Light neutral boson in spinor-connection theory

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It is shown how spinor-connection theory should allow a neutral spinless particle whose mass is much smaller than the neutral-pion mass. It is also shown how the theory prohibits charged particles with similar structure and mass.

I. INTRODUCTION

The field equations of the theory have been given previously[1,2] and electrically neutral and charged pionlike particle solutions have been demonstrated.[1,3] Denoting these pionlike particles by $P^0, P^{\pm}$ we found[3] that the mass ratio of the neutral-to-charged particles, that is, $m(P^0)/m(P^{\pm})$, would closely equal the corresponding mass ratio for pions, that is, $m(\pi^0)/m(\pi^{\pm})$, if the electromagnetic and strong coupling constants have a common value of 1/137.

The purpose of this work is to show that the theory also allows a much lighter, electrically neutral, spinless particle, L^0 say, with anticipated mass $m(L^0) \ll 26$ MeV/c^2. The essential difference between the L^0 and P^0 is that the L^0, unlike the P^0, has a zero torsion field. Apart from this difference in the torsion, the general structure and method of derivation is the same as that used previously.[3] The L^0, like the P^0, carries a strong charge as a source for a short-ranged field intensity. Interestingly, electrically charged, Coulomb field particles, say $L^{\pm}$, with similar structure and mass, are prohibited by the theory.

II. NEUTRAL L^0 SOLUTION

We use the same ansatz as was used previously for the neutral P^0 particle[1] and hence have six real radial functions: P, Q, R, S, f_0, and g_0. The first four functions describe the spinor-tetrad field, which by this ansatz is collapsed to a single pair of Dirac four-spinors having opposite half-integral spin and opposite sign for the electric and weak charges. The remaining two functions, f_0 and g_0, specify a spherically symmetric space-time line element. Scaling these functions as before, the field equations require the same set of eight real radial functions (P_1, Q_1, R_1, S_1, f_1, g_1, K_1, and J_1), where the last two, K_1 and J_1, specify the strong potential field and the strong field intensity, respectively. For convenience we now drop all subscripts of unity and write $(P_1, Q_1, \ldots, J_1) = (P, Q, \ldots, J)$. Neglecting as before[1] terms of relative order $T^{-1} \simeq 10^{-39}$, the field equations for a strong charge $\pm q$ are given by

$$K' = -\frac{fgJ}{\gamma y^2} , \tag{2.1}$$

$$J' = \frac{fB}{16\gamma g} , \tag{2.2}$$

$$P' = \frac{fP}{y} + \frac{f}{2g}\left[K - \frac{3A}{16y^2}\right]Q , \tag{2.3}$$

$$Q' = -\frac{fQ}{y} - \frac{f}{2g}\left[K - \frac{3A}{16y^2}\right]P , \tag{2.4}$$

$$R' = -\frac{fR}{y} - \frac{f}{2g}\left[K + \frac{3A}{16y^2}\right]S , \tag{2.5}$$

$$S' = \frac{fS}{y} + \frac{f}{2g}\left[K + \frac{3A}{16y^2}\right]R , \tag{2.6}$$

$$f' = \frac{f(1-f^2)}{2y} + \frac{f^3J^2}{2y^3} + \frac{f^3}{16yg^2}\left[\frac{3A^2}{32y^2} - KB\right] , \tag{2.7}$$

$$g' = \frac{-g(1-f^2)}{2y} - \frac{gf^2J^2}{2y^3} + \frac{f^2}{16yg}\left[\frac{3A^2}{32y^2} - KB\right] - \frac{f^2}{4y^2}(PQ + RS) , \tag{2.8}$$

where the prime denotes differentiation with respect to the scaled radial variable y, and where

$$A = P^2 + Q^2 - R^2 - S^2 , \tag{2.9}$$

$$B = P^2 + Q^2 + R^2 + S^2 . \tag{2.10}$$

The constant γ is specified as before by

$$\gamma = -(64\pi\alpha)^{-1/2} , \tag{2.11}$$

where α is the coupling constant for the strong field, with $\alpha = q^2(4\pi\hbar c)^{-1} \simeq 1/137$, as needed[3] for $m(P^0)/m(P^{\pm}) = m(\pi^0)/m(\pi^{\pm})$. For the choice of γ as given by (2.11), we must require the normalization condition[1]

$$2\pi \int_0^\infty \frac{fB}{g} dy = 1 . \tag{2.12}$$

The function A in (2.9) determines torsion, since the torsion scalar,[1] $T^{\alpha\beta\gamma}T_{\alpha\beta\gamma}$, contains the factor A^2.

