{p}_{1\perp}^2+q^2)-\frac{m^2\mathbf{k}_{2\perp}}{(p_1k_2)(p_2k_2)}\right.$$
$$\left.+\frac{2}{p_2k_2}(\mathbf{p}_{1\perp}^2+\mathbf{p}_{2\perp}^2+p_1k_2)+(p_1\leftrightarrow -p_2)\right]\sum_{i=1}^{2}|M_i|^2\ , \tag{19}$$

where $\mathbf{p}_{1\perp}$ and $\mathbf{k}_{2\perp}$ are the components of $\mathbf{p}_1$ and $\mathbf{k}_2$ perpendicular to Q, respectively. As shown in Appendix C the cross section is given by

$$d\sigma(p_1k\to p_2P)=\int d^2N_{\text{hard}}(p_1,Q,\Delta\omega_2)d\sigma(Qk\to P)\ ,$$

where d^2N_{hard} may be written as

$$d^2N_{\text{hard}}(q^2,Q_0,\Delta\omega_2)=\frac{\alpha}{\pi}d^2N_0(q^2,Q_0)\left[2+2\left[1+\frac{1+\xi^2}{1-\xi^2}\ln\xi\right]\ln\frac{\Delta\omega_2}{E_1}+\frac{3(1+\xi^2)+2\xi}{2(1-\xi^2)}\ln\xi\right]\ . \tag{20}$$

Written in this form it is apparent that the dependence on $\Delta\omega_2$ is of the same form as in d^2N_{soft}, Eqs. (15) and (16), so that the dependence of $\Delta\omega_2$ drops out when the soft- and hard-photon contributions are added together, as it should.

The correction to the equivalent-photon spectrum is obtained in Appendix C:

$$dN_{\text{hard}}(Q_0,\Delta\omega_2)=\frac{\alpha^2dQ_0}{2\pi^2Q_0}\left[\left[8\ln^2(M/m)-16\ln(M/m)+\frac{2\pi^2}{3}+12\right]\ln\frac{E_1}{\Delta\omega_2}-6\ln^2(M/m)+14\ln(M/m)-\frac{\pi^2}{2}-12\right]\ . \tag{21}$$

VI. THE VACUUM-POLARIZATION RADIATIVE CORRECTION

The radiative correction due to vacuum-polarization effects may again be written down from Ref. 5. It is given by

$$d^2N_{\text{vac}}(q^2,Q_0)=-2\,d^2N_0(q^2,Q_0)\pi_f(q^2)q^2\ , \tag{22}$$

where $\pi_f(q^2)$ is the renormalized vacuum-polarization term:

$$\pi_f(q^2)=\frac{\alpha}{3\pi q^2}\left[\frac{5}{3}-\frac{4\xi}{(1-\xi)^2}\right.$$
$$\left.+\left[1-\frac{2\xi}{(1-\xi)^2}\right]\frac{1+\xi}{1-\xi}\ln\xi\right]\ , \tag{23}$$

where ξ is defined in Eq. (10).

The radiative correction to the spectrum is obtained in

Appendix B which becomes, for $Q_0 \ll E_1$,

$$dN_{\rm vac}(Q_0)=\frac{\alpha^2}{3\pi^2}\frac{dQ_0}{Q_0}\times\left[4\ln^2(M/m)-\tfrac{32}{3}\ln(M/m)+\frac{\pi^2}{3}+\frac{86}{9}\right] \quad (24)$$

which shows that the vacuum polarization increases the number of soft ($Q_0 \ll E_1$) equivalent photons of the fast-moving electron.

VII. THE ELASTIC VERTEX RADIATIVE CORRECTION

When conditions are such that $\Delta\omega_2$, the energy of the emitted photon with momentum k_2, can be kept small $\Delta\omega_2 \ll E_1$, the vertex is essentially elastic $E_1=E_2+\omega_2$, and the radiative correction is given by

$$\begin{aligned} d^2N_{\rm el\ corr}(q^2,Q_0,\Delta\omega_2) &= d^2N_{\rm vir}(q^2,Q_0)+d^2N_{\rm soft}(q^2,Q_0,\Delta\omega_2)+d^2N_{\rm vac}(q^2,Q_0) \\ &= \frac{\alpha}{\pi}d^2N_0(q^2,Q_0)\left\{2\left[1+\frac{1+\xi^2}{1-\xi^2}\ln\xi\right]\ln\frac{E_1}{\Delta\omega_2}-2-\frac{3(1+\xi^2)+2\xi}{2(1-\xi^2)}\ln\xi\right. \\ &\quad \left.-\frac{2}{3}\left[\frac{5}{3}-\frac{4\xi}{(1-\xi)^2}+\left[1-\frac{2\xi}{(1-\xi)^2}\right]\frac{1+\xi}{1-\xi}\ln\xi\right]\right\} . \end{aligned} \quad (25)$$

