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Fermions in the chiral Schwinger model

Shogo Miyake
Department of Engineering Science, Tohoku University, Sendai 980, Japan

Ken-ichi Shizuya
Department of Physics, Tohoku University, Sendai 980, Japan
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We derive an operator solution for the fermion in the chiral Schwinger model with a Wess-Zumino term and study the quantum structure of the model in a manifestly covariant operator formalism. The $U(1)_L$ gauge symmetry restored by the inclusion of the Wess-Zumino term gets spontaneously broken and the gauge field becomes massive. The left-handed fermion is found to be confined. The right-handed fermion, on the other hand, remains a massless free field in spite of the fact that the left- and right-handed sectors of the model are coupled through the anomaly. This massless fermion is interpreted as the Nambu-Goldstone mode associated with the spontaneous breakdown of the global $U(1)_R$ symmetry.

I. INTRODUCTION

Recently there has been considerable interest in the study of the chiral Schwinger model.[1-4] The motivation stems from exploring, in this model, the possibility of constructing a consistent quantum theory out of gauge theories with anomalies. Such a possibility was first pointed out by Jackiw and Rajaraman.[1] On the other hand, Faddeev[5] suggested that the Wess-Zumino term[6] may be included in an anomalous gauge theory to make the latter a physically sensible quantum theory. The analysis of Jackiw and Rajaraman can be understood in this context;[2,3] their effective action is obtained in the unitary gauge, where the Wess-Zumino term disappears.

In this paper we study the full consistency, as a quantum theory, of the chiral Schwinger model with the (negative) Wess-Zumino term. As yet, most approaches to this problem have dealt with the bosonized form of the model. Here our analysis places special emphasis on the fermion and symmetry contents of the model. We construct a fermion operator out of the boson fields in the bosonized theory and show that it behaves properly as the fermion in this model. We carry out the quantization of the model within a manifestly covariant operator formalism with indefinite metric, developed by Nakanishi.[7] Our analysis confirms that this chiral Schwinger model possesses consistent fermion contents and reveals further that one of the chiral fermions (the left-handed one) is confined.

In Sec. II we examine the anomaly structure of the chiral Schwinger model. It turns out that the gauge anomaly spoils the conservation of both $U(1)_L$ and $U(1)_R$ chiral currents.

In Sec. III we consider the chiral Schwinger model with its anomaly canceled by the Wess-Zumino term. We verify, in an operator language, that the local $U(1)_L$ gauge symmetry is restored in this model.

In Sec. IV we quantize the bosonized version of the model. We find that the restored gauge symmetry gets spontaneously broken and that the associated Nambu-Goldstone boson becomes unphysical. The physical spectrum of the model agrees with the one obtained earlier.[1]

In Secs. V and VI we study the fermion content of the model. A fermion operator is constructed, which yields the correct canonical commutation relations and which correctly embodies the anomaly structure of the fermionic chiral currents. The left-handed fermion is found to be confined; its propagator has no pole in momentum space, like the fermion in the Schwinger model.[8] On the other hand, the right-handed fermion remains a massless free field in spite of the fact that the $U(1)_R$ fermion current obeys an anomalous conservation law. This massless fermion is understood as the Nambu-Goldstone mode associated with the spontaneous breakdown of the $U(1)_R$ symmetry. Section VII is devoted to summary and discussion.

II. ANOMALY IN THE CHIRAL SCHWINGER MODEL

The chiral Schwinger model describes a chiral coupling of the fermion ψ to a gauge field A_μ, with the Lagrangian $L=L_0+L_g$:

$$L_0=\bar{\psi}[i\partial\!\!\!/+e\sqrt{\pi}A_\mu\gamma^\mu(1-\gamma^5)]\psi-\tfrac{1}{4}F_{\mu\nu}{}^2 , \tag{2.1}$$

$$L_g=B\partial_\mu A^\mu+\tfrac{1}{2}\alpha B^2 , \tag{2.2}$$

where $\psi=(\psi_1,\psi_2)^t$ and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; $\gamma^0\equiv\sigma^1$, $\gamma^1\equiv i\sigma^2$, $\gamma^5\equiv-\gamma^0\gamma^1=\sigma^3$ so that $\gamma^\mu\gamma^5=\epsilon^{\mu\nu}\gamma_\nu$ with $\epsilon^{01}=-\epsilon^{10}=\epsilon_{10}=1$. (Our metric is $g^{00}=-g^{11}=1$.) The L_g is a gauge-fixing term corresponding to linear covariant gauges with parameter α and B is an auxiliary field. The chiral $U(1)_R\times U(1)_L$ symmetry is manifest in the Lagrangian L; in particular, the right-handed fermion ψ_1 is a free field. The L_0 is also invariant under the local $U(1)_L$ gauge transformations $A_\mu\to A_\mu+\partial_\mu\xi_L$ and

$$\psi \to \exp[ie\sqrt{\pi}\xi_L(1-\gamma^5)]\psi ,$$

although this gauge invariance is spoiled at the quantum level.