The field equations above split naturally into two classes, corresponding to the existence or nonexistence of torsion. If torsion is present, we need just two of the four functions P, Q, R, and S. For example, the P^0 solution[4] given previously arises with $R=S=0$ and $A=B$. By symmetry of the field equations, the torsional solution with $P=Q=0$, and $A=-B$, is apparently physically identical to the previous solution.

In this work, we wish to investigate the case of zero torsion. Actually, a rather cursory numerical search for this case was made previously,[1] but with inconclusive results, due to a lack of guidance provided by a later-developed and more analytical approach.[3] That same analytical approach is used here, in appropriately abbreviated detail.

The symmetry of the field equations permits a globally zero torsion field, $A=0$, in two, apparently physically indistinguishable cases: namely, $(P,Q)=\pm(S,R)$. For definiteness, we choose here the positive sign so that we can write, by (2.9) and (2.10),

$$P=S, \quad Q=R \ , \tag{2.13}$$

$$B=2(P^2+Q^2) \ , \tag{2.14}$$

$$A=0 \ . \tag{2.15}$$

At large radial distances from the particle, that is, $y\to\infty$, the metric field is given by Schwarzschild's vacuum solution

$$g^2=f^{-2}=1-\frac{\mu}{8\pi y} \ , \tag{2.16}$$

where, with the scaling chosen for y, the mass m (in grams) of the particle is[1,3]

$$m=\mu M_0 \ , \tag{2.17}$$

where μ is dimensionless and M_0 is the fundamental mass of the theory. At large y, the remaining field functions differ only slightly from those for the P^0 solution. We find that

$$K=\frac{-a^2}{16\gamma^2y^2}\left[1+\frac{2}{3}\left[\frac{\mu}{8\pi y}\right]+\frac{23}{48}\left[\frac{\mu}{8\pi y}\right]^2+\frac{a^4}{3^3\times 2^{11}\gamma^4y^2}+\cdots\right] , \tag{2.18}$$

$$J=\frac{-a^2}{8\gamma y}\left[1+\left[\frac{\mu}{8\pi y}\right]+\frac{23}{24}\left[\frac{\mu}{8\pi y}\right]^2+\frac{a^4}{3^3\times 2^{10}\gamma^4y^2}+\cdots\right] , \tag{2.19}$$

$$P=\frac{a^3}{96\gamma^2y^2}\left[1+\frac{3}{2}\left[\frac{\mu}{8\pi y}\right]+\cdots\right] , \tag{2.20}$$

$$Q=\frac{a}{y}\left[1+\frac{1}{2}\left[\frac{\mu}{8\pi y}\right]+\frac{5}{16}\left[\frac{\mu}{8\pi y}\right]^2-\frac{a^4}{3\times 2^{11}\gamma^4y^2}+\cdots\right] , \tag{2.21}$$

where a is a constant. The field intensity generated by the strong charge is proportional to Jy^{-2}, and hence varies as y^{-3} far from the source because of (2.19). On the other hand, as $y\to 0$ for consistency of (2.2) and (2.12) with strong charge $\pm q$, we need

$$\lim_{y\to 0} J=J(0)=-(32\pi\gamma)^{-1} \ . \tag{2.22}$$

Having set these boundary values, we can now fill in a semianalytic global solution, with the far-field ($y\to\infty$) solution holding good until we approach very close to the Schwarzschild radius at $y=\eta$, where

$$\eta=\frac{\mu}{8\pi} \ . \tag{2.23}$$

As $y\to\eta$, $y-\eta>0$, we find

$$B\simeq b_0\simeq 6(a/\eta)^2 \ , \tag{2.24}$$

$$K\simeq k_0\simeq -8\pi\alpha(a/\eta)^2 \ , \tag{2.25}$$

while J varies as $\ln(y-\eta)$. We can solve the metric field equations (2.7) and (2.8) if the KB term dominates over the J^2, that is, if

$$\frac{1}{{t_0}^2}\gg\frac{g^2J^2}{\eta^2} \ , \tag{2.26}$$

where the constant ${t_0}^2$ is, with the help of (2.24) and (2.25),

$$\frac{1}{{t_0}^2}\simeq -\frac{KB}{8}\simeq 6\pi\alpha(a/\eta)^4 \ . \tag{2.27}$$

