Higher-derivative gravity, surface terms, and string theory

Robert C. Myers
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 26 February 1987)

We consider adding Euler densities to the Einstein action as new gravity interactions in higher-dimensional theories. When the appropriate surface terms are included, the surface geometry is sufficient data for the boundary-value problem associated with the new Lagrangians. We also discuss the relevance of such interactions in the context of string theory.

Motivated by string theories, Euler densities have come under study as higher-derivative interactions for gravity in more than $3+1$ dimensions.[1,2] The original motivation was the observation that the low-energy effective actions of some string theories include $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ interactions, but the string theories do not include any ghosts which such an interaction produces. The apparent resolution of this problem was that the interaction should actually be the Euler density of four-dimensional manifolds, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}-4R^{\mu\nu}R_{\mu\nu}+R^2$, which leaves the graviton propagator unmodified.[1] This reasoning later came under some criticism,[3,4] but in any event the effects of Euler densities as gravity interactions in higher-dimensional theories have been examined by many authors.[2,5–7] The inclusion of such terms is also necessary in the analysis of $O(\alpha'^3)$ interactions in low-energy string theory.[8]

In this paper we examine the boundary-value problem associated with these actions. We find that with the addition of certain surface terms there is a good boundary-value problem, by which we mean that the action can be extremized while keeping only the surface geometry fixed. Motivated by string theory we attempt to construct a broader class of such *good* interactions. We also discuss the relevance of our results to low-energy string theory. From the point of view of boundary data, it is straightforward to see that despite their exceptional properties, these interactions are not the only ones relevant for string theory.

Investigations of general relativity in a Hamiltonian framework first revealed the necessity to supplement the Einstein action with a surface term[9]

$$I=\frac{1}{16\pi G}\int R\sqrt{-g}\,d^{N+1}x\pm\frac{1}{8\pi G}\int K\sqrt{\pm h}\,d^{N}x\ . \tag{1}$$

Here h_{ab} is the boundary metric and $K=h^{ab}K_{ab}$ is the trace of the second fundamental form. The plus (minus) signs apply to a spacelike (timelike) boundary. The same surface term was also later revealed from considerations of the path-integral approach to quantum gravity.[10,11] One can recognize the need for the surface term in deriving Einstein's equations by extremizing the action against variations δg_{ab} of the metric.[11,12] One fixes the metric on the boundary (i.e., δg_{ab} vanishes there), but in the variation of the Ricci scalar, one encounters a total derivative which produces a surface integral involving the derivative of δg_{ab} normal to the boundary. These normal-derivative terms do not vanish by themselves, but are canceled by the variation of the surface term included in the action above.

Now we will extend these considerations to general Euler density actions. The concept of Euler number has a simple extension to manifolds with boundaries just as for any topological index.[13,14] The Euler number is remarkable though because it can be calculated entirely in terms of *local* integrals over the volume and boundary of the manifolds. We will find that the boundary integrals provide the surface terms necessary for gravitational actions.

The Euler densities are most elegantly constructed in terms of differential forms.[2,5] We consider an $(N+1)$-dimensional space-time with metric $g=\eta_{AB}E^A\otimes E^B$ where E^A, $A=0,1,\ldots,N$ are an orthonormal basis of one-forms, and $\eta_{AB}=\mathrm{diag}(-1,+1,\ldots,+1)$. It is convenient to introduce the forms

$$\epsilon_{A_0\cdots A_m}=\frac{1}{(N-m)!}\epsilon_{A_0\cdots A_m A_{m+1}\cdots A_N}E^{A_{m+1}}\wedge\cdots\wedge E^{A_N}\ , \tag{2}$$

where $\epsilon_{A_0\cdots A_N}$ is the completely antisymmetric tensor with $\epsilon_{0\cdots N}=1$. A useful identity in the following manipulations is

$$E^B\wedge\epsilon_{A_0\cdots A_m}=\delta^B_{A_m}\epsilon_{A_0\cdots A_{m-1}}-\delta^B_{A_{m-1}}\epsilon_{A_0\cdots A_{m-2}A_m}+\cdots+(-)^m\delta^B_{A_0}\epsilon_{A_1\cdots A_m}\ .$$