It is convenient to use Fujikawa's method[9] to calculate the gauge anomaly. The local chiral rotations (with parameters ξ_R and ξ_L) of ψ and $\bar{\psi}$ yield the Jacobian factor $\propto \mathrm{Tr}[(\xi_R-\xi_L)\gamma^5]$, which one may regularize using the cutoff $e^{\tau D}$ with the operator

$$D=[i\partial\!\!\!/+e\sqrt{\pi}A\!\!\!/(1-\gamma^5)]^2 .$$

The regularized Jacobian

$$\mathrm{Tr}[(\xi_R-\xi_L)\gamma^5 e^{\tau D}] \quad (\tau \to +0 \text{ in Euclidean space}) ,$$

which turns into the anomaly, is easily evaluated. As a result, both of the fermion chiral currents

$$J_L^{(0)\mu}=e\sqrt{\pi}\bar{\psi}\gamma^\mu(1-\gamma^5)\psi$$

and

$$J_R^{(0)\mu}=e\sqrt{\pi}\bar{\psi}\gamma^\mu(1+\gamma^5)\psi$$

obey the anomalous conservation laws

$$\partial_\mu J_L^{(0)\mu}=-\partial_\mu J_R^{(0)\mu}=-e^2(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\mu A_\nu . \tag{2.3}$$

The superscript (0) indicates that these currents are self-dual (or anti-self-dual), $\epsilon_{\mu\nu}J_L^{(0)\nu}=-J_{L\mu}^{(0)}$ and $\epsilon_{\mu\nu}J_R^{(0)\nu}=J_{R\mu}^{(0)}$. This duality property is a direct consequence of the matrix identity $\gamma^\mu\gamma^5-\epsilon^{\mu\nu}\gamma_\nu$, specific to two dimensions, which is respected in the path-integral derivation of the anomaly. We have confirmed that the Pauli-Villars regularization leads to the same result.

The form of the anomaly is determined uniquely up to the ambiguity arising from the addition of local counterterms to the vacuum functional.[10] In the present case, one can take into account this ambiguity to replace Eq. (2.3) by the conservation laws

$$\partial_\mu J_L^\mu=ae^2\partial_\mu A^\mu-e^2(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\mu A_\nu , \tag{2.4a}$$

$$\partial_\mu J_R^\mu=e^2(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\mu A_\nu , \tag{2.4b}$$

where a is a real constant parametrizing the ambiguity. [Note here that the gauge field responds to $\mathrm{U(1)}_L$ gauge transformations alone.] Consequently, only the $J_L^\mu-ae^2A^\mu=J_L^{(0)\mu}$ portion of the left-handed current J_L^μ is anti-self-dual while the $J_R^\mu=J_R^{(0)\mu}$ remains self-dual. In view of the duality structures of the currents, we can express them in the form

$$J_L^\mu=ae^2A^\mu+e(g^{\mu\nu}-\epsilon^{\mu\nu})\partial_\nu\phi , \tag{2.5a}$$

$$J_R^\mu=-e(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu\phi , \tag{2.5b}$$

where the field ϕ is defined by the equation

$$\partial^2\phi=-e(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\mu A_\nu . \tag{2.6}$$

The right-hand sides of Eq. (2.4) represent the most general form of the chiral anomaly. Any sensible regularization method should yield the anomaly of this form. It is possible to reproduce this result by means of a point-splitting prescription. The phase factors are needed separately for the (regularized) vector and axial-vector currents:

$$\begin{aligned} J_V^\mu(x;\epsilon)&=e\sqrt{\pi}\bar{\psi}(x+\epsilon)\gamma^\mu\Gamma_V(x+\epsilon,x)\psi(x) ,\\ J_A^\mu(x;\epsilon)&=e\sqrt{\pi}\bar{\psi}(x+\epsilon)\gamma^\mu\gamma^5\Gamma_A(x+\epsilon,x)\psi(x) . \end{aligned} \tag{2.7}$$

It is a simple exercise to verify that the desired form of the anomaly is obtained by the choice of the phase factors:

$$\begin{aligned} \Gamma_V(x+\epsilon,x)&=\exp\left[ie\sqrt{\pi}\int_x^{x+\epsilon}dz^\mu[(1+a)A_\mu+\epsilon_{\mu\nu}A^\nu]\right] ,\\ \Gamma_A(x+\epsilon,x)&=\exp\left[ie\sqrt{\pi}\int_x^{x+\epsilon}dz^\mu[A_\mu+(1-a)\epsilon_{\mu\nu}A^\nu]\right] . \end{aligned} \tag{2.8}$$

The result remains the same if one replaces $\epsilon_{\mu\nu}A^\nu$ by γ^5A_μ in the above.

III. CHIRAL SCHWINGER MODEL WITH THE WESS-ZUMINO TERM

We have studied the anomaly of the chiral Schwinger model in the previous section. Following Faddeev,[5] let us introduce the (negative) Wess-Zumino term L_{WZ} to cancel the anomaly and consider the new Lagrangian

$$\begin{aligned} L^{\mathrm{new}}&=L_0+L_{\mathrm{WZ}}+L_g ,\\ L_{\mathrm{WZ}}&=\tfrac{1}{2}(a-1)(\partial_\mu\theta)^2-e\theta[(a-1)g^{\mu\nu}-\epsilon^{\mu\nu}]\partial_\mu A_\nu , \end{aligned} \tag{3.1}$$

where θ is a scalar field.

