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Compact three-dimensional U(1) gauge theory reexamined

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Convincing evidence of a nonvanishing string tension in the continuum limit of compact three-dimensional U(1) gauge theory is presented. It is based on Monte Carlo measurements of Wilson loops on a 32^3 lattice at $\beta=2.0$ and 2.2. The observed string tensions at these couplings are consistent with the Polyakov theory. Also, a very clean signal of a string vibrational contribution to the potential is observed.

I. INTRODUCTION

Compact U(1) gauge theory in $2+1$ dimensions is the most well-understood nontrivial gauge theory. The pioneering work is due to Polyakov.[1] The partition function with two external charges is that of a Coulomb gas of magnetic monopoles interacting with an electric current loop. This gas is always in the plasma phase. Polyakov showed that at arbitrarily large finite β there is a mass gap and a nonvanishing string tension given by

$$\sigma a^2=\frac{4\sqrt{2}}{\pi\sqrt{\beta_V}}\exp[-\pi^2V(0)\beta_V] , \tag{1}$$

where $V(0)=0.2527$ and β_V is the inverse coupling constant as defined by the Villain action (the latter is a large-β approximation to the Wilson action).

Several numerical studies of three-dimensional U(1) gauge theory [$U(1)_3$] have been performed previously.[2-5] In Ref. 2 the monopole density was studied and the physical picture of confinement was confirmed. References 3 and 4 focused on Wilson loop measurements. In Ref. 3 an exploratory study was made, whereas in Ref. 4 fairly large loops (8×8) were probed with reasonable statistics. The results were nevertheless inconclusive. The reason for this, according to the authors, is that the distance scales probed by the measured Wilson loop sizes are too small in order to disentangle the three-dimensional Coulomb term ($\ln R$) from the linear potential (σR) originating from string formation. Thus to date no conclusive numerical evidence for $\sigma\neq0$ in the continuum limit exists. One should also mention Ref. 5, where a mass gap was established using the Villain action and a dual method. Also the derivative of the string tension was measured and found to be consistent with predictions from Eq. (1). This unclear situation calls for a new measurement of large Wilson loops with high statistics.

Another reason for reexamining $U(1)_3$ theory is that, because of its simplicity, it is often a testing ground for new algorithmic approaches. The conventional method for determining the static force between two charges on the lattice is by measuring Wilson loops. These vacuum expectation values are exponentially damped for confining theories, and hence very time consuming to measure. Ideally one would like to generate configurations including the charges in the action. This gives a complex action, which is impossible to handle with standard updating procedures. Alternative algorithms have therefore been suggested: the dual method[6] and the complex Langevin equation.[7] Being reasonably simple and theoretically well known in the $\beta\to\infty$ limit $U(1)_3$ theory is a good testing ground for these new algorithms. The dual method[6] gives values a factor 2 larger for σ than those of Ref. 4, whereas the complex Langevin approach[7] did not provide any evidence for string formation at all. This situation with regard to new approaches is another strong incentive for a new numerical investigation within the Wilson loop paradigm.

We have measured Wilson loops up to sizes 14×12 on a 32^3 lattice with the Wilson action using the variance reduction technique of Ref. 8. The string tension is carefully extracted using two different techniques, yielding a σ value of the same order of magnitude as that of Ref. 4. Our σ values obtained at $\beta=2.0$ and 2.2 are consistent with Eq. (1) indicating that we are in the continuum limit.

As a by-product we also find strong numerical evidence for the presence of a string vibration term $-\pi(d-2)/24R$ in the potential. This fact is, of course, an additional piece of numerical support for string formation in $U(1)_3$ theory. It turns out that this contribution to the potential is what in Ref. 4 was interpreted as a Coulomb term.

TABLE I. Details of the Monte Carlo runs.

	$\beta=2.0$	$\beta=2.2$
No. of sweeps used for thermalization	>5 000	>5 000
No. of sweeps used for measurements	22 000	11 700
No. of sweeps between two measurements for loops		
Smaller than 8×8	90	90
Larger than 8×8	30	30

This paper is organized as follows. In Sec. II we describe our numerical procedures and the data. Section III contains the string-tension extractions and the results are presented and discussed in Sec. IV.