The correction to the equivalent-photon spectrum is correspondingly given by

$$dN_{el\ \rm corr}(Q_0,\Delta\omega_2)=\frac{\alpha^2}{2\pi^2}\frac{dQ_0}{Q_0}\left[\left[8\ln^2(M/m)-16\ln(M/m)+\frac{2\pi^2}{3}+12\right]\ln\frac{\Delta\omega_2}{E_1}+\tfrac{26}{3}\ln^2(M/m)-\tfrac{190}{9}\ln(M/m)+\tfrac{13}{18}\pi^2+\tfrac{496}{27}\right] . \quad (26)$$

VIII. THE TOTAL RADIATIVE CORRECTION

The total radiative correction to equivalent-photon emission from an electron vertex is obtained by adding the contribution from the hard-photon emission to the elastic radiative correction Eq. (25):

$$d^2N_{\rm tot\ corr}(q^2,Q_0)=d^2N_{\rm el\ corr}(q^2,Q_0,\Delta\omega_2)+d^2N_{\rm hard}(q^2,Q_0,\Delta\omega_2) .$$

It is then apparent from Eqs. (13), (16), and (20) that the contributions to the radiative correction from virtual and hard and soft real photons cancel exactly

$$d^2N_{\rm vir}(q^2,Q_0)+d^2N_{\rm soft}(q^2,Q_0)+d^2N_{\rm hard}(q^2,Q_0)=0 , \quad (27)$$

and we obtain the remarkable result that the total radiative correction is given by the vacuum-polarization contribution alone:

$$d^2N_{\rm tot\ corr}(q^2,Q_0)=-\frac{2\alpha}{3\pi}d^2N_0(q^2,Q_0)\left[\frac{5}{3}-\frac{4\xi}{(1-\xi)^2}+\left[1-\frac{2\xi}{(1-\xi)^2}\right]\frac{1+\xi}{1-\xi}\ln\xi\right] . \quad (28)$$

For the radiative correction to the equivalent-photon spectrum which is the most useful quantity for applications, we find, from Eq. (24),

$$dN_{\rm tot\ corr}(Q_0)=\frac{\alpha^2}{3\pi^2}\frac{dQ_0}{Q_0}\left[4\ln^2(M/m)-\tfrac{32}{3}\ln(M/m)+\frac{\pi^2}{3}+\frac{86}{9}\right] . \quad (29)$$

IX. APPLICATIONS: RADIATIVE CORRECTIONS TO 2γ PROCESSES

We discuss here the application of our theoretical results to processes involving two equivalent photons. The production of a system of particles X in electron-electron collisions or electron-positron collisions,

$$ee \to ee\gamma\gamma \to eeX ,$$

is in most cases of experimental interest well described by the creation of X by two quasireal, equivalent photons. When we include the radiative correction the cross section is given by

$$\begin{aligned} d\sigma(p_1p'_1\to p_2p'_2P_X) = \int &[dN_0(Q_0^1)+dN_{\rm corr}(Q_0^1)] \\ &\times[dN_0(Q_0{}^2)+dN_{\rm corr}(Q_0{}^2)] \\ &\times d\sigma(Q^1Q^2\to P_X) , \end{aligned} \quad (30)$$

where we have assumed head-on collisions of the equivalent photons with momenta Q^1 and Q^2 producing X. If desired the angular distribution of the equivalent photons may be taken into account by the use of $d^2N_0(q^2,Q_0)$, $d^2N_{corr}(q^2,Q_0)$ replacing $dN_0(Q_0)$, $dN_{corr}(Q_0)$ in Eq. (30). The radiative corrections dN_{corr} and d^2N_{corr} are given in Secs. VI and VII for elastic vertex and total radiative corrections, respectively. The cross section $d\sigma(Q^1Q^2 \to P_X)$ is the physical cross section involving real photons including radiative corrections to the process $\gamma\gamma \to X$. To relative order α we have

$$d\sigma(p_1p_1' \to p_2p_2'P_X) = d\sigma_0(p_1,p_1' \to p_2p_2'P_X) + d\sigma_{corr}(p_1p_1' \to p_2p_2'P_X) ,$$

where $d\sigma_0$ is the lowest-order cross section:

$$d\sigma_0(p_1p_1' \to p_2p_2'P_X) = \int dN_0(Q_0^1)dN_0(Q_0{}^2) \times d\sigma(Q^1Q^2 \to P_X) \qquad (31)$$

with $dN_0(Q_0^1)$ given by Eq. (7), and $d\sigma_{corr}$ the radiative correction to relative order α,

$$d\sigma_{corr}(p_1p_1' \to p_2p_2'P_X) = 2\int dN_0(Q_0^1)dN_{corr}(Q_0{}^2) \times d\sigma(Q^1Q^2 \to P_X) , \qquad (32)$$

where $dN_{corr}(Q_0{}^2)$ is given for elastic vertex correction by Eq. (26) and for total correction by Eq. (29).

