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Vacuum averages for arbitrary spin around a cosmic string

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The vacuum averages of the energy densities of massless spin-$\frac{1}{2}$ and spin-1 fields and of the time component of the Bel-Robinson tensor are evaluated around a cosmic string. The results are transcribed into the Rindler wedge and correlated with other calculations.

INTRODUCTION

In a previous work[1] we analyzed the vacuum average $\langle T_{\mu\nu}\rangle$ of the energy-momentum tensor of a massless scalar field in a space-time of $d+1$ dimensions possessing a conical singularity. We now wish to extend the calculation to fields of arbitrary spin. In this case, d will be restricted to three so that the standard higher-spin theory can be employed. The space-time is then that of a straight cosmic string.

The recent paper of Frolov and Serebriany[2] is concerned with spins 0, $\frac{1}{2}$, and 1. Our method is different in detail and we work with any spin.

ARBITRARY SPIN ON THE CONE: GREEN'S FUNCTIONS

We can write the metric on the conical space-time[3,4] in cylindrical coordinates as

$$ds^2=dt^2-dr^2-r^2d\phi^2-dz^2 \ ,$$

where the angle ϕ runs from 0 to β. Introducing a physical angle $\bar{\phi}$, which runs over the range 0 to 2π, the metric is

$$\begin{aligned} ds^2&=dt^2-dr^2-(\beta/2\pi)^2r^2d\bar{\phi}^2-dz^2 \\ &=g_{\mu\nu}dx^\mu dx^\nu \quad (\mu=0,1,2,3) \ . \end{aligned} \tag{1}$$

When discussing spin, it is necessary, first of all, to set up a system of local tetrads. In our earlier work[5] we used the polar system in which the tetrads are parallel to the local $(t,r,\bar{\phi},z)$ coordinate axes.

From now on we concentrate on just the $(r,\bar{\phi})$ plane. All rotations are about the z axis and we shall refer to the coordinate system (1) as a polar one. Both ϕ and $\bar{\phi}$ will be referred to as polar angles.

The formulation of arbitrary spin which we use has been described before.[6-9] There are many equivalent (and inequivalent) ways of writing the equations of motion. Most convenient here is the equation

$$t^{\mu_1\cdots\mu_{2j}}\nabla_{\mu_1}\varphi=0 \ , \tag{2}$$

first given in Ref. 10.

φ is a $(2j+1)$-spinor belonging to the $(j,0)$ representation of the local Lorentz group and the $t^{\mu\cdots\nu}$ are extensions[6] to curved space of the matrices introduced by Weinberg[11] and described further by Williams.[12] (See also Refs. 7–9 and 13.)

The covariant derivative ∇_μ is equal to $\partial_\mu+\Gamma_\mu$. On the cone the connection Γ is given by[5]

$$\Gamma_0=0, \quad \Gamma_1=0, \quad \Gamma_2=-i(\beta/2\pi)J_z, \quad \Gamma_3=0 \ , \tag{3}$$

in polar coordinates and for polar tetrads. J_z is the standard spin-j angular momentum matrix.

For spin $\frac{1}{2}$, $t^\mu=\sigma^\mu$ and Eq. (2) is then the Weyl equation. For a spin-1, neutral field, Eq. (2) is equivalent to Maxwell's equations. (In a Cartesian angular momentum basis $\varphi\approx\mathbf{H}-i\mathbf{E}$.)

In the spin-2 case, φ is the self-dual Weyl conformal tensor and Eq. (2) is the Bianchi identity. The quantity $\varphi^\dagger t^{\mu\nu\rho\sigma}\varphi$ is the Bel-Robinson tensor which obviously generalizes[7] to $\varphi^\dagger t^{\mu\cdots\nu}\varphi$ for arbitrary spin. This tensor is covariantly conserved, by virtue of the equations of motion (2), and in Ref. 9 we show that it generates multiple derivative transformations.