Let ω be the Levi-Civita connection one-form which is compatible with the metric and vanishing torsion.[15,16] The curvature two-form is given by

$$\Omega^A{}_B=D\omega^A{}_B=d\omega^A{}_B+\omega^A{}_C\wedge\omega^C{}_B=\tfrac{1}{2}R^A{}_{BCD}E^C\wedge E^D\ .$$

The Ricci tensor and scalar are defined as $R_{AB}=R^C{}_{ACB}$ and $R=\eta^{AB}R_{AB}$. The Euler number of a compact manifold with $2m$ dimensions is found by integrating

$$\mathcal{L}_m=\Omega^{A_1B_1}\wedge\cdots\wedge\Omega^{A_mB_m}\wedge\epsilon_{A_1B_1\cdots A_mB_m}$$
$$=2^{-m}\delta^{C_1D_1\cdots C_mD_m}_{A_1B_1\cdots A_mB_m}R^{A_1B_1}{}_{C_1D_1}\cdots R^{A_mB_m}{}_{C_mD_m}\epsilon\ , \tag{3}$$

where the generalized δ function $\delta^{C_1\cdots D_m}_{A_1\cdots B_m}$ is totally antisymmetric in both sets of indices. For example, one has

$$\mathcal{L}_1=\Omega^{AB}\wedge\epsilon_{AB}=R\epsilon\ ,$$
$$\mathcal{L}_2=\Omega^{AB}\wedge\Omega^{CD}\wedge\epsilon_{ABCD}$$
$$=(R^{ABCD}R_{ABCD}-4R^{AB}R_{AB}+R^2)\epsilon\ .$$

To extend our discussion to the Euler character of manifolds with boundaries, we must introduce the second fundamental form.[14,15] Let ω be the connection form on a manifold M with boundary ∂M. One can choose Gaussian normal coordinates $\{x^a,y\}$ such that $y=0$ is the local equation for ∂M (Ref. 15). In these coordinates, the line element is

$$ds^2=-dy^2+g_{ab}(x^c,y)dx^adx^b\ ,$$

where we have chosen to consider a spacelike boundary. Now choose a product metric on M, which agrees with the original metric at ∂M:

$$ds^2=-dy^2+g_{ab}(x^c,y=0)dx^adx^b\ .$$

This metric yields a new connection ω_0, which has only tangential components on ∂M. The second fundamental form is then defined as[14]

$$\theta=\omega-\omega_0\ . \tag{4}$$

This form is related to the more familiar tensor K_{AB} usually discussed in the context of general relativity [see Eq. (1)] by

$$\theta^A{}_B=\theta^A{}_{B,C}E^C=(N\cdot N)(N^AK_{BC}-N_BK^A{}_C)E^C\ , \tag{5}$$

where N is the unit normal at ∂M and $(N\cdot N)$ has been retained so that either timelike or spacelike boundaries might be considered. In Gaussian normal coordinates, one has $N=dy$ and $K_{ab}=-\frac{1}{2}(N_{a;b}+N_{b;a})$.

Next the second fundamental form is used to construct the appropriate Chern-Simons form on ∂M (Ref. 14). Define

$$\omega_s=\omega-s\theta$$

which interpolates between ω at $s=0$ and ω_0 at $s=1$. Denote the corresponding curvature as

$$\Omega_s=d\omega_s+\omega_s\wedge\omega_s\ .$$

On the boundary of a $2m$-dimensional manifold, define the Chern-Simons form

$$Q_m=m\int_0^1 ds\,\theta^{A_1B_1}\wedge\Omega_s{}^{A_2B_2}\wedge\cdots\wedge\Omega_s{}^{A_mB_m}\wedge\epsilon_{A_1B_1\cdots A_mB_m}\ . \tag{6}$$