X. RADIATIVE CORRECTIONS TO NEUTRAL-BOSON PRODUCTION IN 2γ PROCESSES

This process was discussed in a previous publication,[8] and we give only a brief outline of the calculation. The radiative correction to the total cross section is from Eq. (32) given by

$$\sigma_{corr}(p_1p_1' \to p_2p_2'P_{B^0}) = 2\int dN_0(Q_0{}^1)dN_{tot\ corr}(Q_0{}^2) \times \sigma(Q^1Q^2 \to P_{B^0}) , \qquad (33)$$

where we have assumed that the neutral boson B^0 with momentum P_{B^0} is recorded irrespective of the energy of a hard photon emitted in the untagged process. The physical 2γ creation cross section is

$$\sigma(Q^1Q^2 \to P_{B^0}) = (2J+1)\frac{8\pi^2}{M}\Gamma_{B^0\to\gamma\gamma}\delta(M^2-m_{B^0}{}^2) \qquad (34)$$

with J, m_B, and Γ the spin, mass, and the 2γ decay width of the boson. With $dN_0(Q_0^1)$, Eq. (7), and $dN_{tot\ corr}(Q_0{}^2)$, Eq. (29) inserted in Eq. (33), the integration is simple with the result, which may be written as

$$\sigma(p_1p_1' \to p_2p_2'P_{B^0}) = \sigma_0(p_1p_1' \to p_2p_2'P_{B^0})(1+\Delta_{tot}) , \qquad (35)$$

where the total radiative correction Δ_{tot} is given by

$$\Delta_{tot} = (2\alpha/3\pi)I/I_0$$

with

$$I = \left[l_2{}^2 - \frac{16}{3}l_2 + \frac{\pi^2}{3} + \frac{86}{9} \right] \times \left[l_1{}^2 + l_1l_2 - \frac{3}{4}l_2 - l_1 - \frac{\pi^2}{6} - \frac{1}{8} \right]$$

and[6]

$$I_0 = \tfrac{2}{3}l_1{}^3 + 2l_1{}^2l_2 + l_1l_2{}^2 - 2l_1{}^2 - 5l_1l_2 - \tfrac{3}{2}l_2{}^2 + \left[\frac{1}{2} - \frac{2\pi^2}{3} \right] l_1 + \left[\frac{19}{4} - \frac{\pi^2}{3} \right] l_2 + \tfrac{21}{4} + \tfrac{5}{6}\pi^2 + 4L_3(1) ,$$

where $l_1 = \ln(4E_1{}^2/m_B{}^2)$, $l_2 = \ln(m_B{}^2/m^2)$, and $L_3(x)$ is the Euler trilogarithm.

As pointed out in Ref. 8, the radiative correction implies that the boson decay width $\Gamma^0_{B^0\to\gamma\gamma}$ inferred from ee collision experiments differs from the real boson decay width $\Gamma_{B^0\to\gamma\gamma}$:

$$\Gamma_{B^0\to\gamma\gamma} = \Gamma_{B^0\to\gamma\gamma}(1-\Delta_{tot}) . \qquad (36)$$

The total radiative correction Δ_{tot} is small and positive, it is given in Fig. 3 for π^0, $\eta'(958)$, and $f^0(1270)$ mesons.

For completeness we also consider the elastic vertex correction. The radiative correction to the cross section is again given by Eq. (33) with $dN_{tot\ corr}(Q_0{}^2)$ replaced by

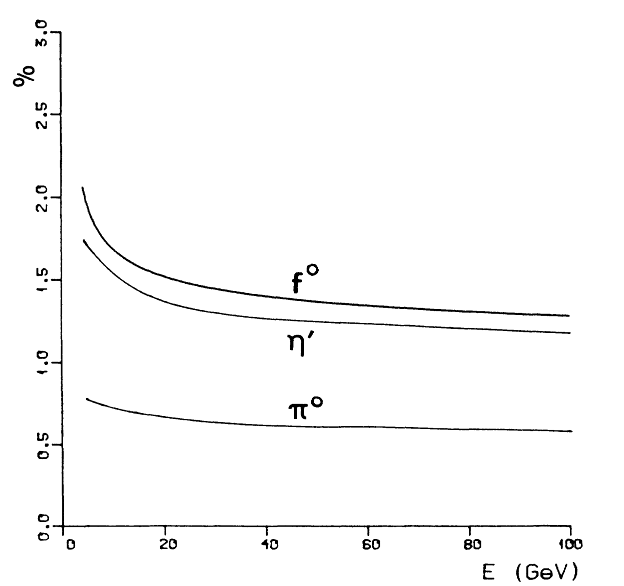

FIG. 3. The radiative correction Δ_{tot} for π^0, $\eta'(958)$, and $f^0(1270)$ mesons.