The $\mathrm{U(1)}_L$ gauge symmetry, though not manifest in the Lagrangian L_0+L_{WZ}, is recovered in this modified chiral Schwinger model. To verify this, let us look into the equations of motion:

$$[i\partial\!\!\!/+e\sqrt{\pi}A\!\!\!/(1-\gamma^5)]\psi=0 , \tag{3.2}$$

$$\partial_\mu F^{\mu\nu}[A]-\partial^\nu B+J^\nu=0 , \tag{3.3}$$

$$(a-1)\partial^2\theta+e[(a-1)g^{\mu\nu}-\epsilon^{\mu\nu}]\partial_\mu A_\nu=0 , \tag{3.4}$$

$$\partial_\mu A^\mu+\alpha B=0 . \tag{3.5}$$

The gauge field is now coupled to the current

$$J^\mu=J_L^\mu+e[(a-1)g^{\mu\nu}+\epsilon^{\mu\nu}]\partial_\nu\theta , \tag{3.6}$$

which, in view of Eqs. (2.4a) and (3.4), is conserved, $\partial_\mu J^\mu=0$. As a consequence, B obeys a massless free-field equation

$$\partial^2B=0 . \tag{3.7}$$

Except for the gauge-fixing equation (3.5), all of the field equations are $U(1)_L$ gauge covariant when θ simultaneously undergoes the transformation $\theta \to \theta - e\xi_L$. Hence, the new theory recovers the gauge symmetry at the quantum level.

In the above, we have examined the equations of motion since we resort to the operator formalism. Alternatively, in the path-integral language, the gauge symmetry restoration is seen at the Lagrangian level: The gauge variation of the Wess-Zumino term precisely cancels the anomalous Jacobian factor arising from the gauge rotations of the fermion fields ψ and $\bar{\psi}$.

IV. BOSONIZED SYSTEM AND ITS CANONICAL QUANTIZATION

Equations (3.3)–(3.7) are combined with Eqs. (2.5a) and (2.6) to form a bosonic set of field equations. Note that these equations are reproduced from the effective Lagrangian

$$L_{\text{boson}} = -\tfrac{1}{4}F_{\mu\nu}{}^2 + \tfrac{1}{2}ae^2 A_\mu{}^2 + \tfrac{1}{2}(\partial_\mu \phi)^2 + eA_\mu(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu \phi + \tfrac{1}{2}(a-1)(\partial_\mu\theta)^2 + eA_\mu[(a-1)g^{\mu\nu} + \epsilon^{\mu\nu}]\partial_\nu\theta + L_g \ . \tag{4.1}$$

Except for the gauge-fixing term L_g, this Lagrangian is invariant under the $U(1)_L$ gauge transformations $A_\mu \to A_\mu + \partial_\mu \xi_L$, $\theta \to \theta - e\xi_L$, and $\phi \to \phi - e\xi_L$. We regard this boson system as a bosonized version of the chiral Schwinger model with a Wess-Zumino term and study its canonical quantization in this section. (In the unitary gauge $\theta = 0$, this system is regarded as a bosonized version of the original chiral Schwinger model.[1]) As pointed out by Nakanishi,[7] one has to introduce indefinite metric for a manifestly covariant quantization of massless scalar fields, as well as gauge fields, in two dimensions.

To determine the asymptotic fields and physical states in this boson system, one has to solve the field equations following from Eq. (4.1). It is a simple exercise to derive the operator solutions

$$aeA_\mu = \epsilon_{\mu\nu}\partial^\nu(\sqrt{a-1}\,V + \eta) + \sqrt{a}\,\partial_\mu(B_0 + X) \ , \tag{4.2}$$

$$\theta = a^{-1}[-\eta + (a-1)^{-1/2}V - \sqrt{a}\,X] \ , \tag{4.3}$$

$$\phi = a^{-1}[(a-1)\eta - \sqrt{a-1}\,V - \sqrt{a}\,X] \ . \tag{4.4}$$

Here V, η, X, and $B_0 \equiv (e\sqrt{a}\,)^{-1}B$ constitute the asymptotic fields of the system and are described by the Lagrangian

$$L_{\text{asy}} = \tfrac{1}{2}(\partial_\mu V)^2 - \tfrac{1}{2}m^2V^2 + \tfrac{1}{2}(\partial_\mu\eta)^2 + \tfrac{1}{2}(\partial_\mu X)^2 - \tfrac{1}{2}[\partial_\mu(X + B_0)]^2 + \tfrac{1}{2}\alpha a e^2 B_0{}^2 \ , \tag{4.5}$$

where

$$m^2 = e^2a^2/(a-1) \ . \tag{4.6}$$

The canonical (and field) commutation relations among the original fields A_μ, θ, ϕ, and B are reproduced from those of the asymptotic fields via the operator solutions (4.2)–(4.4). As a check of the operator solutions, one can easily confirm that the propagators directly obtainable from the original Lagrangian (4.1) are reproduced from the propagators of the asymptotic fields. Our result is a covariant-gauge version of the unitary-gauge result of the Jackiw and Rajaraman.[1] As noted in Ref. 1, in the case $a < 1$, the V field becomes tachyonic ($m^2 < 0$) and the theory is not unitary. Correspondingly, we shall henceforth take $a > 1$.