For example, with $K=K^A{}_A$

$$Q_1=\theta^{AB}\epsilon_{AB}=2(N\cdot N)KN^A\epsilon_A\ ,$$
$$Q_2=2\theta^{AB}\wedge(\Omega^{CD}-\tfrac{2}{3}\theta^C{}_E\wedge\theta^{ED})\wedge\epsilon_{ABCD} \tag{7}$$
$$=4[(N\cdot N)(KR-2K_{AB}R^{AB})$$
$$+\tfrac{2}{3}(K^3-KK_{AB}K^{AB}+2K_{AB}K^{BC}K_C{}^A)]N^D\epsilon_D\ ,$$

where in constructing Q_2, we have used Eqs. (4) and (5) as well as the fact that the nonzero components of $\omega_0{}^A{}_B$ have only tangential indices while those of $\theta^A{}_B$ have one normal index.

The Euler character for a $2m$-dimensional manifold M with a boundary ∂M is

$$I_m=\int_M\mathcal{L}_m-\int_{\partial M}Q_m\ . \tag{8}$$

Using Eq. (2), the definitions of $\mathcal{L}_m$ and Q_m given in Eqs. (3) and (6) may be extended to an arbitrary number of dimensions, and we may consider I_m as a part of the gravity action. For less than $2m$ dimensions, I_m will simply vanish. For exactly $2m$ dimensions, I_m is not very interesting since all of the field-theory interactions will vanish being total derivatives, but it may still play a role in the path-integral approach.[12] It is for more than $2m$ dimensions that I_m contributes nontrivial gravity interactions.

That I_m with their surface terms are appropriate as actions with good boundary-value problems is suggested by the observation that $I_1/16\pi G$ is precisely the Einstein action given in Eq. (1). We examine in detail that this is also true for

$$I_2=\int_M\Omega^{AB}\wedge\Omega^{CD}\wedge\epsilon_{ABCD}$$
$$-2\int_{\partial M}\theta^{AB}\wedge(\Omega^{CD}-\tfrac{2}{3}\theta^C{}_E\wedge\theta^{ED})\wedge\epsilon_{ABCD}\ .$$

We extremize this action in two stages, keeping only the metric on the boundary fixed. First the basis forms E^A are varied while keeping the connection ω fixed. Only the volume integral contributes:

$$\delta_EI_2=\int_M\delta E^F\wedge\Omega^{AB}\wedge\Omega^{CD}\wedge\epsilon_{ABCDF}$$
$$=\int_M\delta E^F\wedge[(R_{ABCD}R^{ABCD}-4R_{AB}R^{AB}+R^2)g_{FG}$$
$$-4(R_{FABC}R_G{}^{ABC}+2R_{FABG}R^{AB}$$
$$-2R_F{}^AR_{AG}+RR_{FG})]\epsilon^G\ .$$

Next we vary the connection ω which yields contributions in both the volume and surface integrals:

$$\delta_\omega I_2 = \int_M 2D(\delta\omega^{AB})\wedge\Omega^{CD}\wedge\epsilon_{ABCD} - 2\int_{\partial M}[\delta\omega^{AB}\wedge(\Omega^{CD}-\tfrac{2}{3}\theta^C{}_E\wedge{}^{ED})\wedge\epsilon_{ABCD} + \theta^{AB}\wedge(d\delta\omega^{CD}+2\delta\omega^C{}_E\wedge\omega^{ED}-\tfrac{4}{3}\delta\omega^C{}_E\wedge\theta^{ED})\wedge\epsilon_{ABCD}]$$

$$= 2\int_M d(\delta\omega^{AB}\wedge\Omega^{CD}\wedge\epsilon_{ABCD}) - 2\int_{\partial M}\delta\omega^{AB}\wedge\Omega^{CD}\wedge\epsilon_{ABCD}$$

$$-2\int_{\partial M}(\tfrac{4}{3}\delta\omega^{nB}\wedge\theta^{nC}+4\delta\omega^{nC}\wedge\theta^{nB}-\tfrac{8}{3}\delta\omega^{nC}\wedge\theta^{nB})\wedge\theta^{nD}\wedge N^A\epsilon_{ABCD}=0 .$$

Here we have used $D\Omega=0=D\epsilon_{ABCD}$ as well as $\delta\theta^{AB}=\delta\omega^{AB}$ on ∂M, and that $\omega_0{}^{AB}$ has only tangential components on ∂M while both θ^{AB} and $\delta\omega^{AB}$ have one normal index denoted by n. Since the variation of the connection canceled exactly, the action can be made stationary against variations of the metric keeping only the boundary metric, but not its normal derivatives fixed.