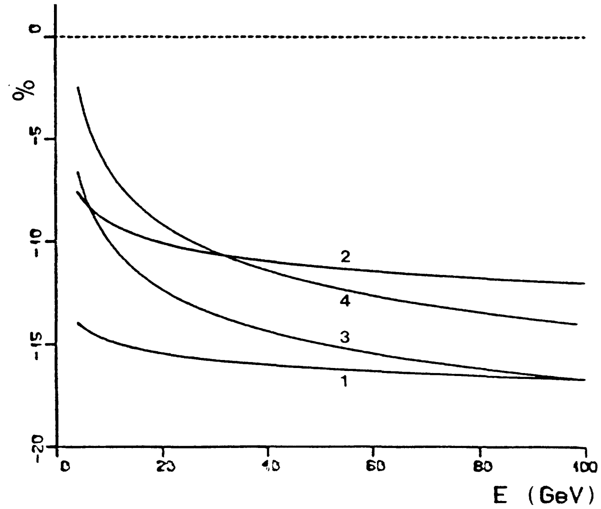

FIG. 4. The elastic (soft-) photon radiative correction $\Delta_{el}(\Delta\omega_2)$ for π^0 and $\eta'(958)$ for various values of $\Delta\omega_2$. Curves 1 and 2: π^0 and $\Delta\omega_2=10$ MeV and 100 MeV, respectively. Curves 3 and 4: η' and $\Delta\omega_2=500$ MeV and 1 GeV, respectively.

$dN_{el\ corr}(Q_0{}^2)$. Curves for $\Delta_{el}(\Delta\omega_2)$ defined in the same way as Δ_{tot} in Eq. (35) are given for π^0 and η' in Fig. 4. In contrast to Δ_{tot},Δ_{el} is negative and numerically much larger.

XI. RADIATIVE CORRECTIONS TO FERMION AND BOSON PAIR PRODUCTION

We calculate in this section the pair production of fermions and bosons including the effect of equivalent-photon radiative corrections. The intrinsic radiative corrections to the photoproduction process $\gamma\gamma\rightarrow$pair particles which must be included in order to obtain the complete radiative correction, is not taken into account in the present calculation.

The radiative-correction cross section is given by Eq. (32) with the 2γ cross section replaced by $\sigma(Q^1Q^2\rightarrow p_+p_-)$, where p_+ and p_- are the momenta of the produced pair particles. For the production of a fermion pair the total cross section is the Dirac cross section[9]

$$\sigma_0(Q^1Q^2\rightarrow p_+p_-)=\frac{\pi\alpha^2}{m_2{}^2}\tau\left\{(2+2\tau-\tau^2)\ln\frac{1+\sqrt{1-\tau}}{\sqrt{\tau}}-(1+\tau)\sqrt{1-\tau}\right\}, \quad (37)$$

where $\tau=4m_2{}^2/M^2$ with

$$M^2=(p_++p_-)^2=(Q_1+Q_2)^2=4Q_0^1Q_0{}^2$$

and m_2 the fermion mass. For a boson pair the cross section is, correspondingly,[10]

$$\sigma_0(Q^1Q^2\rightarrow p_+p_-)=\frac{\pi\alpha^2}{2m_2{}^2}\tau\left\{(\tau^2-2\tau)\ln\frac{1+\sqrt{1-\tau}}{\sqrt{\tau}}+(1+\tau)\sqrt{1-\tau}\right\} \quad (38)$$

with τ given as above with m_2 the boson mass. For simplicity we have neglected any form factor in Eqs. (36) and (37).

The total radiative correction for fermion or boson pair production is given by

$$\Delta_{tot}=\frac{\sigma_{corr}(p_1p_1'\rightarrow p_2p_2'p_+p_-)}{\sigma_0(p_1p_2'\rightarrow p_2p_2'p_+p_-)},$$

where[11]

TABLE I. Coefficients I_n in Eq. (39) for fermion and boson pair production.