We find that f has a turning point at $y\simeq\eta$, where it attains a maximum value of

$$f\simeq\frac{t_0}{2} \tag{2.28}$$

so that, since $f>1$,

$$t_0>2 \ . \tag{2.29}$$

We find also at $y\simeq\eta$, that $g\simeq {t_0}^{-1}$, so that the inequality (2.26) becomes $\eta^2\gg J^2$. From (2.19), $J^2>9a^4(64\gamma^2\eta^2)^{-1}$, so that we anticipate

$$8\eta^2|\gamma|\gg 3a^2 \ . \tag{2.30}$$

Proceeding inside the Schwarzschild radius, we find a solution similar to that[3] for the P^0, $P^{\pm}$. The solution is

$$\frac{yg}{f}\simeq\text{const}\simeq\frac{\eta}{{t_0}^2} \ , \tag{2.31}$$

$$f\simeq Fy^{({t_0}^2+1)/2} \ , \tag{2.32}$$

$$P\simeq(b_0/2)^{1/2}\sin(\phi-\sigma y^2) \ , \tag{2.33}$$

$$Q\simeq(b_0/2)^{1/2}\cos(\phi-\sigma y^2) \ , \tag{2.34}$$

$$K\simeq k_0 \ , \tag{2.35}$$

$$J\simeq -(32\pi\gamma)^{-1}\left[1-\frac{\pi b_0{t_0}^2y^2}{\eta}\right] , \tag{2.36}$$

where F, σ, and ϕ are constants.

To estimate the mass factor μ, we use the normalization condition (2.12). With the help of (2.31), (2.33), (2.34), and (2.14), we obtain

$$1=2\pi\int_0^\infty \frac{fB}{g}dy>2\pi\int_0^\eta \frac{b_0t_0{}^2y}{\eta}dy\simeq\pi t_0{}^2\eta b_0 \ .$$

Noting (2.24) and (2.27), the previous inequality gives

$$(\eta/\alpha)(\eta/a)^2<1 \ . \tag{2.37}$$

Using $t_0>2$ and (2.27), we also obtain

$$6\pi\alpha(a/\eta)^4<\tfrac{1}{4} \ . \tag{2.38}$$

The inequalities (2.37) and (2.38) combine to give $\eta^2<\alpha(24\pi)^{-1}$, so with (2.23) we obtain

$$\mu<(8\pi\alpha/3)^{1/2} \ . \tag{2.39}$$

Using $\alpha\simeq137^{-1}$ as the strong coupling constant, Eq. (2.39) gives $\mu<0.25$. Recalling[3] that for the $P^\pm$, $\mu\simeq2.18$, and that $m(P^\pm)$ should be close to $m(\pi^\pm)$, that is, 140 MeV/c^2, we obtain $m(L^0)<16$ MeV/c^2. This upper limit is very sensitive to the magnitude of the maximum value, $f_{\max}=\frac{1}{2}t_0$, attained by f in the Schwarzschild region. As $f_{\max}$ increases beyond unity, or as t_0 increases beyond 2, the upper mass limit decreases as $t_0{}^{-2}$. Another rather more restrictive upper mass limit can be obtained from (2.30) and (2.37). The result is $\eta<<(8|\gamma|\alpha)/3$, giving $\mu<<0.4$, or $m(L^0)<<26$ MeV/c^2.

In principle, as previously,[3] an accurate value for $m(L^0)$ could be determined by numerical integration. At present, the author has no access to computing facilities.

III. ABSENCE OF $L^\pm$ SOLUTIONS

The L^0, P^0, and $P^\pm$ solutions all have a common underlying structure. In each case the vacuum metric field admits a Schwarzschild singularity. However, in all cases this singularity is just averted by a sudden growth in fields determining the detailed structure of the particle. We show now that torsionless, electrically charged, spinless particles, say $L^\pm$, with this same underlying structure are prohibited by the theory.

If they exist, the $L^\pm$ particles would have gravitational, electric potential, and electric intensity fields at large y which are determined[3] by

$$g^2=f^{-2}=1-\frac{\mu}{8\pi y}+\frac{\alpha}{16\pi y^2} \ , \tag{3.1}$$

$$K=\frac{2\alpha}{y} \ , \tag{3.2}$$

$$J=-\frac{1}{4}\left[\frac{\alpha}{\pi}\right]^{1/2} . \tag{3.3}$$

On the other hand, as $y\to0$,

$$\lim_{y\to0}J=J(0)=0 \ . \tag{3.4}$$

The metric (3.1) is singular for $y>0$ if and only if $\mu\geq(16\pi\alpha)^{1/2}\simeq0.61$, or $m(L^\pm)\geq39$ MeV/c^2. Hence if they do exist, the $L^\pm$ should be much more massive than L^0.

