Long-distance Lienard-Wiechert potentials and $q\bar{q}$ spin dependence

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The long-range spin dependence of the qq interaction is considered in a model in which the confining potential is required to be the static limit of retarded scalar and vector potentials analogous to the Lienard-Wiechert potentials of classical electrodynamics. A generalization of Darwin's method is used to obtain the corresponding Hamiltonian. The long-distance spin-dependent interaction is found to be determined completely by only two potentials: namely, the static scalar and vector potentials. This is to be compared with the four potentials required in Eichten and Feinberg's general formulation. Two different solutions are allowed by Gromes's theorem. In one, the scalar potential can be linear; in the other, it must be logarithmic.

At present the form of the long-distance (LD) spin-dependent interaction between quarks is not known and cannot be obtained from perturbation theory. It can be written in terms of bilinear products of Dirac matrices (the Dirac expansion) or in terms of products of Pauli matrices and the orbital angular-momentum operator (the Pauli expansion), but the potential functions multiplying these matrix operators are not known.[1] Although reasonable assumptions about the r dependence of these LD potential functions can be made, their relative magnitudes have not been established. Our purpose is to give a theoretical, albeit phenomenological, basis for determining these relative magnitudes for the case of scalar and vector static potentials.

The question of the spin dependence of the $q\bar{q}$ force has received considerable attention in the literature[2] but discussions of the LD component are almost inevitably based on a simple assumption for the form of the Dirac expansion. From this assumption the corresponding Pauli expansion can be obtained by some method of reduction. For example, one may assume a Dirac expansion of the form[3] $S+V\gamma_{1\mu}\gamma^{2\mu}$ in the Bethe-Salpeter equation,[4] or generalized Breit interactions,[5] $(V_1+\beta_1\beta_2 S_1)\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_1\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_2$ and $(V_2+\beta_1\beta_2 S_2)\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2$, may be used in the Breit equation. Some authors also include a tensor term.[6] In each case the spin dependence is generated through the corresponding Pauli expansion.[7]

The most general treatment is given by Eichten and Feinberg[8] who obtain expressions for the potential functions in the Pauli expansion where the potentials are expressed in terms of integrals over the magnetic and electric color fields, but these fields are not easily evaluated. In a similar approach Gromes[9] uses Lorentz invariance to derive a relationship between the static potential and the two spin-orbit potentials in the Pauli expansion. An important consequence of his theorem is the necessity of including a long-distance component in the spin-dependent interaction. Pantaleone, Tye, and Ng[1] generalize the formalism in Refs. 8 and 9 and study the consistency of the parameters used in various potential-model calculations.[10] They conclude that no long-distance correction is necessary for the Υ system but that some type of nonperturbative relativistic corrections is needed for the charmonium system.

We suggest that the appropriate form for the Dirac expansion of the LD spin-dependent interaction can be derived by an approach that begins with the classical interaction energy between two relativistic particles. In general, the interaction energy is velocity dependent and may be expanded in powers of v^2/c^2: i.e.,

$$U=U^{(0)}(r)+U^{(2)}(\mathbf{r},\mathbf{v}_i)+O(v^4/c^4)\ , \tag{1}$$

where $U^{(0)}(r)$ is the static potential and $U^{(2)}(\mathbf{r},\mathbf{v}_i)$ is the velocity-dependent potential of order v^2/c^2. In QCD it is convenient to discuss the short-distance (SD) and long-distance (LD) behaviors of the interaction separately. The interaction may therefore be divided into four parts: (1) the SD static potential $U^{(0)}_{\rm SD}$; (2) the SD velocity-dependent potential $U^{(2)}_{\rm SD}$; (3) the LD static potential $U^{(0)}_{\rm LD}$; and (4) the LD velocity-dependent potential $U^{(2)}_{\rm LD}$. The quantized version of the first two potentials $U^{(0)}_{\rm SD}$ and $U^{(2)}_{\rm SD}$ can be derived from single-gluon-exchange diagrams which give (for the $q\bar{q}$ state)

$$U^{(0)}_{\rm SD}=-\tfrac{4}{3}(\alpha_s/r)\ , \tag{2}$$

and

$$U^{(2)}_{\rm SD}=\tfrac{4}{3}(\alpha_s/2r)(\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2+\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_1\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_2)\ , \tag{3}$$

