

Coupled-channel treatment of Cabibbo-angle-suppressed $(D, D_s^+) \rightarrow PP$ decays

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The Cabibbo-angle-suppressed processes of the type $(D, D_s^+) \rightarrow PP$ are discussed in a coupled-channel formalism. We explain satisfactorily all the measured rates for processes of the type $(D^0, D^+) \rightarrow PP$, including $D^0 \rightarrow \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^-$. Predictions are made for all the other as yet unmeasured rates, including those for $D_s^+ \rightarrow PP$.

There is now a considerable amount of data on the Cabibbo-angle-suppressed charm decays into hadrons.¹⁻³ An outstanding problem in the understanding of two-body hadronic decays of D^0 is the ratio

$$B(D^0 \rightarrow K^+ K^-) / B(D^0 \rightarrow \pi^+ \pi^-).$$

This ratio is 3.5 ± 1.1 if computed from Ref. 4, or it may be computed from Mark III data:³ $B(D^0 \rightarrow K^+ K^-) = 0.6 \pm 0.10 \pm 0.08$ and $B(D^0 \rightarrow \pi^+ \pi^-) = 0.16 \pm 0.05 \pm 0.03$. Mark III data have recently⁵ been updated: $B(D^0 \rightarrow K^+ K^-) = 0.51 \pm 0.09 \pm 0.06$ and $B(D^0 \rightarrow \pi^+ \pi^-) = 0.14 \pm 0.04 \pm 0.03$. An SU(3)-symmetric amplitude with physical masses in the phase space would make this ratio $= 0.85$. We expect final-state interactions and the ensuing interferences to play an important role in the eventual resolution of this paradox. It was shown in Ref. 6 that SU(3) symmetry, together with final-state interactions, goes a long way towards explaining this anomalous result. The problem is somewhat more complicated since in the isospin (I)=0 state, $\pi^+ \pi^-$ and $K^+ K^-$ channels would couple. Similarly, in $I=1$ state, $K^+ K^-$ and $\pi^0 \eta$ (and possibly $\pi^0 \eta'$) channels would couple. Thus, a proper treatment of the problem ought to involve a coupled-channel calculation. Such calculations have been attempted^{7,8} unsuccessfully in the past. The failure of these calculations was largely due to an inadequate treatment of the $I=1$ $K\bar{K}$ channel.

In this paper we have studied the Cabibbo-angle-suppressed decays of the type $(D, D_s^+) \rightarrow PP$ (P = pseudoscalar meson), including the final-state interactions. We treat $I=0$, $D^0 \rightarrow \pi\pi$ and $D^0 \rightarrow K\bar{K}$ decays in a coupled-channel formalism. Similarly, we treat $D^0 \rightarrow K\bar{K}$ and $D^0 \rightarrow \pi\eta$ as coupled channels in an $I=1$ final state. We find our results to be very encouraging. We can explain all the data where data exist and we make predictions on all the $(D, D_s^+) \rightarrow PP$ modes where data do not as yet exist.

We start with the weak Hamiltonian for Cabibbo-angle-suppressed hadronic charm decays:

$$H_w = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \\ \times \left\{ \frac{1}{2}(C_+ + C_-)[(\bar{u}d)(\bar{c}c) - (\bar{u}s)(\bar{c}c)] \right. \\ \left. + \frac{1}{2}(C_+ - C_-)[(\bar{d}d)(\bar{u}c) - (\bar{s}s)(\bar{u}c)] \right\}, \quad (1)$$

where $(\bar{u}d)$, etc., represent left-handed hadronic currents and θ_C is the Cabibbo angle. We define the following convenient combinations of the QCD coefficients C_+ and C_- :

$$C_1 = \frac{1}{3}(2C_+ + C_-), \quad C_2 = \frac{1}{3}(2C_+ - C_-). \quad (2)$$

In the past⁹ we have used

$$C_+ = 0.69, \quad C_- = 2.09, \quad C_+^2 C_- = 1. \quad (3)$$

This choice corresponds to $C_1/C_2 = -4.9$. We have investigated the sensitivity of the fit to the ratio C_1/C_2 and found that $C_1/C_2 = -3.0$ leads to a satisfactory fit to data.

To proceed with the calculation, we first compute the decay amplitudes in the factorization approximation⁹ without final-state interactions. The method is detailed in Ref. 9. We list *some* of the decay amplitudes below. A factor of $(G_F/\sqrt{2})\sin\theta_C \cos\theta_C f_+(0)$ has been omitted. $f_0(s)$ is the annihilation form factor:

$$A(D^0 \rightarrow \pi^+ \pi^-) = -\sqrt{2}C_1 f_\pi(m_D^2 - m_\pi^2), \\ A(D^0 \rightarrow \pi^0 \pi^0) = C_2 f_\pi(m_D^2 - m_\pi^2), \\ A(D^0 \rightarrow K^+ K^-) = \sqrt{2}C_1 f_K(m_D^2 - m_K^2), \\ A(D^0 \rightarrow \bar{K}^0 K^0) = 0, \quad (4) \\ A(D^+ \rightarrow \pi^0 \pi^+) = -(C_1 + C_2) f_\pi(m_D^2 - m_\pi^2), \\ A(D^+ \rightarrow \bar{K}^0 K^+) = \sqrt{2}C_1 f_K(m_D^2 - m_K^2), \\ A(D_s^+ \rightarrow K^0 K^+) = -\sqrt{2}C_1 [f_\pi(m_{D_s^+}^2 - m_K^2) \\ + f_{D_s^+} f_0(m_{D_s^+}^2) / f_+(0)], \\ A(D_s^+ \rightarrow K^+ \pi^0) = \frac{1}{\sqrt{2}} [C_2 f_\pi(m_{D_s^+}^2 - m_\pi^2) \\ + \sqrt{2}C_1 f_{D_s^+} f_0(m_{D_s^+}^2) / f_+(0)].$$