That the desired cancellation is achieved for all I_m is a result of the topological origin of the terms. The Chern-Simons form Q_m is constructed in $2m$ dimensions to yield[14]

$$\mathcal{L}_m(\omega)-\mathcal{L}_m(\omega_0)=dQ_m(\omega,\omega_0) .$$

Since the product metric at the boundary cannot generally be extended throughout the manifold, $\mathcal{L}_m(\omega_0)$ is not well defined, but considering a small variation of the connection ω, one will have $\delta\mathcal{L}_m=d\delta Q_m$ to yield the desired cancellation. These results survive when the forms are extended to arbitrary dimensions using Eq. (2).

That these actions yield simple boundary-value problems is a rather extraordinary property, which is not true of an arbitrary combination of Riemann tensors. To see how it would fail, consider two higher-derivative interactions for a scalar field:

$$I_A=\int d^{N+1}x\,\phi\nabla_a\phi\nabla^a\phi\nabla^2\phi ,$$

$$I_B=\int d^{N+1}x(\phi\nabla^2\phi)^2 .$$

Since these interactions contain second derivatives of ϕ, their variations include surface terms involving normal derivatives of $\delta\phi$. One finds that I_A can be extremized with respect to variations of ϕ with only $\delta\phi=0$ at the boundary, if the action is supplemented by the surface integral

$$S_A=\int d^Nx[\tfrac{2}{3}\phi(N\cdot\nabla\phi)^3-\phi\nabla_a\phi\nabla^a\phi N\cdot\nabla\phi] ,$$

where N is again the unit normal to the boundary. No surface terms exist though which can be used to eliminate $N\cdot\nabla(\delta\phi)$ from the variation of I_B. The problem arises because I_B contains terms which are quadratic in second derivatives of ϕ (with respect to a particular coordinate). On the other hand, I_A only has terms which are linear in second derivatives of ϕ. Essentially such terms can be eliminated by partial integration. Examining the second form of the Euler densities given in Eq. (3), one might have easily seen that it must be possible to produce a simple boundary-value problem. The Riemann tensors contain second derivatives of the metric, but the antisymmetry of the generalized δ function ensures that at most one factor will have a second derivative with respect to one particular coordinate.[7] One might ask whether it is possible to construct new interactions making nontrivial use of gauge invariance to produce a good boundary-value problem.

Again, string theory motivates this question. In the scattering amplitude for three gravitons, one finds a contribution in bosonic string theory containing six momenta, which does not appear in the corresponding superstring amplitude[1]

$$k^1\cdot\epsilon_3\cdot k^1 k^2\cdot\epsilon_1\cdot k^2 k^3\cdot\epsilon_2\cdot k^3 ,$$

where k^i and ϵ_i are, respectively, the momenta and polarization tensors of the three gravitons. This amplitude would arise from an interaction proportional to

$$\partial_e\partial_f h^{ab}\partial_a\partial_b h^{cd}\partial_c\partial_d h^{ef} . \qquad (9)$$

Naively, this term appears to be more than linear in second derivatives of the graviton field, h^{ab}. Consider time derivatives, for example. Variations with respect to h^{ij} and h^{0i} reveal that Eq. (9) is only linear in their second time derivatives. the variation with respect to h^{00} yields an unsatisfactory term:

$$\sim 3\partial_0\partial_0(\delta h^{00})(\partial_0\partial_0 h^{00})^2+\cdots . \qquad (10)$$

Therefore Eq. (9) appears to be cubic in $\partial_0\partial_0 h^{00}$, but in fact this is a gauge artifact. Imposing the gauge condition $\partial_a h^{ab}=0$ reduces Eq. (20) to

$$\sim 3\partial_0\partial_0(\delta h^{00})(\partial_0\partial_i h^{i0})^2+\cdots$$

which will not lead to problems with the surface terms. Therefore the gauge invariance of the theory ensures that Eq. (9) is a good interaction.