Supposing now that $\mu\geq(16\pi\alpha)^{1/2}$, the field equations (2.1)–(2.12) still apply. We notice that with $A=0$ in Eq. (2.7), the function f will be singular near the Schwarzschild region unless the $-KB$ term is positive, or equivalently, unless K is negative. However, K cannot be negative, as follows from (2.1), (2.2), (2.10), (2.11), and (3.2)–(3.4). Hence, the $L^\pm$ do not occur.

A point worthy of note is that the common underlying structure of P^0, L^0, and $P^\pm$ raises an intriguing possibility that the structure of all elementary particles might be such that Schwarzschild-type metric singularities are avoided in every case, but just at the last minute, so to speak. The condition $m\geq39$ MeV/c^2 just noted for the $L^\pm$, if it were general, would then prohibit the electron. However, the condition is not general, but very specific to spherically symmetric, electrically charged, spinless particles with vacuum metric (3.1). Electric particles with spin are currently being studied by the author. The cylindrically symmetric fields needed are tediously complicated, but the condition $m\geq39$ MeV/c^2 is absent.

IV. CONCLUDING REMARKS

Light electrically neutral bosons have been previously proposed: for example, the X^0 particle.[5] It seems, however, that the X^0 and L^0 have little in common, apart from small mass, zero spin, and present lack of empirical verification. The X^0 involves scalar or pseudoscalar particle fields which do not occur in spinor-connection theory. Here the L^0 is built up from two members of a four-fermion field. Furthermore, the L^0 is neutral in the electrical sense only, since it carries a strong charge. Indeed by the symmetry of the theory, there should also be another L^0-type particle carrying the weak charge. However, we have no idea of the value of the weak field coupling constant α and hence can make no estimate for the mass of such a weak particle.

Also, it seems unlikely that the L^0 could fulfill the conjectured role of the X^0, which is to decay to an electron-positron pair. Presumably, a decay $L^0\to e^++e^-$ would violate strong charge conservation. Pair production might possibly arise by L^0, $\bar{L}^0$ annihilation, although any detailed mechanisms (solutions) for either decay or annihilation processes are unknown. However, all of the L^0, P^0, and $P^\pm$ particles are unstable in the sense that their solutions demand a total absence of fields arising from external sources.

The L^0 might play a quite different decay role. There are several obstacles to identifying the pionlike $P^{0,\pm}$ particles with the actual pions $\pi^{0,\pm}$. The most obvious obstacle[1] is that the pionlike decay $P^0\to\gamma+\gamma$ violates charge conservation. A sufficiently light L^0 particle raises the possibility of the decay $P^0\to L^0+\gamma+\gamma$ as a substitute for $\pi^0 \rightarrow \gamma + \gamma$. This speculated neutral decay exhibits an interesting intrinsic feature of the theory: namely, nonconservation of torsion, with P^0 being torsional and L^0 torsionless. If we form the pseudovector $B^\mu = E^{\alpha\beta\gamma\mu} T_{\alpha\beta\gamma}$, where $T_{\alpha\beta\gamma}$ is the torsion field, we find that $B^\mu{}_{;\mu} = 3i\Lambda(2T)^{-1}\langle \psi \Gamma_{05} \psi \rangle$. This nonconservation law for B^μ is easily derived from the field equations[1,2] for the spinor-tetrad and torsion fields.[6]

[1]J. T. Lynch, Phys. Rev. D **31**, 1287 (1985).

[2]J. T. Lynch, Class. Quantum Gravit. **3**, 103 (1986).

[3]J. T. Lynch, Phys. Rev. D **35**, 2372 (1987).

[4]It is worth noting that in Refs. 1 and 3 the field equations corresponding to the present Eqs. (2.3) and (2.4) appeared with the B function in the place of the present A function. That switch is of no consequence, however, since in that context we had $A = B$, with $R = S = 0$.

[5]A. Schäfer *et al.*, Mod. Phys. Lett. **A1**, 1 (1986).

[6]The nonconservation of the neutral torsion current B^μ was not realized in Ref. 2, wherein it was wrongly mentioned in passing that $B^\mu{}_{;\mu} = 0$.