The commutation relations among V, η, B, and X are easily read from the Lagrangian (4.5). In what follows, we take the Landau gauge $\alpha = 0$ and avoid (inessential) complications due to the dipole-ghost part[7] of X. Then, η and X are massless fields of positive norm whereas $(B_0 + X)$ is a massless field of negative norm; V is a scalar field of mass m and positive norm. The (nontrivial) field commutation relations are given by

$$\begin{aligned} [V(x), V(y)] &= i\Delta(x-y;m^2) \ , \\ [\eta(x), \eta(y)] &= [X(x), X(y)] \\ &= -[B_0(x), X(y)] = iD(x-y) \ , \\ [B_0(x), B_0(y)] &= 0 \ , \end{aligned} \tag{4.7}$$

where $\Delta(x;m^2)$ and $D(x) \equiv \Delta(x;0) = -\tfrac{1}{2}\epsilon(x^0)\theta(x^2)$ are the usual D functions in two dimensions.

In terms of the asymptotic fields, the conserved current (3.6) is rewritten as

$$J^\mu = ea(a-1)^{-1/2}\epsilon^{\mu\nu}\partial_\nu V + \partial^\mu B \ .$$

Accordingly, the (left-handed) charge operator is given by

$$Q_L = \int dx^1 J^0(x) = \int dx^1 \partial^0 B \ . \tag{4.8}$$

In view of the commutation relations (4.7), the asymptotic fields V, η, and B commute with Q_L, but X does not:

$$[Q_L, X(x)] = ie\sqrt{a} \ . \tag{4.9}$$

This implies that the $U(1)_L$ symmetry is spontaneously broken in the present model. The X is regarded as the associated Nambu-Goldstone boson, which, being gauge variant, is unphysical. This conclusion does not necessarily contradict Coleman's theorem,[11] which excludes the use of indefinite metric. The local gauge transformations of the original fields A_μ, θ, and ϕ are generated by the operator

$$G_\xi = \int dx^1 [\xi_L \partial_0 B - (\partial_0 \xi_L)B] \ . \tag{4.10}$$

In the present model, the physical states are projected out of the asymptotic states by the following subsidiary conditions:

$$B^{(+)}\,|\,\text{phys}\rangle = \Omega^{(+)}\,|\,\text{phys}\rangle = 0 \ , \tag{4.11}$$

where

$$\Omega^{(+)} \equiv \lim_{L\to\infty} \int_{-L}^{L} dx^1 \partial_0 \eta^{(+)}(x) \ . \tag{4.12}$$

For a massless field, the separation of its positive- and negative-frequency parts needs some care, as explained later. In Eq. (4.11) the first condition is the standard one while the second condition[7] eliminates a negative-norm component in the massless field $\eta^{(\pm)}$. The B is a physical field of zero norm. The massive boson V and the massless boson η represent physical excitations belonging to the physical subspace of positive norm, in agreement with the spectrum found in Ref. 1.

For massless fields $f=(\eta,B,X)$, the positive- and negative-frequency parts are defined by regularizing the infrared divergences

$$f^{(\pm)}(x)=-i\int dz^1 D^{(\pm)}(x-z)\overleftrightarrow{\partial}_0 f(z)\ , \tag{4.13}$$

where

$$f\overleftrightarrow{\partial}g\equiv(\partial f)g-f(\partial g),\ D^{(-)}(x)\equiv-D^{(+)}(-x)\ ,$$

and

$$D^{(+)}(x)=(2\pi)^{-1}\int dp^1(2p^0)^{-1}[e^{-ip\cdot x}-\theta(\kappa-p^0)]\ . \tag{4.14}$$

Here $\kappa>0$ is an infrared cutoff introduced by Klaiber.[12] For the massive field V, $V^{(\pm)}$ are defined in terms of the usual $\Delta^{(\pm)}(x;m^2)$ functions. The commutation relations among $f^{(\pm)}(x)$ are given in the Appendix.

V. THE FERMION FIELD

In this section we construct the fermion field out of the asymptotic boson fields and study its properties. In deriving the fermion solution, dual fields play an important role. Here, we first summarize their properties. A free massless field $f(x)$ has its dual $\tilde{f}(x)$ defined by

$$\partial_\mu f=\epsilon_{\mu\nu}\partial^\nu\tilde{f}\quad\text{or}\ \tilde{f}(x)=\int_{-\infty}^{x^1}dz^1\partial_0 f(x^0,z^1)\ . \tag{5.1}$$

The $\tilde{f}(x)$ is a nonlocal functional of $f(x)$ and obeys a free field equation $\partial^2\tilde{f}=0$. As is clear from the definition, the combination $f+\tilde{f}$ is a function of $x^+\equiv x^0+x^1$ (i.e., a left mover) while $f-\tilde{f}$ is a function of $x^-\equiv x^0-x^1$. The positive- and negative-frequency parts $\tilde{f}^{(\pm)}(x)$ are defined by Eq. (4.13) with $f\to\tilde{f}$. A dual field does not commute with the original field at spacelike separations. The commutation relations involving dual fields are summarized in the Appendix.