where the annihilation potential has been omitted for simplicity. The third potential $U^{(0)}_{\rm LD}$ is commonly assumed to be linear on the basis of descriptions in terms of flux tubes[11] or relativistic strings,[12] but a logarithmic or small power-law behavior is also used due to its success in phenomenological calculations. The fourth potential $U^{(2)}_{\rm LD}$, which is a primary source of the LD spin dependence, is considered in the literature at the classical level by Woodcock and Havas,[13] and in a more restricted sense by Bopp as quoted by Bagge.[14] The implications of their results for the quantum-mechanical case are discussed elsewhere.[5] Possible forms for the quantum potential $U^{(2)}_{\rm LD}$ are considered by various authors.[5,15,16] However the Pauli expansion of the scalar interaction

given by Olsen and Miller is inconsistent with the results of Woodcock and Havas[13] as shown in Ref. 5, and the results of Ref. 16 for the most general form for the vector interaction represent, in fact, only a special case.[5]

A derivation of $U_{\rm LD}^{(2)}$ similar to the deviation on Eqs. (2) and (3) is not possible because perturbation theory is not valid at large distances. However, another approach may be considered. For example, Eqs. (2) and (3) can be obtained without using the perturbative expansion of quantum field theory. The original derivation of Eq. (3) in 1929 is based on a classical expression for the interaction energy between two electric charges obtained by Darwin.[17] His method does not use perturbation theory. It starts with the Lienard-Wiechert potentials of classical electrodynamics, $\phi=e/s$ and $\mathbf{A}=\mathbf{v}(t_r)\phi/c$, where $s=R-\mathbf{R}\cdot\mathbf{v}(t_r)/c$, $R=c(t-t_r)$, and t_r is the retarded time. Darwin shows to order v^2/c^2 that the two-time dependence of the potentials can be reduced to a dependence on quantities evaluated at a single time t and that a classical Lagrangian can be constructed which is independent of the particle's acceleration. Breit[18] transcribed Darwin's results into quantum-mechanical form and obtained Eq. (3).

A direct application of Darwin's method is difficult, if not impossible, in chromodynamics because of the nonlinearity of the theory. Therefore, we assume first that the principal effect of the nonlinearity is the confinement behavior of the interaction at large distances and, second, that for each particle an effective potential field can be constructed which simulates the effect of the flux tube on the other particle. The description of the effective fields is linear in that the potential energy of one particle is regarded as a consequence of that particle moving through the effective field of the other particle. We impose only two conditions on the LD effective field: (1) it must transform as a Lorentz scalar or a Lorentz four-vector and (2) it must be a retarded field, i.e., one that depends on both t and the retarded time $t_r=t-R/c$.

The Lienard-Wiechert potentials provide the insight needed for the construction of the effective retarded potentials satisfying the above three conditions. We notice that the invariant magnitude of the four-vector Lienard-Wiechert potential is $[e/s\gamma(t_r)]^2$, where $\gamma=(1-v^2/c^2)^{-1/2}$. Clearly, $s\gamma(t_r)$ is an invariant. Since a scalar potential is, by definition, a Lorentz invariant, the most general choice for the retarded scalar interaction which behaves as $S(r)$ in the static limit is

$$\Lambda=S(\gamma s)\ . \tag{4}$$

Likewise a retarded four-vector potential $\Phi=(\phi,\mathbf{A})$ can be constructed by requiring that $\Phi\cdot\Phi=\phi^2-\mathbf{A}\cdot\mathbf{A}$ is an invariant. It follows that

$$\phi=\gamma V(\gamma s) \tag{5}$$

and

$$\mathbf{A}=\frac{\mathbf{v}(t_r)}{c}\phi\ , \tag{6}$$

where γ is evaluated at the retarded time t_r. Equations (4)–(6) form the basis for the calculation of the two-particle interaction energy. They reduce to the Lienard-Wiechert potentials of electrodynamics when $S=0$ and $V=\alpha/\gamma s$.

