H dibaryon in the naive quark model with arbitrary N_c

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It is shown that the H dibaryon (a proposed $\Lambda\Lambda$ bound state) is not bound in the naive quark model for $N_c > 3$. Flavor-SU(3)-symmetry breaking is also discussed.

The possibility that multiquark hadrons may exist has stimulated a lot of theoretical and experimental work. In particular, many papers have been devoted to the study of the hypothetical H dibaryon after Jaffe discovered[1,2] that the strong hyperfine interaction between the six quarks in the $\Lambda\Lambda$ channel should produce a genuine six-quark state below the $\Lambda\Lambda$ threshold. The binding energy of this state was estimated to be 50–80 MeV (the actual number depends on whether the calculation is performed in the naive quark model[3] or in the bag model[1,2]). More elaborate calculations performed within the standard models with coupled-channel effects taken into account[4] and other estimates (see Ref. 5) have added an uncertainty of ± 100 MeV to the above value thus raising the question of whether H is bound at all. Recently the much studied chiral-soliton model has predicted that H should be bound.[6–8] On the other hand, from the experimental point of view there is as yet no evidence for the existence of the H (Refs. 5, 9, and 10).

A theoretical clarification of the situation would follow from the knowledge of the relevant solution of the QCD equations. Since such solutions are beyond our reach much of the attention has been recently directed toward an understanding of the $1/N_c$ expansion of QCD. Models expected to be equivalent to QCD (N_c) for $N_c \to \infty$ (Ref. 11) have been studied in some detail.[12,13] It has been noted[14,15] that in this limit the predictions of the chiral-soliton model and those of the naive quark model coincide. This coincidence strengthens the belief that both models correctly describe the salient features of QCD(∞). Although the question of the stability of the H dibaryon in the framework of the chiral-soliton model has already been studied[6–8] only a specific version of this model [e.g., with baryon quantized as an octet of $SU(3)_F$ (Ref. 15)] has been considered. Furthermore, the resulting conclusions depend on the values of model parameters. Thus, it is interesting to study the H dibaryon in the framework of the naive quark model with an arbitrary N_c as well.

In this paper such a study is performed. It is shown that for any $N_c > 3$ the H dibaryon is heavier than the $\Lambda\Lambda$ state. This result is independent of any parameters and follows directly from the group-theoretical assignment of hadrons into the appropriate multiplets. It is further shown that this instability cannot be removed by relaxing the assumption of SU(3)-flavor symmetry.

In the Fermi-Breit approximation to hyperfine interactions the gluon-exchange-induced shift in energy [assume $SU(3)_F$ symmetry] is given by

$$\Delta E = -a\Sigma = -a \sum_{i<j} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \ . \tag{1}$$

Here $\frac{1}{2}\lambda$ and $\frac{1}{2}\sigma$ are the generators of the SU(N_c) (color) and SU(2) (spin) groups and a is a constant depending on quark mass, the strong coupling constant, and the S-wave function at the origin $\Psi(0)$. The expectation value of the Σ operator in the state R belonging to specific representations of the color-spin-SU($2N_c$), color-SU(N_c), and spin-SU(2) groups may easily be expressed as

$$\langle R \mid \Sigma \mid R \rangle = 4C(2N_c,R) - 2C(N_c,R) - \frac{4}{N_c}S(S+1) - \frac{3n}{N_c}(N_c{}^2-1) \ , \tag{2}$$

where $C(N,R)$ is a Casimir operator for the appropriate representation of the SU(N) group in question and n is the number of quarks in the considered state.

For $N_c = 2k+1$ the Λ hyperon is composed of k up quarks, k down quarks, and a single strange quark.[13] The up and down quarks form together an SU(2)-flavor singlet with the Young tableau $(0,k)$ and the symmetry of the Λ hyperon itself is described by the Young tableau $(1,k)$. In order to ensure the antisymmetrization of the entire wave function of Λ, the corresponding color-spin representation must be that given by the associated Young tableau B (i.e., flavor Young tableau reflected around its main diagonal) which we denote in a compact notation by $[k+1,k]$ according to the number of boxes in subsequent columns. In the H dibaryon—composed of $2k$ up quarks, $2k$ down quarks, and two strange quarks—the up and down quarks form together an SU(2)-flavor singlet with the Young tableau $(0,2k)$. Upon the multiplication by (2,0), i.e., by a pair of strange quarks one obtains three flavor multiplets out of which H could possibly be composed in the most general SU(3)-flavor symmetry breaking case. In order to ensure the antisymmetrization of the entire color-spin-flavor wave function the H must therefore be composed of states belonging to the following representations of color-spin:

$$R_1 = [2k,2k,2], \quad R_2 = [2k+2,2k] \ , \qquad R_3 = [2k+1,2k,1] \ . \tag{3}$$

Of these only R_1 and R_2 contain in their decomposition terms which transform as color and spin singlets [one such term for R_1 (R_2) only] and are therefore of interest to us. For $N_c=3$ ($k=1$) R_1 (R_2) reduces to previously studied representation of color-spin: **490** (**189**). Calculation of the Casimir operators for arbitrary N gives for relevant representations of SU(N) the expressions gathered in Table I. (These expressions can be obtained from the general formula given in Ref. 16 as well.) From Eq. (2) and Table I we obtain for (color- and spin-singlet members of) R_1 and R_2:

$$\langle R_1 \mid \Sigma \mid R_1 \rangle = -2N_c^2+12N_c+6 \,, \qquad \langle R_2 \mid \Sigma \mid R_2 \rangle = -2N_c^2+4N_c-2 \,, \tag{4a}$$

and for spin-$\frac{1}{2}$ Λ state we get

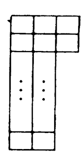
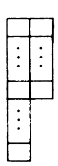
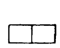
$$\langle B \mid \Sigma \mid B \rangle = 3\left[N_c - \frac{1}{N_c}\right] . \tag{4b}$$