We are now ready to study the Dirac equation (3.2). There, only the left-handed projection $A_L^\mu\equiv(g^{\mu\nu}+\epsilon^{\mu\nu})A_\nu$ is coupled to the fermion ψ. The A_L^μ is expressed in terms of the operator solution (4.2) as

$$eA_L^\mu=(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu(-\phi+\eta+a^{-1/2}B_0)\ , \tag{5.2}$$

where ϕ is given by Eq. (4.4). We can therefore gauge away this A_L^μ and write the fermion field ψ (roughly) as

$$\psi(x)=\exp[i\sqrt{\pi}(1-\gamma^5)\xi_0(x)]\psi_0(x)\ , \tag{5.3}$$

where $\xi_0=-\phi+\eta+a^{-1/2}B_0$ and ψ_0 is a free spinor field. As is well known, a free spinor can be expressed in terms of a free massless boson field χ and its dual $\tilde{\chi}$ in the form

$$\psi_0(x)=\exp\{i\sqrt{\pi}[\chi(x)+\gamma^5\tilde{\chi}(x)]\}u\ , \tag{5.4}$$

where $u=(u_1,u_2)^t$ is a two-component constant.

Note that one can include into the phase ξ_0 the dual-field contribution of the form $\alpha_1(\eta-\tilde{\eta})+\alpha_2(B_0-\tilde{B}_0)+\alpha_3(X-\tilde{X})$; this does not affect A_L^μ in Eq. (5.2) since $(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu\propto\partial/\partial x^+$. We fix the field χ and the parameters α_1, α_2, and α_3 in such a way that the left- and right-handed currents in Eq. (2.5) are reproduced. We present the fermion operator constructed in this manner:

$$\psi(x)=:\exp\{i\sqrt{\pi}[-(1-\gamma^5)\phi(x)+\chi(x)+\tilde{\chi}(x)]\}:u\ , \tag{5.5}$$

$$\chi(x)=\eta(x)+a^{-1/2}B_0(x)\ . \tag{5.6}$$

Here, the ϕ term guarantees that $\psi(x)$ behaves correctly under both global and local $U(1)_L$ transformations. In Eq. (5.5), the normal ordering :( ): of operators is defined according to the rule, $F^{(-)}G^{(-)}G^{(+)}F^{(+)}$ with $F=\chi+\tilde{\chi}$ and $G=\phi$.

Regularization is needed to define the fermion current operators. We define the vector and axial-vector currents by a point-splitting prescription:

$$\begin{aligned}J_V^\mu(x;\epsilon)&=e\sqrt{\pi}\Gamma_V(x+\epsilon,x):\bar{\psi}(x+\epsilon)\gamma^\mu\psi(x):\ ,\\ J_A^\mu(x;\epsilon)&=e\sqrt{\pi}\Gamma_A(x+\epsilon,x):\bar{\psi}(x+\epsilon)\gamma^\mu\gamma^5\psi(x):\ .\end{aligned} \tag{5.7}$$

Here, the ordering of operators is specified as

$$:\bar{\psi}(x+\epsilon)\gamma^\mu\psi(x):\equiv\tfrac{1}{2}(\gamma^\mu)_{\alpha\beta}[\bar{\psi}(x+\epsilon)_\alpha,\psi(x)_\beta]\ ,$$

etc. One can verify [using Eq. (5.19) below] that, with the *same* phase factors as those in Eq. (2.8), the correct chiral currents are obtained from the operators $J_L^\mu(x;\epsilon)=\frac{1}{2}[J_V^\mu(x;\epsilon)-J_A^\mu(x;\epsilon)]$ and $J_R^\mu(x;\epsilon)=\frac{1}{2}[J_V^\mu(x;\epsilon)+J_A^\mu(x;\epsilon)]$ in the symmetric $\epsilon^\mu\to 0$ limit.[13] Thereby the normalization of $u=(u_1,u_2)^t$ is fixed:

$$\begin{aligned}u_1u_1^*&=\mu/(2\pi)\ ,\\ u_2u_2^*&=\mu(2\pi)^{-1}\exp\{(e^2/m^2)[\ln(m^2/4\mu^2)+2\gamma]\}\ ,\end{aligned} \tag{5.8}$$

where γ is Euler's constant and $\mu\equiv\kappa e^\gamma$ is an infrared cutoff introduced to define $D^{(\pm)}(x)$ in Eq. (4.14).

The operator $\psi(x)$ satisfies the Dirac equation in the sense that

$$i\partial\!\!\!/\psi+e\sqrt{\pi}:A\!\!\!/(1-\gamma_5)\psi:=0\ , \tag{5.9}$$

where the symbol : : implies that the operator A_μ lies between $G^{(-)}$ and $G^{(+)}$ in the normal-ordered expression for ψ.