In flat space, with Cartesian coordinates, the quantization of the arbitrary-spin field results in the commutation (or anticommutation) relations[14-16]

$$\begin{aligned} &[\varphi_m(x),\varphi_n(x')]_\mp=0 \ , \\ &[\varphi_m(x),\varphi_n^\dagger(x')]_\mp=i^{2j-1}\bar{t}_{mn}^{\mu\cdots\nu}\partial_\mu\cdots\partial_\nu\Delta(x,x') \\ &\qquad\qquad\qquad\equiv i^{2j-1}\mathbf{T}\Delta \ , \end{aligned}$$

where $\Delta(x,x')$ is the scalar commutator function and $\bar{t}$ is obtained from t by reversing the sign of those components having an odd number of spatial indices.[11] The other Green's functions are obtained from the corresponding scalar ones by action of the operator $\mathbf{T}$. For example, the Feynman Green's function S is given by

$$S(x,x')=i^{2j-1}\bar{t}^{(\mu)}\partial_{(\mu)}D(x,x') \ , \tag{4}$$

D being the scalar Feynman function.

For our case, and in our terminology, S is the Green's function for $\beta=2\pi$ in Cartesian coordinates with respect to Cartesian tetrads.

Our method of obtaining the scalar Green's function of period β was to insert the flat (i.e., $\beta=2\pi$) Green's function D into a contour integral over a complex polar angle. However, it is not possible simply to substitute this form into (4) to obtain the corresponding spinor result because the transformation properties would not be correct. We have to ensure that the integrand is taken with respect to the appropriate tetrad system. The explicit expression for the spinor propagator on the cone with respect to polar tetrads is

$$S_\beta^P(x,x')=i^{2j-1}\bar{t}^{(\mu)}\nabla_{(\mu)}\int_A D(\phi-\phi'-\alpha)U(\phi-\phi'-\alpha)\times\frac{\exp(2\pi i\alpha\delta/\beta)}{2\beta i\sin(\pi\alpha/\beta)}d\alpha\ , \quad (5)$$

where $U(\phi)$ is the standard spin-j representation matrix for a rotation through ϕ about the z axis, $U(\phi)=\exp(i\phi J_z)$. D is the massless, scalar propagator in Minkowski space. We have indicated its polar angle dependence only. The contour A is as in our earlier papers.[1,17,18]

We have included a general phase factor in the construction of S_β. For neutral fields, which are our main concern here, just set δ equal to zero for half odd-integer spin and equal to a half for integer spin.

If ϕ', say, is increased by β then we see from (5) that S_β^P undergoes a phase change of $-\exp(2\pi i\delta)$. The spin-$\frac{1}{2}$ field, for example, changes sign (if $\delta=0$). This is correct since the polar tetrad performs a complete 2π rotation.

S_β^P thus has the correct periodicity and it can be checked that it has the desired singularity when $x=x'$.

It is easy to prove the following basic result:

$$U^{-1}(\phi)\bar{t}^{(\mu)}\nabla_{(\mu)}U(\phi)=\bar{t}^{0(\nu)}\partial_{(\nu)}\ , \quad (6)$$

where the operator on the right is identical to that in (4); i.e., it is of flat-space, Minkowski form with *Cartesian* coordinates x and y related to r and ϕ (*not* $\bar{\phi}$) in the usual way. We indicate this by the superscript zero.

Our expression for the spinor propagator is then contained in

$$S_\beta^F(x,x')\equiv U^{-1}(\phi)S_\beta^P(x,x')U(\phi')=i^{2j-1}\bar{t}^{0(\mu)}\partial_{(\mu)}D_\beta(x,x')\ , \quad (7)$$

where the matrix D_β is given by

$$D_\beta(x,x')=\int_A D(\phi-\phi'-\alpha)U(-\alpha)\frac{\exp(2\pi i\alpha\delta/\beta)}{2i\beta\sin(\pi\alpha/\beta)}d\alpha\ . \quad (8)$$

Equation (7) is as close as we can get, formally, to (4).