Thus

$$\Delta E(R_1) - 2\Delta E(\Lambda) = a\left[2N_c^2 - 6N_c - 6 - \frac{6}{N_c}\right] , \qquad \Delta E(R_2) - 2\Delta E(\Lambda) = a\left[2N_c^2 + 2N_c - 6 - \frac{6}{N_c}\right] , \tag{5}$$

and for any $N_c > 3$ both the above differences are positive. Only for R_1 and $N_c=3$ one gets a negative energy difference. Thus the H dibaryon (R_1) in the SU(3)-flavor-symmetric model is not bound for $N_c > 3$.

TABLE I. Casimir operators for relevant SU(N) representations.

R (Young tableau)	$C(N,R)$
 m, 2 *[m,m,2]*	$[N^2(2m+2)+N(-2m^2+4m+6)-(2m+2)^2]/2N$
 m, n *[m,n]*	$[N^2(m+n)+N(m+3n-m^2-n^2)-(m+n)^2]/2N$
 *(2,0)*	$(N-1)(N+2)/N$
 *[2,0]*	$(N-2)(N+1)/N$

To treat the case of broken-SU(3)-flavor symmetry it is helpful[3] to perform the calculations not in the R_1,R_2 basis but in the basis in which the strange-quark pair has definite spin: $S_s=0$ or 1. In both states of the H dibaryon sector (labeled $|S_s=0\rangle$, $|S_s=1\rangle$) the strange (nonstrange) quarks are in the representation [2,0] ($[2k,2k]$) of the color-spin group. For the $S_s=0$ state the strange quarks are symmetric (2,0) in color hence the nonstrange quarks must belong to the $[2k,2k]$ representation of SU(N_c) if the whole system is to be color neutral. Knowledge of the color-spin and color-symmetry properties of the nonstrange and strange groups of quarks suffices to calculate the hyperfine interaction within these groups. Using formula (2) and Table I one obtains

$$\langle S_s=0 \mid \Sigma \mid S_s=0 \rangle_{\text{nonstrange}} = 2\left[1-\frac{1}{N_c}\right](-N_c^2+3N_c+3) \,, \qquad \langle S_s=0 \mid \Sigma \mid S_s=0 \rangle_{\text{strange}} = -6\left[1-\frac{1}{N_c}\right] . \tag{6}$$

The hyperfine interaction between the strange and nonstrange groups of quarks vanishes because of their vanishing spins.

For the $S_s=1$ state the strange quarks are antisymmetric in color; hence, the nonstrange quarks must belong to the $[2k+1,2k-1]$ representation of SU(N_c). Calculating the hyperfine interaction Σ within the nonstrange (strange) group of quarks with the help of Eq. (2) and Table I one obtains

$$\langle S_s=1 \mid \Sigma \mid S_s=1 \rangle_{\text{nonstrange}} = 2\left[-N_c^2+4N_c+2-\frac{7}{N_c}\right] , \qquad \langle S_s=1 \mid \Sigma \mid S_s=1 \rangle_{\text{strange}} = -2\left[1+\frac{1}{N_c}\right] . \tag{7}$$

The trace and determinant of the interaction cannot depend on the choice of basis: $\{|S_s=0\rangle, |S_s=1\rangle\}$ vs $\{R_1,R_2\}$. By comparing (6) and (7) with (4a) we get, therefore,

$$\langle S_s=1 \mid \Sigma \mid S_s=1 \rangle_{\text{strange-nonstrange}} = 8\left[1+\frac{2}{N_c}\right] , \qquad \langle S_s=1 \mid \Sigma \mid S_s=0 \rangle_{\text{strange-nonstrange}} = \pm 4\sqrt{(N_c-1)(N_c+3)} . \tag{8}$$

As indicated in (8) only the contribution from the exchange of a gluon between strange and nonstrange quarks is here nonvanishing. SU(3)-flavor symmetry is broken by suppressing the contribution from the strange-nonstrange (strange) hyperfine interactions in (6)–(8) by $\epsilon = m_u/m_s$ (ϵ^2). Since we are interested in the $N_c \to \infty$ limit we neglect below all those terms which do not contribute to the leading and next-to-leading (in N_c) terms in the resulting eigenvalues:

$$\lambda_1 = -2N_c{}^2 + (8+4\epsilon)N_c + O(1)\ ,$$
$$\lambda_2 = -2N_c{}^2 + (8-4\epsilon)N_c + O(1)\ . \qquad (9)$$

From (9) and (4b) it is seen that in the limit $N_c \rightarrow \infty$ the H dibaryon is not bound in the SU(3)-flavor-symmetry-breaking case either.

Recent interest in models expected to be equivalent to the large-N_c limit of quantum chromodynamics is usually justified by the hope that they may shed some light on the physically relevant case $N_c = 3$. Calculations in the chiral-soliton model seem to show that the H should exist. It is therefore very interesting to note that in the naive quark model with arbitrary N_c the H dibaryon is not bound.

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