In the low-energy effective action of the string, one expects Eq. (9) to arise as the leading term in the expansion of a covariant term such as $R_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\lambda\rho}R^{\lambda\rho\mu\nu}$. The question becomes whether there exists a covariant completion of Eq. (9) which is at most linear in second derivatives of h^{ab}. We approach this question using the usual $N+1$ decomposition of general relativity.[16] In this framework, the gauge invariance is expressed as saying that g_{00} and g_{0i} are completely arbitrary on an initial time slice. Therefore we need only eliminate higher-order terms in $\partial_0\partial_0 g_{ij}$. There are eight distinct contractions of three Riemann tensors and two independent terms with two Riemann tensors and two covariant derivatives. It is a simple exercise to find linear combinations of those terms in which all terms cubic or quadratic in higher time derivatives of g_{ij} are eliminated. The final result is that the only such combination is $\mathcal{L}_3$. Unfortunately $\mathcal{L}_3$ does not contain a three-graviton interaction,[2] and hence cannot provide the desired interaction. Similarly if one examines four derivative terms, one finds that only $\mathcal{L}_2$ is linear in $\partial_0\partial_0 g_{ij}$. It is likely that $\mathcal{L}_m$ is the only covariant $2m$ derivative interaction linear in $\partial_0\partial_0 g_{ij}$ since any other new density would be an unknown topological term.

Hence while the bosonic string theory should have a good boundary-value problem, we find that the low-energy effective action does not. This apparent paradox is resolved by the fact that the string's boundary data include an infinite set of massive fields as well as the massless fields which appear in the low-energy action. In terms of these local fields, the string field equations are an infinite set of ordinary differential equations which may be written schematically as[17]

$$\alpha'\nabla^2\Psi=\Psi*\Psi+M*\Psi+M*M \;, \quad (11)$$

$$(\alpha'\nabla^2+N)M=\Psi*\Psi+M*\Psi+M*M \;. \quad (12)$$

Here Ψ and M represent massless and massive fields, respectively. N is an integer and hence the massive fields in Eq. (12) have mass squared of the order of the string tension $1/\alpha'$. One should also note that the interactions include derivative couplings. The effective low-energy theory is constructed as a perturbative expansion in $\alpha'\nabla^2$ (or $\alpha' p^2$). First one inverts the kinetic operator in Eq. (12) to yield

$$M=\frac{1}{N}\left[1-\frac{\alpha'\nabla^2}{N}+\left[\frac{\alpha'\nabla^2}{N}\right]^2+\cdots\right]\times(\Psi*\Psi+M*\Psi+M*M) \;.$$

Then one solves for the fields M by iteratively substituting for M on the right-hand side. This process produces an infinite expansion for M in both $\alpha'\nabla^2$ and the massless fields Ψ. Substituting this result into Eq. (11) yields a differential equation in terms of the massless fields only. Typically these expansions are truncated with a finite number of derivatives, and gauge invariance is used to determine the expansion in Ψ (Ref. 17). Having eliminated the massive fields, the boundary data must still include their corresponding expansions in terms of massless fields and derivatives. Therefore the boundary-value problem for the effective low-energy action becomes increasingly complicated as the expansion in derivatives is carried to higher orders (i.e., the boundary data will include higher derivatives of the massless fields).