Because the scalar Green's function has only simple poles, D_β can be evaluated easily by residues, exactly as in Ref. 18. The resulting expression is a little long and, since it is not needed in the calculation of the vacuum averages, will not be given here. For spin $\frac{1}{2}$ it agrees with the formula of Frolov and Serebriany.[2]

VACUUM EXPECTATION VALUES

For spins 0, $\frac{1}{2}$, and 1 we can find a local expression for the energy-momentum tensor $T_{\mu\nu}$ as a bilinear function of the fields and so can evaluate the vacuum average $\langle T_{\mu\nu}\rangle$ by the standard method using the Green's function S. The calculation will be given shortly.

If j is bigger than 1 the expression for $T_{\mu\nu}$ is contentious. One cannot find a *local* expression in terms of just the φ fields. Potentials are necessary and then there is the question of gauge invariance. For these higher spins we shall be content to evaluate the vacuum average of the Bel-Robinson tensor $\langle\varphi^\dagger t^{(\mu)}\varphi\rangle$ for $(\mu)=(0)$ or, equivalently, of $\langle\varphi^\dagger\varphi\rangle$.

For spin $\frac{1}{2}$, in our conventions, $\langle T_{\mu\nu}\rangle$ is given by

$$\langle T_{\mu\nu}\rangle=-\tfrac{1}{2}\lim_{x'\to x}\mathrm{Tr}[\sigma_{(\mu}(\nabla_{\nu)}-\nabla'_{\nu)})S(x,x')]\ ,$$

where S is given by (7). For simplicity we calculate $\langle T_{00}\rangle$,

$$\langle T_{00}\rangle=-[\mathrm{Tr}\partial_0 S_\beta^P]=-[\mathrm{Tr}\partial_0 S_\beta^F]\ , \quad (9)$$

the coincidence limit being indicated by the angular brackets.

This limit diverges. Our general procedure for eliminating the divergence is to deform the contour A into a loop around $\alpha=0$, which by residues, gives the flat Green's function $S_{2\pi}$, and into a remainder Γ.

The divergence resides in the $S_{2\pi}$ part which is discarded leaving an integral which is finite in the coincidence limit.

Substitution of (7) into (9) thus yields

$$\langle T_{00}\rangle=-2i\partial_0\partial_0\int_\Gamma D(\alpha)\cos(\alpha/2)\frac{\cos(2\pi\alpha\delta/\beta)}{2\beta i\sin(\pi\alpha/\beta)}d\alpha\ ,$$

where we have used σ traces and have already set $\phi'=\phi$, $r'=r$, and $z'=z$. [Only the cosine part of the exponential in (8) contributes because the σ traces associate the sine part with $\partial/\partial z$ and this coincidence limit vanishes.]

If the time derivatives are taken and t' set equal to t we find, after the product of the two cosines is written as a sum,

$$\langle T_{00}\rangle=\frac{B}{32\pi^2r^4}[W_4(1/2B+\delta)+W_4(1/2B-\delta)]\ .$$

We have set $B=2\pi/\beta$.

$W_N(\delta)$ is given by the loop integral

$$W_N(\delta)=-i(-2)^{-N/2}\oint\frac{\cos(2\pi\alpha\delta/\beta)}{\sin(\alpha/2)^N\sin(\pi\alpha/\beta)}d\alpha\ ,$$

which we have evaluated earlier[1] in terms of Bernoulli polynomials. A few examples follow:

$$BW_2(\delta)=-2\pi\left[\frac{1}{3}-\frac{B^2}{2}D_2\right]\ ,$$

$$BW_4(\delta)=-\pi\left[\frac{11}{45}-\frac{B^2}{3}D_2+\frac{B^4}{14}D_4\right]\ ,$$