Integral	Fermion production	Boson production
I_0	$\frac{14}{9}$	$\frac{4}{9}$
I_1	$\frac{28}{9}\ln2-\frac{43}{9}$	$\frac{8}{9}\ln2-\frac{11}{9}$
I_2	7.16	1.47
I_3	−27.11	−5.00

$$\begin{aligned}\sigma_{corr}=\frac{\alpha^5}{\pi^2m_2{}^2}\Bigg\{&\left[2l_E\bar{l}_2+2l_E{}^2-\frac{3}{2}\bar{l}_2-2l_E-\frac{\pi^2}{3}-\frac{1}{4}\right]\left[\left[\frac{1}{3}\bar{l}_2{}^2-\frac{16}{9}\bar{l}_2+\frac{\pi^2}{9}+\frac{86}{27}\right]I_0-(\tfrac{2}{3}\bar{l}_2-\tfrac{16}{9})I_1\right]\\&+\left[\frac{1}{3}\bar{l}_2^2-\frac{16}{9}\bar{l}_2+\frac{\pi^2}{9}+\frac{86}{27}\right][2(l_E+\bar{l}_2)-\tfrac{1}{2}]I_1\\&+\left[\frac{2}{3}l_E{}^2-\frac{4}{3}\bar{l}_2{}^2-\frac{2}{3}l_E\bar{l}_2+\frac{61}{18}\bar{l}_2+\frac{26}{9}l_E-\frac{35}{36}-\frac{\pi^2}{9}\right]I_2+\tfrac{1}{3}[2(l_E+\bar{l}_2)-\tfrac{1}{2}]I_3\Bigg\}\end{aligned} \quad (39)$$

with $l_E=\ln(E_1{}^2/m_2{}^2)$ and $\bar{l}_2=\ln(4m_2{}^2/m^2)$ and where I_n is given in Table I for fermion and boson pair production.

The uncorrected cross section is for fermion pair production given by[6]

$$\begin{aligned}\sigma_0(p_1p_1'\rightarrow p_2p_2'p_+p_-)=\frac{\alpha^4}{9\pi m_2{}^2}[&\tfrac{28}{3}l_E{}^3+28l_E{}^2\bar{l}_2+14\bar{l}_2{}^2l_E-56l_E{}^2+(28\ln2-64)\bar{l}_2{}^2\\&-(184-56\ln2)l_E\bar{l}_2-42.7l_E+156.8\bar{l}_2+498.4]\ .\end{aligned} \quad (40)$$

TABLE II. The radiative correction Δ (in %) to neutral-boson production and charged-fermion and -boson pair production in untagged 2γ processes.

Electron energy	5 GeV	15 GeV	70 GeV	100 GeV
$ee \to ee\pi^0$	0.78	0.70	0.61	0.59
$ee \to ee\eta'$	1.73	1.44	1.22	1.18
$ee \to eef^0$	1.96	1.58	1.32	1.28
$ee \to ee\mu^+\mu^-$	1.17	1.05	0.92	0.90
$ee \to ee\pi^+\pi^-$	1.22	1.08	0.94	0.92
$ee \to ee\pi^+\pi^-$ (Ref. 4)	1.04 ± 0.6	0.70 ± 0.61	0.72 ± 0.48	

The corresponding cross section for boson pair production is found to be given by

$$\sigma_0(p_1p_1'\to p_2p_2'p_+p_-)=\frac{\alpha^4}{18\pi m_2{}^2}[\tfrac{8}{3}l_E{}^3+8l_E{}^2\bar{l}_2+4\bar{l}_2{}^2l_E-8l_E{}^2+(8\ln 2-17)\bar{l}_2{}^2$$

$$+(16\ln 2-42)l_E\bar{l}_2-43.1l_E+16.7\bar{l}_2+149.3]\ . \tag{41}$$

Values for Δ_{tot} for pair production of $\mu^+\mu^-$ and $\pi^+\pi^-$ are given in Table II. We include in the table the numerical results of Defrise[4] for $\pi^+\pi^-$ production. Our results which contain rather small uncertainties, of the order $(m_2/E_1)^2$ as discussed in Ref. 8, agree to some extent with Defrise's results within his rather large limits of uncertainties.

XII. SINGLE-TAG RESONANCE PRODUCTION

The calculations of the radiative corrections in Secs. IX and X are based on the full range of angles and momentum transfers. In experiments, however, tagging of secondary electrons are often used. Our theory is easily adapted to single- or double-tag situations. As an example we discuss briefly neutral-boson production in a single-tag experiment. We assume that the momentum transfer of one of the electrons is confined by the maximum and minimum acceptance angles θ_{max} and θ_{min} by

$$q_{min}{}^2<q^2<q_{max}{}^2\ , \tag{42}$$

where now $(q_{min,max})^2=E_1E_2(\theta_{min,max})^2$ and where the angles satisfy $m^2\ll E_1E_2\theta^2\ll m_{B^0}{}^2$. The modified equivalent-photon spectra are clearly obtained by replacing the full integration over q^2 by an integration over the confined region. From Eq. (7) we find the modified spectrum