Darwin[17] obtained the second-order potential $U^{(2)}$ for electrodynamics by considering the Lagrangian for a single particle moving through the potential of the other particle. This method is not valid for arbitrary potentials because it involves the addition of a term to the Lagrangian that destroys the relativistic invariance of the corresponding action. However this problem does not arise in the construction of the two-particle Lagrangian. Let Λ_{ij} and Φ_{ij} be the scalar and four-vector potentials, respectively, of the jth particle evaluated at the site of the ith particle and expand Λ_{ij} and Φ_{ij} about the time t. This is equivalent to an expansion in powers of v^2/c^2, and it can be shown that

$$\Lambda_{ij}=S(r)+rS'(r)[\mathbf{r}\cdot\mathbf{a}_j+(\hat{\mathbf{r}}\cdot\mathbf{v}_j)^2]/2c^2\ , \tag{7}$$

$$\Phi_{ij}=V(r)+\{V(r)v_j^2+rV'(r)[\mathbf{r}\cdot\mathbf{a}_j+(\hat{\mathbf{r}}\cdot\mathbf{v}_j)^2]\}/2c^2\ , \tag{8}$$

and $\mathbf{A}_{ij}=V(r)\mathbf{v}_j/c$, where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and $\mathbf{v}_j,\mathbf{a}_j$ are evaluated at time t. For $V(r)=\alpha/r$ Eq. (8) is identical to the corresponding expression obtained by Darwin.[17] The invariant action I_{12} for particles 1 and 2 can be constructed from the Lorentz scalars Λ_{ij}, m_i, and $u_i\cdot\Phi_{ij}$, where $u_i=(\gamma_i c,\gamma_i\mathbf{v}_i)$ is the four-vector velocity of the ith particle. This suggests the following form for the action:

$$I=-\sum(m_ic^2+\Lambda_{ij}/2+u_i\cdot\Phi_{ij}/2c)d\tau_i\ ,$$

where $d\tau_i=dt/\gamma_i$ is the proper time of the ith particle. However, the above expression depends on the accelerations of the particles. This is the reason another term must be added to I_{12}. This additional action term must be the integral of a total derivative, it must be Lorentz invariant to order $1/c^2$, and, of course, it must remove the acceleration dependence. The appropriate term is

$$I=\frac{1}{4c^2}\frac{d}{dt}\{r[V'(r)+S'(r)]\mathbf{r}\cdot(\mathbf{v}_2-\mathbf{v}_1)\}dt\ .$$

The above expression differs from the one used by Darwin[17] for $V=\alpha/r$ by the presence of the velocity v_1. But it is precisely the inclusion of v_1 that makes I_{12} invariant to order $1/c^2$. The resulting action is $I'_{12}=I_{12}+I=L_{12}dt$ and the two-particle Lagrangian is

$$L_{12}=-\sum_{i\neq j}(m_ic^2+\Lambda_{ij}/2+u_i\cdot\Phi_{ij}/2c)/\gamma_i+(1/4c^2)\frac{d}{dt}[r(V'+S')\mathbf{r}\cdot(\mathbf{v}_2-\mathbf{v}_1)]$$

$$=-m_1c^2/\gamma_1-m_2c^2/\gamma_2-V-S-H_V-H_S\ ,$$

where

$$H_V = -\left[(rV'+2V)\mathbf{v}_1\cdot\mathbf{v}_2+(r^2V''+rV')\hat{\mathbf{r}}\cdot\mathbf{v}_1\hat{\mathbf{r}}\cdot\mathbf{v}_2-\sum_i[(rV'+V)v_i^2+(r^2V''+2rV')(\hat{\mathbf{r}}\cdot\mathbf{v}_i)^2]/2\right]\Big/2c^2 , \tag{9}$$

$$H_S = -\left[rS'\mathbf{v}_1\cdot\mathbf{v}_2+(r^2S''+rS')\hat{\mathbf{r}}\cdot\mathbf{v}_1\hat{\mathbf{r}}\cdot\mathbf{v}_2-\sum_i[(rS'-S)v_i^2+(r^2S''+2rS')(\hat{\mathbf{r}}\cdot\mathbf{v}_1)^2]/2\right]\Big/2c^2 . \tag{10}$$

The generalized momentum is $p_i=\partial L/\partial q_i$ and the corresponding Hamiltonian is $H_{12}=\sum p_i\dot{q}_i-L_{12}$. Carrying out the indicated operations, we find that

$$H_{12}=[(m_1c^2)^2+(cp_1)^2]^{1/2}+[(m_2c^2)^2+(cp_2)^2]^{1/2}+U_{\rm LD} , \tag{11}$$

where $U_{\rm LD}=S+V+H_V+H_S$ is the long-distance interaction energy.