$$BW_6(\delta)=+\frac{\pi}{2}\left[\frac{191}{945}-\frac{4B^2}{15}D_2+\frac{B^4}{24}D_4-\frac{B^6}{720}D_6\right]\ ,$$

where the D polynomials are given by

$$D_2(\delta)=4\delta^2-\tfrac{1}{3}\ ,$$

$$D_4(\delta)=16\delta^4-8\delta^2+\tfrac{7}{15}\ ,$$

$$D_6(\delta)=64\delta^6-80\delta^4+28\delta^2-\tfrac{31}{21}\ .$$

Then for $\langle T_{00}\rangle$ we obtain, if $\delta=0$,

$$\langle T_{00}\rangle = -(5760\pi^2 r^4)^{-1}(B^2-1)(7B^2+17)\ , \qquad (10)$$

which agrees with the result of Frolov and Serebriany,[2] who use the four-component formalism. The other components of $\langle T_{\mu\nu}\rangle$ follow from symmetry arguments.

We note the polynomial relations

$$BW_2(1/2B) = -\tfrac{1}{2}BW_2(\tfrac{1}{2})\ ,$$

$$BW_4(1/2B) = -\tfrac{7}{8}BW_4(\tfrac{1}{2}) - \tfrac{1}{4}BW_2(\tfrac{1}{2})\ ,$$

and

$$BW_6(1/2B) = \tfrac{1}{8}BW_6(\tfrac{1}{2}) - \tfrac{17}{64}BW_4(\tfrac{1}{2}) - \tfrac{31}{128}BW_2(\tfrac{1}{2})\ .$$

Turning now to spin 1, the energy-momentum tensor of a massless field is $\frac{1}{2}\varphi^\dagger t^{\mu\nu}\varphi$. We therefore might as well discuss the spin-j generalization and attempt to evaluate the average

$$\langle \varphi^\dagger t^{(\mu)}\varphi\rangle = -i[\mathrm{Tr}\, t^{0(\mu)} S_\beta^F(x,x')]$$
$$= -i(-1)^j[\mathrm{Tr}\, t^{0(\mu)}\bar{t}^{0(\nu)}\partial_{(\nu)}D_\beta(x,x')]\ .$$

Some technicalities are now necessary.

D_β is given by (8) and contains the matrix factor $\exp(-i\alpha J_z)$. The required traces can be evaluated but, for simplicity, we restrict ourselves to the totally temporal case, $(\mu)=(0)$. If $j=1$ this yields the average of the energy density $(\mathbf{E}^2+\mathbf{H}^2)/2$.

Since $t^{0(0)}=1$ we need the coincidence limit

$$-i(-1)^j[\mathrm{Tr}\,\bar{t}^{0(\nu)}\exp(-i\alpha J_z)\partial_{(\nu)}D_\beta(x,x')] \qquad (11)$$

and, if use is made of the results of Ref. 19, it is easily shown that this reduces to

$$-i(-1)^j 2^{2j}\left[\left[\cos\alpha\frac{\partial}{\partial t} + i\sin\alpha\frac{\partial}{\partial z}\right]^{2j} D_\beta\right]\ , \qquad (12)$$

when we impose the massless condition $\partial^\mu\partial_\mu D_\beta(x,x') = 0 \ (x\neq x')$.

We now allow the differential operator in (12) to pass through the integral in Eq. (8) and act on the Minkowski Green's function D. ϕ' can be immediately set equal to ϕ and r' to r so that the relevant coincidence limit is

$$\left[\left[\cos\alpha\frac{\partial}{\partial t} + i\sin\alpha\frac{\partial}{\partial z}\right]^{2j}[(t-t')^2-(z-z')^2 + 2r^2(\cos\alpha-1)]^{-1}\right]\ ,$$

which is easily reduced to

$$\frac{(-1)^{2j}(2j)!}{(2r^2)^{j+1}(-1+\cos\alpha)^{j+1}}$$

for j integral, and to zero if j is half odd integral.