Having derived the fermion operator ψ, let us next show that it behaves properly as the fermion in the present chiral model. Using the commutation relations among the boson fields and their duals, one can cast the products of the fermion fields ψ_α and $\psi_\alpha^\dagger$ $(\alpha=1,2)$ in the form

$$\psi_\alpha(x)\psi_\alpha^\dagger(y)=\tau_\alpha(z)I_\alpha(x,y)\quad(\alpha=1,2)\ , \tag{5.10}$$

$$\psi_\alpha^\dagger(y)\psi_\alpha(x)=\tau_\alpha(-z)I_\alpha(x,y)\ , \tag{5.11}$$

$$\tau_1(z)=u_1u_1^*\exp\{2\pi[D^{(+)}(z)+\tilde{D}^{(+)}(z)]\}\ , \tag{5.12}$$

$$I_1(x,y)=:\exp\{i\sqrt{\pi}[Y(x)-Y(y)]\}:\ , \tag{5.13}$$

$$\tau_2(z)=u_2^* u_2 \exp\{4\pi(a-1)a^{-2}[\Delta^{(+)}(z;m^2)-D^{(+)}(z)] +2\pi[D^{(+)}(z)-\tilde{D}^{(+)}(z)]\} , \qquad (5.14)$$

$$I_2(x,y)=:\exp\{i\sqrt{\pi}[-2(\phi(x)-\phi(y))+(Y(x)-Y(y))]\}: , \qquad (5.15)$$

where $z^\mu=x^\mu-y^\mu$ and $Y(x)=\chi(x)+\tilde{\chi}(x)$. In $I_2(x,y)$, the nomal ordering is defined according to the previous rule with $F\to(Y(x)-Y(y))$ and $G\to(\phi(x)-\phi(y))$; analogously for $I_1(x,y)$. As for the $D^{(\pm)}$ and $\tilde{D}^{(\pm)}$ functions, see the Appendix. Noting Eq. (5.8), we find that $\tau_1(z)=-i(2\pi)^{-1}(z^0+z^1-i0)^{-1}$. As a consequence, we obtain the anticommutation relation

$$\{\psi_1(x),\psi_1^\dagger(y)\}=\delta(z^0+z^1) . \qquad (5.16)$$

The product $\psi_1(x)\psi_1(y)$ is cast in the form (5.10) with $\tau_1(z)\to 1/\tau_1(z)$ and

$$I_1\to:\exp[i\sqrt{\pi}(Y(x)+Y(y))]: ;$$

hence $\{\psi_1(x),\psi_1(y)\}=0$ follows immediately.

The commutation relations among the left-handed fermion ψ_2 are examined in a similar fashion. Eventually, we see that $\psi(x)$ obeys the correct canonical (equal-time) commutation relations

$$\{\psi_\alpha(x),\psi_\beta^\dagger(y)\}_{\rm ET}=\delta_{\alpha\beta}\delta(x^1-y^1) , \qquad (5.17)$$

$$\{\psi_\alpha(x),\psi_\beta(y)\}_{\rm ET}=0 , \qquad (5.18)$$

where $\alpha,\beta=1$ or 2. In addition, $\psi(x)$ commutes with A_μ, θ, and B (at equal times), which are the fundamental fields appearing in the aboriginal fermionic form of the model. On the contrary, ψ does not commute with ϕ; this fact poses no problem since ϕ appears only in the bosonized version of the model.

Equations (5.10)–(5.15) yield a compact expression for the field product

$$:\bar{\psi}(x+\epsilon)\gamma^\mu(1+\gamma^5)\psi(x): \;= -\frac{i}{\pi}(g^{\mu\nu}+\epsilon^{\mu\nu})(\epsilon_\nu/\epsilon^2)I_1(x,x+\epsilon) . \qquad (5.19)$$

For the left-handed combination, replace $(g^{\mu\nu}+\epsilon^{\mu\nu})$ by $(g^{\mu\nu}-\epsilon^{\mu\nu})$ and I_1 by I_2.

It is enlightening to calculate the fermion propagator. The propagator of the left-handed component ψ_2 takes the form[14]

$$\langle 0 | T\psi_2(x)\psi_2^\dagger(y) | 0\rangle=\theta(z^0)\tau_2(z)-\theta(-z^0)\tau_2(-z) = -\frac{i}{2\pi}\frac{z^0+z^1}{z^2-i0}h(-z^2) , \qquad (5.20)$$

$$h(w)=\exp\{(e^2/m^2)[2K_0(m\sqrt{w})+\ln(\tfrac{1}{4}e^{2\gamma}m^2w)]\} , \qquad (5.21)$$

where $z^\mu=x^\mu-y^\mu$ and $z^2=(z^0)^2-(z^1)^2$. Since $h(-z^2)\to 1$ as $z^2\to -0$, the short-distance behavior of the ψ_2 is the same as a free propagator. On the contrary,

$$h(-z^2)\to[\tfrac{1}{4}e^{2\gamma}m^2(-z^2)]^\zeta \text{ as } z^2\to-\infty , \qquad (5.22)$$

with $\zeta=(a-1)/a^2$; $0<\zeta\le\frac{1}{4}$. The ψ_2 propagator therefore has no pole in momentum space. It behaves like $(p^0+p^1)/(p^2)^{1+\zeta}$ for $p^2\to-0$. Thus, ψ_2 is confined (at long distances) but is asymptotically free (at short distances). On the other hand, the right-handed fermion ψ_1 remains a massless free field, as implied by the field equation (5.9). The ψ_1 propagator is given by Eq. (5.20) with $(z^0+z^1)\to(z^0-z^1)$ and $h(-z^2)\to 1$.