$$dN_0^{tag}(Q_0)=dN_0(Q_0,q_{max}{}^2)-dN_0(Q_0,q_{min}{}^2)$$

$$=\frac{\alpha}{\pi}\frac{dQ_0}{Q_0}\left[1+\frac{E_2{}^2}{E_1{}^2}\right]\ln(q_{max}/q_{min}) \tag{43}$$

and the corresponding total radiative correction from Eq. (29):

$$dN_{tot\,corr}^{tag}(Q_0)=\frac{4\alpha^2}{3\pi^2}\frac{dQ_0}{Q_0}[\ln^2(q_{max}/m)-\tfrac{8}{3}\ln(q_{max}/m)-\ln^2(q_{min}/m)+\tfrac{8}{3}\ln(q_{min}/m)]$$

$$=\frac{4\alpha^2}{3\pi^2}\frac{dQ_0}{Q_0}\ln(q_{max}/q_{min})\left[\ln\left[\frac{q_{max}q_{min}}{m^2}\right]-\frac{8}{3}\right]\ . \tag{44}$$

For single tag we find, instead of Eq. (33),

$$\sigma_{corr}^{tag}(p_1p_1'\to p_2p_2'P_{B^0})=\int[dN_0^{tag}(Q_0^1)dN_{tot\,corr}(Q_0{}^2)+dN_0(Q_0{}^2)dN_{tot\,corr}^{tag}(Q_0^1)]\sigma(Q^1Q^2\to P_{B^0}) \tag{45}$$

which gives

$$\Delta_{tot,tag}=\frac{2\alpha}{3\pi}\left[\ln\left[\frac{q_{max}q_{min}}{m^2}\right]-\frac{8}{3}+\frac{1}{2}f(m_{B^0}{}^2/m^2)\right] \tag{46}$$

with

$$f(m_{B^0}{}^2/m^2)=\frac{l_2{}^2-\frac{16}{3}l_2+\frac{\pi^2}{3}+\frac{86}{9}}{l_2+l_1-1}$$

with l_1 and l_2 as given in Sec. IX:

$$l_1=\ln(4E_1{}^2/m_{B^0}{}^2),\quad l_2=\ln(m_{B^0}{}^2/m^2)\ .$$

As an example we compute the radiative correction for f^0 production in the single-tag mode of the Pluto detector.[12] Here $\theta_{\min}=23$ mrad and $\theta_{\max}=70$ mrad. With $E_1=15.5$ GeV we find

$$\Delta_{\text{tot,tag}}(f^0)=2.4\% \ ,$$

to be compared with an untagged radiative correction at the same energy $\Delta_{\text{tot}}(f^0)=1.6\%$ from Table II. In general it is reasonable to assume that tagging will increase the radiative corrections.

APPENDIX A

The radiative correction due to the anomalous magnetic part of the virtual-photon vertex Eq. (8) is obtained as in Eq. (1) with $T_0^{\mu\nu}$ replaced by

$$\begin{aligned} T_{\text{magn}}^{\mu\nu}&=\text{Tr}\left[(\gamma p_2+m)\gamma^\mu(\gamma p_1+m)\sigma^{\nu\alpha}\frac{i}{2m}Q_\alpha F_2(q^2)\right.\\ &\qquad\left.-(p_1\leftrightarrow p_2,\nu\leftrightarrow\mu)\right]\\ &=-4q^2g^{\mu\nu}F_2(q^2)\ , \end{aligned} \tag{A1}$$

where we have left out terms proportional to Q^ν which vanish by gauge invariance when multiplied with M_ν. This gives

$$d^2N_{\text{magn}}(q^2,Q_0)=\frac{\alpha\, dq^2dQ_0Q_0^2}{\pi q^2Q_0E_1^2}F_2(q^2)\left[1+\frac{q^2}{M^2}\right]^{-2} \tag{A2}$$

which is of the order Q_0^2/E_1^2 relative to $dN_{\text{vir}}^2\ (q^2,Q_0)$ Eq. (13) and therefore negligible.

APPENDIX B

The q^2 integration of Eq. (13)

$$dN_{\text{vir}}(Q_0)=2\int d^2N_0(q^2,Q_0)F_1(q^2)$$

becomes, for $Q_0\ll E_1$, when we introduce ξ by Eq. (10),

$$\begin{aligned} dN_{\text{vir}}(Q_0)=\frac{2\alpha}{\pi}dN_0(Q_0)\left[\ln\frac{m}{\lambda}-1\right]+\frac{\alpha^2}{\pi^2}\left[i_1\left[2\ln\frac{m}{\lambda}-\frac{3}{2}\right]-i_3+\frac{\pi^2}{6}i_5-i_7+2i_9+2i_{11}\right]\\ -\frac{\alpha^2Q_0^2}{\pi^2E_1^2}\left[i_2\left[2\ln\frac{m}{\lambda}-\frac{3}{2}\right]-i_4+\frac{\pi^2}{6}i_6-\frac{1}{2}i_8+2i_{10}+2i_{12}\right]\ , \end{aligned} \tag{B1}$$