If all the factors are combined, the required vacuum expectation value is given by

$$\langle\varphi^\dagger t^{(0)}\varphi\rangle = -\frac{2^{j-7}(2j)!}{\pi^3 r^{2j+2}}BW_{2j+2}(\delta) \qquad (13)$$

for j integral. If the field is neutral, $\delta=\frac{1}{2}$.

The fact that the coincidence limit is independent of the angle α in the $\exp(-i\alpha J_z)$ term of (11) can be shown to imply that only one component of φ contributes to the average (13); i.e., $\langle\varphi^\dagger\varphi\rangle = \langle\varphi_0^\dagger\varphi_0\rangle$, where we are using a standard, spherical angular momentum basis so that $\varphi=\{\varphi_m\}$ with $-j\le m\le j$.

If $j=1$ we quickly read off from (13) that

$$\langle T_{00}\rangle = -(720\pi^2 r^4)^{-1}(B^2-1)(B^2+11)\ , \qquad (14)$$

if $\delta=\frac{1}{2}$, again agreeing with Ref. 2. Our method avoids questions of gauge.

The vacuum average of $T_{\mu\nu}$ for a conformal scalar in a wedge with *Dirichlet* boundary conditions has been known for over ten years. The expression is given in Ref. 20, for example. The method of calculation was similar to that of the present paper except that the formula for the Green's function in the wedge was used and the Minkowski value subtracted explicitly. This Green's function was originally calculated by Lukosz[21] who assumed that the angle of the wedge was an integer fraction of π and used the method of images. Our procedure was the complex contour one, valid for all angles.

A slightly different method, involving modes, was used by Deutsch and Candelas[22] who also discussed the Maxwell field.

Periodic boundary conditions are easier to apply than Dirichlet ones and the corresponding vacuum averages on the cone follow as simple modifications of the wedge expressions given Eq. (14) for $j=1$. For $j=0$ we get

$$\langle T_{00}\rangle = -(1440\pi^2 r^4)^{-1}(B^4-1) \qquad (15)$$

(cf. Brown, Ottewill, and Page[23]).

This formula has been obtained directly by Helliwell and Konkowski[24] using the method of Ref. 22. Other calculations have been performed by Smith[25] and Linet.[26]

THE RINDLER CASE

If ϕ is reinterpreted as an imaginary time v and other adjustments made to the coordinates, the metric for the cosmic string (1) becomes that of the Rindler wedge:

$$ds^2 = \xi^2 dv^2 - d\xi^2 - dx^2 - dy^2$$

(ξ playing the role of r). Thus the vacuum average of $T_{\mu\nu}$ in the Rindler wedge can be found from that around a cosmic string. For example, the identification

$$\langle T_0{}^0\rangle_{\mathrm{Rindler}} = \langle T_2{}^2\rangle_{\mathrm{string}} = -3\langle T_0{}^0\rangle_{\mathrm{string}}$$

can be made where the last equality follows from symmetry and tracelessness.

If we set $\beta=\infty$, $\langle T_0{}^0\rangle_{\mathrm{Rindler}}$ will be the average in the Rindler vacuum minus that in the Minkowski one.[27] [In our work[18] we considered (as is well known) the Minkowski vacuum to be a Rindler state at the finite (local) temperature $1/2\pi\xi$ and so computed the Minkowski average minus the Rindler one.]

From the previous results we have, therefore,

$$\langle T_0{}^0\rangle_{\rm Rindler}=\pi^2T^4/30-1/480\pi^2\zeta^4 \quad (j=0)$$
$$=7\pi^2T^4/120+\zeta^2T^2/48-17/1920\pi^2\zeta^4 \quad (j=\tfrac{1}{2})$$
$$=\pi^2T^4/15+\zeta^2T^2/6-11/240\pi^2\zeta^4 \quad (j=1)\ ,$$

where we have set T equal to $1/\beta\zeta$.