The fermionic form of the model defined by the Heisenberg field equations (3.2)–(3.8) is now solved. The massless physical field η in the bosonized theory is used up to form the chiral fermions and never shows up as it is in the fermionic form of the model. The left-moving combination $\eta+\tilde{\eta}$ makes up the free right-handed fermion ψ_1. The $\eta-\tilde{\eta}$ combination controls the short-distance behavior of ψ_2, although it is confined, as seen from the $D-\tilde{D}$ term in Eq. (5.14). The (positive-norm) physical spectrum of this fermion model consists of a massive boson V and a free chiral fermion (plus a confined chiral fermion). The θ field, being gauge variant, does not contribute to the physical spectrum.

VI. GLOBAL $U(1)_R$ SYMMETRY

In this section, we examine the $U(1)_R$ symmetry of the model. The apparent nonconservation of the $U(1)_R$ fermion current in Eq. (2.4) does not imply the (explicit) breakdown of global $U(1)_R$ symmetry. Indeed, one can define a conserved current as

$$J_R'^\mu=J_R^\mu-e^2(g^{\mu\nu}+\epsilon^{\mu\nu})A_\nu=-e(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu\chi$$

without spoiling its self-duality, where $\chi=\eta+a^{-1/2}B_0$. Accordingly, the conserved right-handed charge is given by $Q_R'=\int dx^1 J_R'^0(x)$. This Q_R' commutes with the asymptotic fields V and B, but fails to commute with η and X:

$$[Q_R',\eta(x)]=-\sqrt{a}\,[Q_R',X(x)]=ie . \qquad (6.1)$$

As a result

$$[Q_R',\phi(x)]=[Q_R',\tfrac{1}{2}(\chi+\tilde{\chi})]=ie . \qquad (6.2)$$

It is now easy to show that Q_R' correctly generates[15] the $U(1)_R$ transformations of ψ_α, A_μ, and θ:

$$\begin{aligned}&[Q_R',\psi_1]=-2e\sqrt{\pi}\psi_1 ,\\ &[Q_R',\psi_2]=[Q_R',A_\mu]=[Q_R',\theta]=0 .\end{aligned} \qquad (6.3)$$

Equation (6.1) implies that the $U(1)_R$ symmetry is spontaneously broken in this model and that $\Pi\equiv(1+a)^{-1/2}(-\sqrt{a}\,\eta+X)$ is the associated Nambu-Goldstone boson. We have already seen that X is the Nambu-Goldstone boson associated with the spontaneously broken $U(1)_L$ symmetry. Thus the spontaneous breakdown of the $U(1)_R$ symmetry offers an explanation why η is massless. More precisely, the left-moving combination $(\eta+\tilde{\eta})$ is regarded as the associated $U(1)_R$ massless mode since $[Q_R',(\eta-\tilde{\eta})]=0$ holds. In this sense, the right-handed fermion ψ_1 is the Nambu-Goldstone mode associated with the spontaneous breakdown of the $U(1)_R$ symmetry.

VII. SUMMARY AND DISCUSSION

In this paper we have studied the quantization of the chiral Schwinger model with a Wess-Zumino term in a manifestly covariant operator formalism with emphasis on the fermion and symmetry contents of the model. The relevant features of this (modified) chiral Schwinger model are summarized as follows.

(1) The model restores local $U(1)_L$ gauge symmetry at the quantum level.

(2) The restored $U(1)_L$ gauge symmetry gets spontaneously broken. The gauge field becomes massive. In the present covariant formalism, the Nambu-Goldstone boson appears in association with this spontaneous symmetry breakdown but it becomes unphysical.

(3) The left-handed fermion ψ_2, which interacts with the gauge field, is confined while the right-handed fermion ψ_1 is a massless free field.

(4) The global $U(1)_R$ symmetry of the model also gets spontaneously broken. This explains why the right-handed fermion ψ_1 remains a massless field; ψ_1 is the Nambu-Goldstone mode associated with this spontaneous $U(1)_R$-symmetry breakdown.

The observations (3) and (4) regarding the fermion are the central results of our analysis. The fermion spectrum found here (especially the confinement of one of the chiral fermions) could hardly be inferred from the particle spectrum of the bosonized version of the model alone.

The right-handed fermion ψ_1 is a free field in the original Lagrangian (2.1). Its free-field nature, however, is not quite obvious since the left- and right-handed fermions get coupled through the quantum gauge anomaly. Indeed, the coupling between the left- and right-handed sectors of the model is explicitly seen either in the bosonic Lagrangian (4.1) or in the fermion operator (5.5). Thus it is a nontrivial fact that ψ_1 behaves like a free massless fermion.