II. MONTE CARLO CALCULATIONS

Using the familiar Wilson action and the Metropolis algorithm for updating, we have measured Wilson loops on a 32^3 lattice at $\beta=2.0$ and 2.2. The variance was reduced by use of the technique of Ref. 8, which amounts to replacing link variables U_l in the measured objects by their local averages

$$\overline{U}_1=\frac{\int dU\, U \exp[\beta \operatorname{Re}(UX_l)]}{\int dU \exp[\beta \operatorname{Re}(UX_l)]}, \quad U_lX_l=\sum_{\Box \supset l} U_\Box . \tag{2}$$

TABLE II. Wilson loop values $W(T,R)$ with errors at (a) $\beta=2.0$ and (b) $\beta=2.2$.

(a)

T \ R	2	3	4	5	6	7	8	9	10
2	0.496 67								
	0.000 40								
3	0.375 84	0.261 66							
	0.000 51	0.000 61							
4	0.286 41	0.184 78	0.121 95						
	0.000 54	0.000 61	0.000 58						
5	0.218 80	0.131 14	0.081 18	0.050 87					
	0.000 55	0.000 57	0.000 52	0.000 45					
6	0.167 24	0.093 25	0.054 21	0.032 06	0.019 13				
	0.000 52	0.000 51	0.000 44	0.000 37	0.000 30				
7	0.127 92	0.066 36	0.036 25	0.020 25	0.011 44	0.006 51			
	0.000 46	0.000 44	0.000 37	0.000 30	0.000 24	0.000 18			
8	0.097 67	0.047 09	0.024 13	0.012 71	0.006 74	0.003 60	0.001 92		
	0.000 40	0.000 35	0.000 28	0.000 21	0.000 16	0.000 11	0.000 09		
9	0.074 69	0.033 53	0.016 15	0.008 03	0.004 02	0.002 04	0.001 02	0.000 51	
	0.000 35	0.000 29	0.000 22	0.000 16	0.000 12	0.000 09	0.000 07	0.000 06	
10	0.057 15	0.023 89	0.010 81	0.005 07	0.002 41	0.001 15	0.000 54	0.000 25	0.000 12
	0.000 30	0.000 23	0.000 17	0.000 12	0.000 09	0.000 07	0.000 05	0.000 04	0.000 03

(b)

T \ R	2	3	4	5	6	7	8	9	10	11	12
2	0.550 81										
	0.000 33										
3	0.436 80	0.324 83									
	0.000 43	0.000 55									
4	0.348 73	0.244 88	0.175 59								
	0.000 47	0.000 45	0.000 50								
5	0.279 12	0.185 54	0.127 01	0.088 04							
	0.000 51	0.000 60	0.000 63	0.000 59							
6	0.223 68	0.140 95	0.092 28	0.061 41	0.041 19						
	0.000 49	0.000 56	0.000 57	0.000 53	0.000 47						
7	0.179 37	0.107 26	0.067 22	0.043 00	0.027 77	0.018 02					
	0.000 49	0.000 57	0.000 56	0.000 51	0.000 45	0.000 31					
8	0.143 89	0.081 72	0.049 03	0.030 16	0.018 77	0.011 82	0.007 48				
	0.000 49	0.000 54	0.000 53	0.000 48	0.000 42	0.000 35	0.000 32				
9	0.115 40	0.062 21	0.035 77	0.021 19	0.012 74	0.007 76	0.004 74	0.002 91			
	0.000 46	0.000 50	0.000 48	0.000 43	0.000 36	0.000 29	0.000 25	0.000 16			
10	0.092 56	0.047 38	0.026 10	0.014 90	0.008 65	0.005 09	0.003 02	0.001 79	0.001 07		
	0.000 43	0.000 45	0.000 42	0.000 37	0.000 30	0.000 23	0.000 19	0.000 12	0.000 09		
11	0.074 25	0.036 09	0.019 05	0.010 48	0.005 89	0.003 35	0.001 91	0.001 10	0.000 63	0.000 36	
	0.000 40	0.000 40	0.000 37	0.000 32	0.000 25	0.000 18	0.000 15	0.000 09	0.000 07	0.000 05	
12	0.059 58	0.027 50	0.013 92	0.007 39	0.004 00	0.002 20	0.001 22	0.000 68	0.000 38	0.000 21	0.000 12
	0.000 37	0.000 35	0.000 32	0.000 27	0.000 21	0.000 15	0.000 10	0.000 07	0.000 05	0.000 04	0.000 03
13	0.047 81	0.020 97	0.010 18	0.005 21	0.002 73	0.001 46	0.000 78	0.000 43	0.000 24	0.000 13	0.000 07
	0.000 34	0.000 31	0.000 27	0.000 23	0.000 17	0.000 12	0.000 08	0.000 05	0.000 04	0.000 03	0.000 02
14	0.038 37	0.015 99	0.007 45	0.003 68	0.001 87	0.000 96	0.000 50	0.000 26	0.000 14	0.000 07	0.000 04
	0.000 30	0.000 27	0.000 23	0.000 19	0.000 14	0.000 08	0.000 06	0.000 04	0.000 03	0.000 02	0.000 02