A useful check of Eqs. (9) and (10) is provided by the work of Woodcock and Havas[13] who obtain the most general single-time, classical Lagrangian derivable from a Lorentz-invariant variational principle. The Lagrangian is determined by an interaction kernel $U_{\rm WH}$ depending on the two-body invariants ω, ξ, χ, and σ, where σ is the four-dimensional separation of the two particles, $\omega=u_1\cdot u_2$,

$$\xi=\gamma_2\{c(t_1-t_2)-\mathbf{v}_2(t_2)\cdot[\mathbf{r}_1(t_1)-\mathbf{r}_2(t_2)]/c\} ,$$

and χ is given by the above expression for ξ, but with $1\leftrightarrow2$. The interaction kernel that corresponds to our Lienard-Wiechert potentials is

$$U_{\rm WH}=\tfrac{1}{2}\omega^l[f(\xi)+f(\chi)]\delta(\sigma) ,$$

where $l=0$ gives H_S, $l=1$ gives H_V, $f(\xi)$ is an arbitrary even function of ξ, and the static potential is $f(r)/r$.

The final step is the quantization of Eq. (11). Generally the transition from classical to quantum mechanics is straightforward, but this is not the case for H_V and H_S. The problem is that classically $\mathbf{p}_i/m_i$ differs from $\mathbf{v}_i$ only by terms of order $1/c^2$. Since these variables appear in the second-order part of $H_{\rm int}$ and $H_{\rm int}$ is only valid to that order, $\mathbf{p}_i/m_i$ and $\mathbf{v}_i$ in $H_{\rm int}$ are effectively identical at the classical level. However at the quantum level for spin-$\frac{1}{2}$ particles, they are represented by completely different operators, i.e., $\mathbf{p}_i\rightarrow-ih\nabla_i$ and $\mathbf{v}_i\rightarrow c\boldsymbol{\alpha}_i$. To our knowledge there is no fundamental basis for resolving this ambiguity. However the correct form for the quantum-mechanical H_V is known for $V=\alpha/r$, and we can use this to determine the correct quantization of the first two terms in H_V. Comparing Eqs. (3) and (9), we find that $\mathbf{v}_1\cdot\mathbf{v}_2$ and $\hat{\mathbf{r}}\cdot\mathbf{v}_1\hat{\mathbf{r}}\cdot\mathbf{v}_2$ should be represented by $c^2\boldsymbol{\alpha}_1\cdot\boldsymbol{\alpha}_2$ and $c^2\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_1\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_2$, respectively (at least in H_V). It can be argued that for the last two terms in both Eq. (9) and Eq. (10), $\mathbf{v}_i$ should be replaced by $-i\nabla_i/m_i$. The other alternative, i.e., replacing $\mathbf{v}_i$ by $c\boldsymbol{\alpha}_i$, has the effect of making the v_i^2 and $(\hat{\mathbf{r}}\cdot\mathbf{v}_i)^2$ terms equivalent in importance (order) to the zero-order potential $V(r)+S(r)$. Recall that $\alpha_i^2=3$ and $(\hat{\mathbf{r}}\cdot\boldsymbol{\alpha}_i)^2=1$. In that case the static potential in the quantum-mechanical calculation would be different from the static potential in the classical problem, and we assume that this is not the case.

For the quantization of the first two terms in Eq. (10) we consider two possibilities: (i) The same quantization procedure should be used for both Hamiltonians; (ii) all velocities $\mathbf{v}_K$ in H_S should be replaced by $-i\nabla_K/m_K$. All other quantization schemes yield results which are not consistent with Gromes's theorem. In both of the above cases a factor $\beta_1\beta_2$, which is represented by unity at the classical level, must be inserted before every scalar term.