This free chiral fermion is characteristic of the present chiral Schwinger model. The "interacting" sector, composed of the gauge field and the left-handed fermion, on the other hand, shares the same features (gauge-field mass generation and fermion confinement) as the Schwinger model. It is intriguing to observe that, for the special value $a=2$ of the parameter a, the ψ_2 propagator in Eq. (5.20) becomes identical in structure to the corresponding fermion propagator in the Schwinger model. The gauge fields then share the same mass ($m^2=4e^2$) in both models. (For $a=2$, the exponent ζ attains the maximum value $\frac{1}{4}$.) It will be interesting to ask whether this maximal resemblance may reveal a possible connection between the two models.

APPENDIX

In this appendix we summarize the commutation relations among the asymptotic fields. The commutation relations among the massless fields η, X, and B_0 in Eq. (4.7) are generally written as

$$[f(x),g(y)]=icD(x-y)\ , \tag{A1}$$

where $c=\pm 1$ and $D(x)=-\frac{1}{2}\epsilon(x^0)\theta(x^2)$. Their positive- and negative-frequency parts $f^{(\pm)}$ and $g^{(\pm)}$ defined by Eq. (4.13) obey the commutation relations of the form

$$\begin{aligned}&[f^{(\pm)}(x),g^{(\mp)}(y)]=cD^{(\pm)}(x-y)\ ,\\&[f^{(\pm)}(x),g^{(\pm)}(y)]=0\ .\end{aligned} \tag{A2}$$

The $D^{(\pm)}(x)$ is defined by Eq. (4.14) and its explicit form is given in Eq. (A5) below. For the massive field V, replace $D^{(\pm)}(x-y)$ by $\Delta^{(\pm)}(x-y;m^2)$ in Eq. (A2).

Every massless free field has its dual defined by Eq. (5.1). Dual fields obey the commutation relations of the form

$$\begin{aligned}&[\tilde{f}(x),\tilde{g}(y)]=icD(x-y)\ ,\\&[f(x),\tilde{g}(y)]=ic[\tilde{D}(x-y)+\tfrac{1}{2}]\ ,\end{aligned} \tag{A3}$$

where $\tilde{D}(x)\equiv-\frac{1}{2}\epsilon(x^1)\theta(-x^2)$. For $[\tilde{f},g]$, replace $\frac{1}{2}$ by $-\frac{1}{2}$ in Eq. (A3). The $\tilde{f}^{(\pm)}(x)$ are defined by Eq. (4.13) with $f\to\tilde{f}$. It is clear from Eq. (A2) that $\tilde{f}^{(\pm)}$ and $\tilde{g}^{(\pm)}$ obey the same commutation relations as $f^{(\pm)}$ and $g^{(\pm)}$. Note, however, the commutation relations

$$\begin{aligned}&[f^{(\pm)}(x),\tilde{g}^{(\mp)}(y)]=c[\tilde{D}^{(\pm)}(x-y)+i\tfrac{1}{8}]\ ,\\&[f^{(\pm)}(x),\tilde{g}^{(\pm)}(y)]=ic\tfrac{1}{8}\ .\end{aligned} \tag{A4}$$

Here $D^{(\pm)}(x)$ and $\tilde{D}^{(\pm)}(x)$ are given by

$$\begin{aligned}&D^{(+)}(x)\pm\tilde{D}^{(+)}(x)=-(2\pi)^{-1}\{\ln[\mu(x^0\pm x^1-i0)]+i\tfrac{1}{2}\pi\}\ ,\\&D^{(-)}(x)\pm\tilde{D}^{(-)}(x)=(2\pi)^{-1}\{\ln[\mu(x^0\pm x^1+i0)]-i\tfrac{1}{2}\pi\}\ ,\end{aligned} \tag{A5}$$

where $\mu\equiv\kappa e^{\gamma}$ in terms of Klaiber's infrared cutoff κ. (γ is Euler's constant.) Note that $iD(x)=D^{(+)}(x)+D^{(-)}(x)$; analogously for $\tilde{D}$, $\tilde{D}^{(+)}$, and $\tilde{D}^{(-)}$.

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[13]Without the phase factors (i.e., $\Gamma_V = \Gamma_A = 1$), $J_R^\mu(x;\epsilon) \to -\frac{1}{2}e(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu(\chi+\tilde{\chi})$; see Eq. (5.19). Thus it is through the phase factors that the massive-field contribution $\propto \phi$ to the right-handed current is introduced; the phase factors are indispensable for a correct definition of the currents.

[14]In general, the positive-frequency part $\tilde{f}^{(+)}$ of a dual field $\tilde{f}$ does not annihilate the vacuum, $\tilde{f}^{(+)}(x)|0\rangle = \frac{1}{4}\Omega_f^{(-)}|0\rangle \neq 0$, where $\Omega_f^{(-)}$ is a constant operator; see Ref. 7 for details. However, the difference $[\tilde{f}^{(+)}(x)-\tilde{f}^{(+)}(y)]$ does annihilate the vacuum. For this reason, $\langle 0|I_\alpha(x,y)|0\rangle = 1$ $(\alpha=1,2)$ for $I_\alpha(x,y)$ defined in Eqs. (5.13) and (5.15).

[15]The nonconserved original charge operator $Q_R = \int dx^1 J_R^0(x)$ does not commute with all asymptotic boson fields. It turns out, however, that the commutation relations in Eqs. (6.2) and (6.3) hold with $Q_R' \to Q_R$ (at equal times).