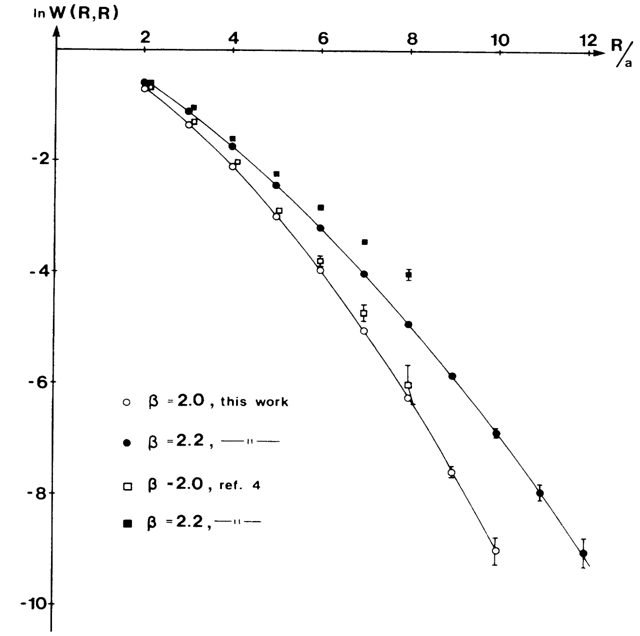

FIG. 1. $\ln W(R,R)$ as a function of R from this work and from Ref. 4 at $\beta=2.0$ and 2.2. The curves show the results of fits to Eq. (4).

In the case of U(1), we can write

$$\overline{U}_l = \frac{X_l^*}{d} \frac{I_1(\beta d)}{I_0(\beta d)}, \quad d = |X_l| , \tag{3}$$

where I_0 and I_1 are modified Bessel functions, which we can tabulate for a given β. Since we want to focus on long distances, we have chosen to measure larger loops more frequently than smaller ones. Such a choice is further motivated by the fact that autocorrelations are

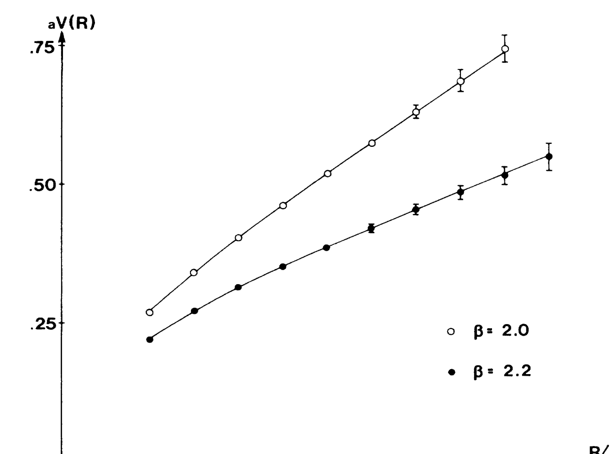

FIG. 2. The static potential $V(R)$. The errors have been estimated by dividing the data into 20 bins and regarding the corresponding values of $V(R)$ as independent measurements. The curves are fits to Eq. (6).