The most striking difference between these two possible quantization schemes is the different types of spin dependence which they imply. The reduction to Schrödinger-Pauli form has been carried out elsewhere.[5] If the spin-independent terms are omitted for simplicity, the results may be written as

$$\begin{aligned}H_V+H_S\rightarrow{}&\frac{1}{4r}(\epsilon'+2W_1')\\&\times\left[\frac{\boldsymbol{\sigma}_1}{m_1^2}+\frac{\boldsymbol{\sigma}_2}{m_2^2}\right]\cdot\mathbf{L}+\frac{W_2'}{2m_1m_2r}\boldsymbol{\sigma}\cdot L\\&+\left[\frac{V'}{r}-V''\right]\frac{T}{4m_1m_2}+\left[\frac{\nabla^2V+W}{6m_1m_2}\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\\&-\frac{S_1}{2m_1m_2r^2}(\boldsymbol{\sigma}_1\cdot\mathbf{L}\boldsymbol{\sigma}_2\cdot\mathbf{L}+\boldsymbol{\sigma}_2\cdot\mathbf{L}\boldsymbol{\sigma}_1\cdot\mathbf{L}) ,\end{aligned} \tag{12}$$

where $T=\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_1\hat{\mathbf{r}}\cdot\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2/3$. In case (i) one finds that

$$S_1=\frac{r}{2}\frac{d}{dr}(rS') ,$$

$$W_2=V-\left[\frac{r^2}{2}S''+2rS'\right] ,$$

$$W=-3[rS'p^2-(S+rS'')i\hat{\mathbf{r}}\cdot\mathbf{p}] ,$$

and in case (ii), $S_1=W=0$ and $W_2=V$. In both cases $\epsilon=V+S$ and $W_1=-S$.

Gromes[9] has derived a theorem relating the static and spin-orbit potentials, which may be written as

$$\epsilon'+W_1'=W_2' . \tag{13}$$

Although the primes in Eq. (13) are usually omitted, it is important to include them in case (i). Gromes's theorem has the somewhat unexpected consequence of determining the form of the scalar potential in case (i). Equation (13) is satisfied for any $V(r)$, and it reduces to the following equation for $S(r)$:

$$\frac{d}{dr}\left[\frac{r^2}{2}S''+2rS'\right]=0\ . \tag{14}$$

The solution to Eq. (14) is the logarithmic potential[19]

$$S(r)=a\ \ln(r)+b\ . \tag{15}$$

Notice also that Eq. (15) implies that $S_1=0$, so the quadratic spin-orbit term does not contribute to Eq. (12). That is appropriate since the general form for the QCD potential given in Ref. 8 does not contain such a term. In case (ii) Gromes's theorem is satisfied for any $V(r)$ and $S(r)$.

For the special case of pure scalar confinement the spin-dependent part of $H_{\rm int}$ is quite simple: i.e.,

$$H_{\rm spin}=-\frac{S'(r)}{4r}\left[\frac{\boldsymbol{\sigma}_1}{{m_1}^2}+\frac{\boldsymbol{\sigma}_2}{{m_2}^2}\right]\cdot\mathbf{L}+\frac{W}{6m_1m_2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ , \tag{16}$$

where in case (i) $W=-3ap^2$ and $S(r)=a\ \ln(r)+b$ and in case (ii) $W=0$ and $S(r)$ is arbitrary.

For pure vector confinement the spin-dependent part of Eq. (12) is

$$H_{\rm spin}=\frac{V'}{4r}\left[\frac{\boldsymbol{\sigma}_1}{{m_1}^2}+\frac{\boldsymbol{\sigma}_2}{{m_2}^2}\right]\cdot\mathbf{L}+\frac{V'}{2m_1m_2r}\boldsymbol{\sigma}\cdot\mathbf{L}+\left[\frac{V'}{r}-V''\right]\frac{T}{m_1m_2}+\frac{\nabla^2V}{6m_1m_2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ . \tag{17}$$

Although the above is intended as a description of the long-distance part of the QCD interaction, it is worth noting that if one sets $V=\alpha/r$, Eq. (17) reduces to usual Fermi-Breit interaction.

In conclusion we have shown how to include retardation effects in the long-distance part of the QCD interaction. This has been done by assuming a simple ansatz for the long-distance Lienard-Wiechert-type potential and using a method based on the original derivation given by Darwin for electrodynamics. The classical interaction Hamiltonian which we obtain is consistent with the results of Woodcock and Havas,[13] and its quantized version satisfies Gromes's theorem.[9] The description of the long-distance QCD interaction is simplified considerably by our result since the Hamiltonian contains only two potentials, $V(r)$ and $S(r)$, rather than the four potentials introduced in Ref. 8.

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