where the integrals i_n are

$$(i_1,i_3,i_5,i_7,i_9,i_{11})=\int_{\xi_{\min}}^{\xi_{\max}}d\xi\frac{(1+\xi^2)\xi}{(1-\xi)^2[(m/M)^2+\xi]^2}\left[\ln\xi,\frac{\xi}{1+\xi^2}\ln\xi,1,\ln^2\xi,\ln\xi\ln(1+\xi),L_2(-\xi)\right] \tag{B2}$$

and

$$(i_2,i_4,i_6,i_8,i_{10},i_{12})=\int_{\xi_{\min}}^{\xi_{\max}}d\xi\frac{1+\xi^2}{(1-\xi)^4}\left[\ln\xi,\frac{\xi}{1+\xi^2}\ln\xi,1,\ln^2\xi,\ln\xi\ln(1+\xi),L_2(-\xi)\right]\ , \tag{B3}$$

where $\xi_{\min}=m^2/4E_1E_2$, $\xi_{\max}=E_2/E_1$. The results of the integrations are

$$\begin{aligned} &i_1=-2\ln^2(M/m)+2\ln(M/m)+2\ln(Q_0/E_1)-2-\frac{\pi^2}{6},\quad i_3=\ln(Q_0/E_1)-1,\\ &i_5=2\ln(M/m)+2\ln(E_1/Q_0)-3,\quad i_7=\tfrac{8}{3}\ln^3(M/m)-4\ln^2(M/m)+\frac{\pi^2}{3}[2\ln(M/m)+1],\\ &1_9=L_3(-1)+2\ln(Q_0/E_1)+\frac{\pi^2}{4}-\ln^2 2-2\ln 2,\\ &i_{11}=L_3(-1)-\frac{\pi^2}{6}\frac{E_1}{Q_0}-2\ln 2\ln(Q_0/E_1)+\ln^2 2+2\ln 2\ , \end{aligned} \tag{B4}$$

$$i_2=-1,\quad i_4=-\tfrac{1}{2},\quad i_6=\frac{2}{3}\frac{E_1}{Q_0},\quad i_7=1,\quad i_8=0,\quad i_{10}=-\ln 2,\quad i_{12}=\frac{\pi^2}{12}\left[1-\frac{2}{3}\frac{E_1}{Q_0}\right]+\ln^2\ . \tag{B5}$$

When these results are inserted into (B1) one obtains Eq. (14).

The soft-photon radiative correction $dN_{\text{soft}}(Q_0)$ is obtained by integrating Eq. (15) over q^2. The integral is, for $Q_0\ll E_1$,

$$dN_{\rm soft}(Q_0)=-\frac{2\alpha}{\pi}dN_0(Q_0)\ln\frac{\Delta\omega_2 m}{\lambda E_1}-\frac{\alpha^2}{2\pi^2}\frac{dQ_0}{Q_0}\left\{4\left[i_1-\left[\frac{Q_0}{E_1}\right]^2 i_2\right]\ln\frac{\Delta\omega_2 m}{\lambda E_1}+G\right\}, \tag{B6}$$

where

$$G=\int_{\xi_{\min}}^{\xi_{\max}}d\xi\frac{1+\xi^2}{(1-\xi)^2}\left[\frac{\xi}{[(m/M)^2+\xi]^2}-\frac{Q_0{}^2}{E_1{}^2(1-\xi)^2}\right]$$
$$\times\left[2\ln\xi\,\ln(1+\xi)-\ln^2\xi+L_2\left[\frac{1-E_2/E_1\xi}{1-\xi^2}\right]+L_2\left[\frac{E_2-E_1\xi}{E_2(1-\xi^2)}\right]-L_2\left[\frac{\xi(E_2/E_1-\xi)}{1-\xi^2}\right]-L_2\left[\frac{\xi(E_1-E_2\xi)}{E_2(1-\xi^2)}\right]\right]$$
$$=-\tfrac{8}{3}\ln^3(M/m)+4\ln^2(M/m)+8L_3(-1)-\frac{\pi^2}{3},$$

with the result which gives Eq. (17).