We agree with the criticism[3,4,17] of the argument that $\mathcal{L}_2$ should appear in the low-energy action.[1] Essentially the ghost poles are immaterial since they occur at a momentum scale which is beyond the validity of the expansion described above. In any event the usual low-energy calculations involve on-shell quantities, and the off-shell behavior remains ambiguous allowing the freedom of field redefinitions. We argue though that in fact $\mathcal{L}_2$ is the correct choice for the low-energy action. String field theory determines the off-shell behavior, which will then dictate the form of the low-energy action. The string field equations are well understood in light-cone gauge,[18] and for closed strings have the form given in Eqs. (11) and (12). Since upon integrating out the massive fields no new terms linear in the graviton appear in the field equations, the propagator is unmodified and hence the quadratic Euler density must occur in the low-energy action. In a theory of open and closed strings, there is an interaction involving the overlap of a single open string and a single closed string, which adds new terms linear in the fields to Eqs. (11) and (12). In that case, a more careful analysis is required to determine the correct result. These arguments come with the following caveat. In string theory, a massless, traceless, symmetric tensor field occurs which we assume is to be identified with the graviton h^{ab}, describing perturbations of the metric of a background space-time. One might equally well make some other identification though, say, $h^{ab}+\alpha' R^{ab}$. This is exactly the freedom of field redefinitions mentioned above under a slightly new guise. At this juncture the full geometry underlying string theory is not understood, so that although the first choice is perhaps the most natural, it still lacks motivation.

Now we make a few more remarks with regard to low-energy string theory. For many string theories, there are at least two more massless bosonic fields: an antisymmetric tensor B_{ab} and the dilaton ϕ. The antisymmetric field strength, $H_{abc}=\nabla_a B_{bc}+\nabla_b B_{ca}+\nabla_c B_{ab}$, can be combined with the Levi-Civita connection to yield a generalized connection with torsion which seems to play a role in the low-energy action:[17,19]

$$\bar{\omega}^{\,A}{}_{B,C}=\omega^{A}{}_{B,C}+H^{A}{}_{BC} \;.$$

Using this connection, generalizations of the curvature and second fundamental forms can be constructed, which upon substitution into I_m yield new actions with good boundary-value problems (in particular, the case $m=2$ is of interest). Varying B_{ab} only produces a variation $\delta\bar{\omega}$ for which most terms cancel, but the B_{ab} equation receives a contribution since one encounters

$$D\epsilon_{A_0\cdots A_m}=(N-m)T^D\wedge\epsilon_{A_0\cdots A_m D}\neq 0$$

in $N+1$ dimensions. Here $T^A=DE^A=-H^{A}{}_{BC}E^B\wedge E^C$ is the torsion two-form. Since the antisymmetric field strength is a covariant tensor, many other *good* and simpler interactions can be written for B_{ab} (for example, $H_{abc}H^{cde}H_{def}H^{fab}$). A conformal transformation of the metric involving the dilaton introduces a Weyl part in the connection,[17,19] and leads to a further generalization of I_m. Finally, it should also be possible to supersymmetrize these interactions as was done for $\mathcal{L}_2$ in Ref. 20.

Ultimately as was discussed above, most of the higher-derivative graviton couplings appearing in the low-energy effective string action are not of the form I_m. In fact at $O(\alpha'^3)$ for the superstring or heterotic string,[3,17,21] and at $O(\alpha'^2)$ in the bosonic string,[1] interactions arise which are not in this class. If one were considering a quantum field theory for higher-dimensional gravity, one may still ask if these new interactions might play a useful role. Since I_m do not modify the boundary-value data, in a perturbative expansion one avoids unitarity problems but the nonrenormalizable nature of the theory remains. One might still hope (as one usually does for Einstein gravity) that the weak-coupling perturbation expansion is not relevant, and consider contributions of I_m to the nonperturbative structure of the theory. Here it may be relevant to make a number of observations. First, the structure of black-hole solutions of the new field equations is not greatly modified.[6,7] Therefore any nonperturbative role which black holes play will be largely unchanged. The new $\mathcal{L}_m$ interactions will induce interesting effects in black-hole thermodynamics, similar to those studied in Refs. 8 and 22, which will help solve the problems associated with black-hole evaporation. Finally in the path-integral approach to quantum gravity, one finds that the Einstein action has a conformal instability.[23,11] A conformal transformation of a gravitational action with additional Euler densities (or any higher curvature terms) would produce higher powers of derivatives of the conformal factor. Since the new interactions may be added with arbitrary coefficients, one may be able to eliminate the conformal instability with an appropriate linear combination of I_m.