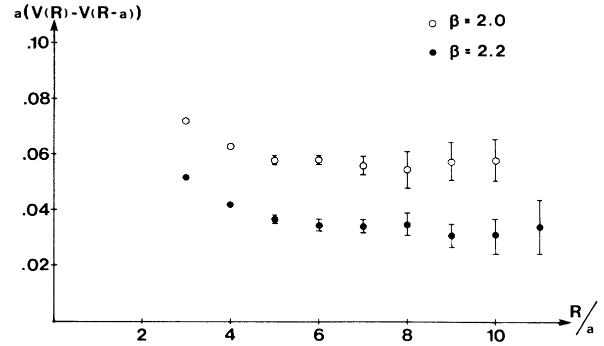

FIG. 3. The difference $V(R)-V(R-a)$. The errors were obtained in the same way as the errors in $V(R)$.

more long ranged for the smaller loops.[9] Further details of the Monte Carlo (MC) runs are found in Table I.

Our results for the Wilson loops are given in Tables II(a) and II(b). The quoted errors are corrected for autocorrelations. We may note that the statistics are considerably improved, by roughly a factor of 10 for larger loops, as compared to previous measurements.[4]

III. THE STRING-TENSION EXTRACTION

We have used two different methods for extracting σ from the Wilson loop measurements. The first method follows Ref. 4, where quadratic loops were fitted to the form

$$-\ln W(R,R) = \sigma R^2 + PR + c \ . \tag{4}$$

As seen from Fig. 1, this form gives a good description of our data. The statistics obtained in Ref. 4 did not allow the authors to exclude the possibility that the curvature may originate from a perturbative term $R \ln R$. To make sure that this is not the case in our data we have performed fits for different lower cuts in R. Excluding $R=2$, and in the case of $\beta=2.2$ also $R=3$, our results are stable to variations of this lower cut. This suggests that we get reliable values for σ. In Fig. 1 we also give the results for the quadratic loops obtained in Ref. 4. Especially for larger loops, we see that our values differ considerably from theirs. This might be a thermalization effect. Whereas the authors of Ref. 4 state that at

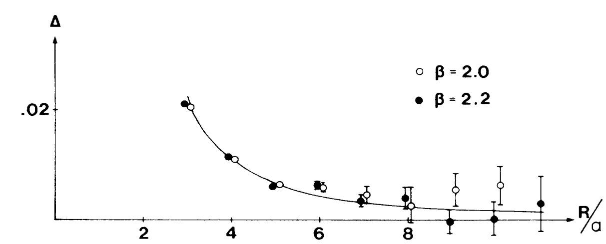

FIG. 4. The quantity $\Delta = a[V(R)-V(R-a)]-\sigma a^2$, with σa^2 given by Eq. (7). The curve corresponds to a vibrational term $-\pi(d-2)/24R$ in the potential.

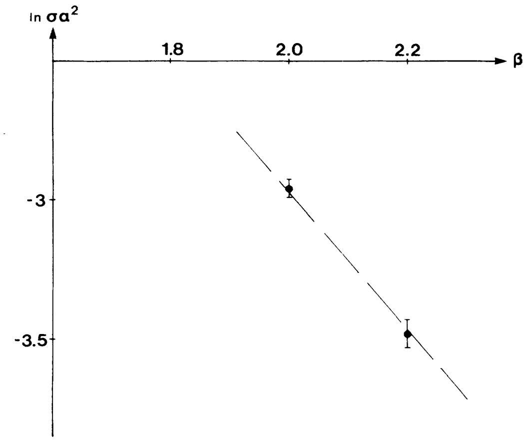

FIG. 5. $\ln\sigma a^2$ as a function of β. The dashed line indicates the slope obtained from Eq. (1).

least 400 sweeps are required to thermalize larger loops we find it necessary to use around 5000.