The vacuum-polarization radiative correction is obtained in the same way. When ξ is introduced as a variable the integration of Eq. (22) over q^2 may be written, for $Q_0 \ll E_1$,

$$dN_{\rm vac}(Q_0)=-\frac{2\alpha^2}{3\pi^2}\left[\frac{5}{3}j_1-4j_2+j_3-2j_4-\left[\frac{Q_0}{E_1}\right]^2(\tfrac{5}{3}j_5-4j_6+j_7-2j_8)\right], \tag{B7}$$

where the integrals j_n are given by

$$(j_1,j_2,j_3,j_4)=\int_{\xi_{\min}}^{\xi_{\max}}d\xi\frac{1+\xi}{1-\xi}\frac{\xi}{[(m/M)^2+\xi]^2}\left[1,\frac{\xi}{(1-\xi)^2},\frac{1+\xi}{1-\xi}\ln\xi,\frac{\xi(1+\xi)}{(1-\xi)^3}\ln\xi\right] \tag{B8}$$

and

$$(j_5,j_6,j_7,j_8)=\int_{\xi_{\min}}^{\xi_{\max}}d\xi\frac{1+\xi}{(1-\xi)^6}((1-\xi)^3,\xi(1-\xi),(1+\xi)(1-\xi)^2\ln\xi,\xi(1+\xi)\ln\xi) \tag{B9}$$

with the results

$$j_1=2\ln(M/m)-2\ln(Q_0/E_1)-1,\quad j_2=(E_1/Q_0)^2-E_1/Q_0,$$
$$j_3=-2\ln^2(M/m)+2\ln(M/m)-\frac{\pi^2}{6}-4+4\ln(Q_0/E_1),\quad j_4=\tfrac{1}{3}\ln(Q_0/E_1)-2(E_1/Q_0)^2+2E_1/Q_0-\tfrac{4}{9} \tag{B10}$$

and

$$\left[\frac{Q_0}{E_1}\right]^2(j_5,j_6,j_7,j_8)=\left[1,\frac{1}{2}\left[\frac{E_1}{Q_0}-1\right]^2,-2,-\left[\frac{E_1}{Q_0}-1\right]^2-\frac{1}{6}\right]. \tag{B11}$$

With these results inserted into Eq. (B7), Eq. (24) is obtained.

APPENDIX C

We calculate here the hard-photon radiative correction, starting with Eq. (19) in the text. In the high-energy, small-angle improved Weizsäcker-Williams method of Sec. II, invariants such as k_2p_1,Q^2, etc., are considerably smaller than energy terms such as $E_1{}^2,E_1\omega_2$, etc. By the use of kinematics, it is easy to show that for $Q_0 \ll E_1$ and small values $Q^2,\mathbf{Q}^2=Q_0{}^2$, one obtains

$$\mathbf{p}_{1\perp}{}^2=\frac{E_1{}^2q^2}{Q_0{}^2},\quad \mathbf{p}_{2\perp}{}^2=\frac{E_2^2q^2}{Q_0{}^2},\quad \mathbf{k}_{2\perp}{}^2=\frac{\omega_2{}^2q^2}{Q_0{}^2},$$

and the cross section becomes

$$d\sigma(p_1k\to p_2P)=\int d^2N_{\rm hard}(p_1,Q)d\sigma(Qk\to P) \tag{C1}$$

with

$$d^2N_{\rm hard}(p,Q)=\frac{\alpha^2}{2\pi^3}\frac{dq^2dQ_0}{q^2Q_0}(1+q^2/M^2)^{-2}$$
$$\times\int\frac{d\omega_2dx\,dy}{\omega_2(2xq^2-q^4-y^2)^{1/2}}\frac{1}{s^2-y^2}\left[\left[1+\frac{E_2{}^2}{E_1{}^2}\right]q^2+4m^2\frac{E_2}{E_1}-4m^2\frac{E_2}{E_1}\frac{s^2+y^2}{s^2-y^2}\right], \tag{C2}$$

where we have introduced, following Bethe's calculation of the bremsstrahlung cross section with screening,[13] the variables

$$x = u + v, \quad y = u - v ,$$

where

$$u = 2\mathbf{p}_1^2(1-\cos\theta_1) = 2\frac{E_1}{\omega_2} p_1 k_2 ,$$

$$v = 2\mathbf{p}_2^2(1-\cos\theta_2) = 2\frac{E_2}{\omega_2} p_2 \cdot k_2 ,$$

and $s = x + 2m^2$. The x and y integrations are performed ($w^2 = 2xq^2 - q^4$)

$$\int_{1/2q^2}^{\infty} dx \int_{-w}^{w} dy \frac{1}{(w^2-y^2)^{1/2}(s^2-y^2)} = -\frac{\pi}{m^2}\frac{\xi}{(1-\xi)}\ln\xi ,$$

$$\int_{1/2q^2}^{\infty} dx \int_{-w}^{w} dy \frac{s^2+y^2}{(w^2-y^2)^{1/2}(s^2-y^2)^2} = -\frac{\pi}{2m^2} ,$$

followed by a simple integration over ω_2, which gives Eq. (20). The ξ integration of Eq. (20) is inferred from Appendix B with the result Eq. (21).

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