The author thanks Gary Horowitz for useful comments. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and also by NSF Grant No. PHY82-17853, supplemented by funds from NASA.

---

[1]B. Zwiebach, Phys. Lett. **156B**, 315 (1985).

[2]B. Zumino, Phys. Rep. **137**, 109 (1986).

[3]D. J. Gross and E. Witten, Nucl. Phys. **B277**, 1 (1986).

[4]S. Deser and A. N. Redlich, Phys. Lett. **176B**, 350 (1986); A. A. Tseytlin, *ibid.* **176B**, 92 (1986).

[5]F. Müller-Hoissen, Phys. Lett. **163B**, 106 (1985).

[6]J. T. Wheeler, Nucl. Phys. **B268**, 737 (1986); **B273**, 737 (1986); D. G. Boulware and S. Deser, Phys. Lett. **175B**, 409 (1986); D. L. Wiltshire, *ibid.* **169B**, 36 (1986); H. Ishihara, *ibid.* **179B**, 217 (1986); A. Tomimatsu and H. Ishihara, Nagoya University report (unpublished); C. Aragone, Phys. Lett. **186B**, 151 (1987).

[7]D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).

[8]R. C. Myers, ITP report, 1986 (unpublished).

[9]J. W. York, Jr., Phys. Rev. Lett. **28**, 1082 (1972).

[10]G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).

[11]S. W. Hawking, in *General Relativity,* edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1985).

[12]S. W. Hawking, Phys. Rev. D **18**, 1747 (1978).

[13]S. Chern, Ann. Math. **46**, 674 (1945).

[14]T. Eguchi, P. G. Gilkey, and A. J. Hanson, Phys. Rep. **66**, 213 (1980).

[15]Y. Choquet-Bruhat, C. DeWitt-Morette, and M. Dillard-Bleick, *Analysis, Manifolds and Physics* (North-Holland, New York, 1982).

[16]R. Arnowitt, S. Deser, and C. Misner, Phys. Rev. **117**, 1595 (1960); **118**, 1100 (1960); **122**, 997 (1961).

[17]D. J. Gross and J. M. Sloan, ITP report, 1987 (unpublished).

[18]M. Kaku and Kikkawa, Phys. Rev. D **10**, 1110 (1974); **10**, 1823 (1974); M. Kaku, *ibid.* **10**, 3943 (1974); E. Cremmer and J.-L. Gervais, Nucl. Phys. **B76**, 209 (1974); **B90**, 410 (1975); M. B. Green and J. H. Schwarz, *ibid.* **B218**, 43 (1983); Phys. Lett. **140B**, 33 (1984); Nucl. Phys. **B243**, 475 (1984); M. B. Green, J. H. Schwarz, and L. Brink, *ibid.* **B219**, 437 (1983); D. J. Gross and V. Periwal, *ibid.* **B287**, 1 (1987).

[19]J. Scherk and J. H. Schwarz, Phys. Lett. **52B**, 347 (1974); R. I. Nepomechie, Phys. Rev. D **32**, 3201 (1985).

[20]L. J. Romans and N. P. Warner, Nucl. Phys. **B273**, 320 (1986); A. H. Chamseddine and P. Nath, Phys. Rev. D **34**, 3769 (1986); S. Deser, in *Supersymmetry and its Applications,* edited by G. W. Gibbons, S. W. Hawking, and P. K. Townsend (Cambridge University Press, Cambridge, England, 1986).

[21]M. T. Grisaru, A. E. M. van de Ven, and D. Zanon, Phys. Lett. **173B**, 423 (1986); Nucl. Phys. **B277**, 388 (1986); **B277**, 409 (1986).

[22]C. G. Callan, R. C. Myers, and M. J. Perry, DAMTP report, 1986 (unpublished).

[23]G. W. Gibbons, S. W. Hawking, and M. J. Perry, Nucl. Phys. **B138**, 141 (1978).