Another and more frequently used method for the string-tension extraction is to establish the linear behavior at large R for the static potential

$$V(R)=-\lim_{T\to\infty}\frac{1}{T}\ln W(T,R) . \tag{5}$$

Compared to the previous one, this method has the advantage of not only making use of quadratic loops, thereby improving the statistics. We have chosen to fit our results for $V(R)$ to the form

$$V(R)=\sigma R+c-\alpha/R \tag{6}$$

(see Fig. 2) expected at large R in a fluctuating string picture.[10] We find that this form allows for good fits, which are stable to exclusions of small R's. From the point of view of the string-tension determination, the assumption of a vibrational term in Eq. (6) is not important. Extracting σ directly from the slope at large R does not significantly change the results.

In order to exhibit more clearly the behavior of $V(R)$ as a function of R, we have also plotted the difference $V(R)-V(R-a)$ (see Fig. 3). A constant value of this difference signals the dominance of a linear term and this is indeed what we observe at large R. Furthermore, the deviations from a constant behavior seen at small R are very well described by the vibrational term in Eq. (6), taking for α the value $\pi(d-2)/24$ as predicted by scalar string theory.[10] This is illustrated in Fig. 4, where we have subtracted from the difference $V(R)-V(R-a)$ a constant piece corresponding to the linear term. One should note that whereas the string tension (in lattice units) decreases by almost a factor $\frac{1}{2}$ when going from $\beta=2.0$ to $\beta=2.2$ the magnitude of the deviations at small R remains essentially unaltered. Such scaling behavior is expected if the deviation originates from a universal $1/R$ term in the potential.

TABLE III. Results for σa^2 from different fits.

Fit to	$\beta=2.0$	$\beta=2.2$
Eq. (4)	0.054(3)	0.032(3)
Eq. (6), α free	0.053(3)	0.031(3)
Eq. (6), α fixed	0.052(2)	0.031(2)

IV. RESULTS

We thus see clear evidence for area-law behavior for the Wilson loops. For final values of σ we have chosen to fit to Eq. (6) keeping $\alpha=\pi(d-2)/24$ fixed. We then arrive at

$$\sigma a^2=\begin{cases}0.052(2), & \beta=2.0 ,\\ 0.031(2), & \beta=2.2 .\end{cases} \tag{7}$$

Fitting Eq. (4) or Eq. (6) with α as a free parameter gives slightly larger but consistent values of σ (see Table III). Since we are using the full Wilson action and not the Villain approximation, which has a different β parameter, we cannot directly check whether these numbers scale according to Eq. (1). However, one expects that the string-tension slope in the two theories should be identical as the continuum limit is approached. In Fig. 5 we show the logarithm of the values of Eq. (7) together with the slope predicted by Eq. (1). As can be seen from this figure that our data are consistent with the slope of Eq. (1). From this we conclude that our measurements are performed in the continuum region and that the values agree with the Polyakov theory.[1]

When fitting all three parameters in Eq. (6), we obtain, for the vibrational coefficient,

$$\alpha=0.11(4) \text{ and } 0.13(4)$$

for $\beta=2.0$ and 2.2, respectively. This result is in agreement with the string-model prediction $\pi(d-2)/24 \approx 0.13$.

As mentioned above, alternative numerical approaches have been applied to compact $U(1)_3$ theory. In Ref. 6 a dual algorithm using the Wilson action yielded $\sigma=0.11$ and 0.055 for $\beta=2.0$ and 2.2, respectively. These values disagree by a factor of 2 with our results and do not follow the slope of Eq. (1). A possible explanation of this discrepancy could be that this algorithm fails to efficiently generate configurations outside the strong-coupling domain.

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