The zero-temperature values agree with the calculations of Candelas and Deutsch.[27] Some comments on these values will be found in Refs. 28 and 29.

The finite-temperature correction (the T-dependent terms) can be obtained[28] by conformal transformation from that on the open Einstein universe. The fact that the correction is a finite polynomial in T reflects the circumstance that the relevant short-time ("heat kernel") expansion terminates on the three-dimensional pseudosphere, and is *exact.* (It terminates on all odd-dimensional spheres.)

The Planck T^4 term corresponds to the Weyl volume term a_0 and the T^2 term to the a_1 coefficient, which vanishes for spin zero. The remaining a_n coefficients all vanish in the massless case.[29]

Similarly, the polynomials in β occurring, for even dimensions, in our earlier paper[1] on the conformal scalar in conical space-times, are directly related to the terminating short-time expansions on the odd-dimensional pseudospheres. (If we did not wish to make the continuation to Rindler space we would have to speak of a "short-angle" expansion.)

Clearly the conformal sphere coefficients can be obtained easily from the cone expressions and vice versa.

The nontermination on even-dimensional pseudospheres shows up in the nonpolynomial nature of the relevant vacuum averages in Ref. 1.

The energy thermal distribution on the open Einstein universe was given in Ref. 28 and the arithmetic errors were corrected in Ref. 29. The formula can also be found in Brown, Ottewill, and Page.[23] It is supposed to hold for all spins but an explicit field theory derivation is lacking for spins greater than 1. When added to a corresponding zero-temperature form, and translated into conical space-time, it gives Eq. (4.6) of Ref. 23, which subsumes (10), (14), and (15).

At the moment we have no corresponding analysis concerning the Bel-Robinson tensor for $j>1$. Hacyan[30] has considered some properties of arbitrary spin fields, involving the Bel-Robinson tensor, along an accelerated world line. Work on spin-$\frac{1}{2}$ fields is contained in Ref. 31. A discussion of the Unruh-Rindler effect can be found in Takagi.[32]

We note that retaining a flux parameter δ, which becomes imaginary in the Rindler case, is equivalent to including a chemical potential.[33] Of course, for massless particles we would expect the chemical potential to be zero.

CONCLUSION AND COMMENTS

Using a particular form of the higher spin massless equations we have evaluated the vacuum average of the time component of the Bel-Robinson tensor around a cosmic string, Eq. (13). For spin 1 this is the average of the energy density. We have also rederived the spin-$\frac{1}{2}$ expression for $\langle T_{\mu\nu}\rangle$. The formalism allows for the effect of a magnetic flux through the singularity axis.

In our previous calculation[1] we also encountered the $W_N(\delta)$ polynomials but there the N index increased with the dimension of space-time whereas here it increases with spin. Calculationally this is because the higher-spin theory (in four dimensions) involves higher derivatives and the spin-zero Green's functions in higher (even) dimensions are obtained by differentiating the four-dimensional one.

Finally we must mention a problem we have ignored so far. We refer to the well-known difficulties arising when massless particles are coupled minimally to gauge fields.[34] Problems with coupling to gravity possibly occur if $j\geq 2$ but for $j\geq 1$ with electromagnetic coupling. In the present instance, because the curvature and flux are confined to the axis of the string, so are the difficulties.

For example, for a massless spin-1 field φ coupled to a magnetic flux through the string, the consistency conditions[34] require that, at $r=0$, φ be perpendicular to the string. This is satisfied because, if there is a nonzero flux, the field actually vanishes on the string.

To see why, go to the gauge in which the field undergoes a phase change $\exp(2\pi i\delta)$ on circling the singularity axis.[17,35] The usual argument[36] of shrinking the circulating contour to zero shows that φ must vanish on the axis. (This result applies also to twisted fields on orbifolds.)

The gravitational case is not so easily dismissed and needs further work.

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