In writing (4) we have neglected terms of the type $m_P^2 f_-(m_P^2)$ compared to terms of the type $m_D^2 f_+(m_P^2)$ and assumed $f_+(m_P^2) \simeq f_+(0)$. Notice that the conserved-vector-current (CVC) hypothesis has forbidden annihilation contribution in all $(D^0, D^+) \rightarrow PP$ decays. However, an annihilation term is present in the

amplitudes for $D_s^+ \rightarrow K\pi$ decays.

(D^0, D^+) decays. To demonstrate the method of coupled-channel unitarization we use $D^0 \rightarrow \pi\pi$ and $D^0 \rightarrow K\bar{K}$ as illustrative examples. The isospin structure of these amplitudes is

$$\begin{aligned} A(D^0 \rightarrow \pi^+ \pi^-) &= (\frac{2}{3})^{1/2} A_0^{\pi\pi, U} + \frac{1}{\sqrt{3}} A_2^{\pi\pi, U}, \\ A(D^0 \rightarrow \pi^0 \pi^0) &= \frac{1}{\sqrt{3}} A_0^{\pi\pi, U} - (\frac{2}{3})^{1/2} A_2^{\pi\pi, U}, \\ A(D^0 \rightarrow K^+ K^-) &= \frac{1}{\sqrt{2}} (A_0^{K\bar{K}, U} + A_1^{K\bar{K}, U}), \\ A(D^0 \rightarrow K^0 \bar{K}^0) &= \frac{1}{\sqrt{2}} (A_0^{K\bar{K}, U} - A_1^{K\bar{K}, U}), \end{aligned} \quad (5)$$

where $A_i^{\pi\pi, U}$, etc., are the unitarized amplitudes in the $I=i$ final state defined by

$$A_i^{\pi\pi, U} = |A_i^{\pi\pi}| \exp(i\delta_i^{\pi\pi}), \text{ etc.}$$

The superscripts $\pi\pi$ and $K\bar{K}$ refer to the decay channels. In (5), $I=0$ channels will couple. First, by setting $\delta_i=0$ and equating the corresponding amplitudes in (4) and (5), we determine the nonunitarized $A_i^{\pi\pi}$ and $A_i^{K\bar{K}}$. The unitarized amplitudes $A_0^{\pi\pi, U}$ and $A_0^{K\bar{K}, U}$ are then determined through the matrix equation

$$\underline{A}_0^U(s) = \underline{D}^{-1}(s) \underline{A}_0(s), \quad (6)$$

where $\underline{A}_0(s)$ is a column with entries $A_0^{\pi\pi}(s)$ and $A_0^{K\bar{K}}(s)$, $\underline{A}_0^U(s)$ is a column with entries $A_0^{\pi\pi, U}(s)$ and $A_0^{K\bar{K}, U}(s)$, $\underline{D}^{-1}(s)$ is a 2×2 matrix defined below, and s is the center-of-mass (energy)² in the $\pi\pi$ channel which will finally be set as $s = m_D^2$.

Using a K -matrix parametrization of the scattering S matrix we parametrize the matrix $\underline{D}(s)$ as⁷

$$\underline{D}(s) = \underline{1} - i \underline{\rho}(s) \underline{K}(s), \quad (7)$$

where $\underline{\rho}(s)$ is a diagonal matrix

$$\underline{\rho}(s) = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \quad (8)$$

with $k = \frac{1}{2}(s - 4m_\pi^2)^{1/2}$ and $k' = \frac{1}{2}(s - 4m_K^2)^{1/2}$. The K matrix, a real matrix, is chosen to have a pole representing $f_0(1300)$, which couples⁴ to both $\pi\pi$ and $K\bar{K}$ channels in the $I=0$ state,

$$\underline{K}(s) = \underline{\Gamma} / (s - m^2), \quad m = 1.3 \text{ GeV} \quad (9)$$

with

$$\underline{\Gamma} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}. \quad (10)$$

In the ‘‘factorization’’ approximation, $\det \underline{\Gamma} = 0$, which we use, the partial widths of $f_0(1300)$ take on a simple form:

$$\begin{aligned} \Gamma(f_0 \rightarrow \pi\pi) &= \frac{-\Gamma_{11} k}{m}, \\ \Gamma(f_0 \rightarrow K\bar{K}) &= \frac{-\Gamma_{22} k'}{m}. \end{aligned} \quad (11)$$

We use⁴ $\Gamma_{f_0} = 250$ MeV with a 90% branching ratio into $\pi\pi$ and a 10% branching ratio into $K\bar{K}$ to determine Γ_{11} and Γ_{22} both of which are < 0 . Γ_{12} ($=\Gamma_{21}$) is then determined from the condition $\det \underline{\Gamma} = 0$. We choose Γ_{12} and $\Gamma_{21} > 0$. We find $\Gamma_{11} = -0.46$ GeV, $\Gamma_{22} = -0.078$ GeV. Γ_{12} is given by $\pm(\Gamma_{11}\Gamma_{22})^{1/2}$. The unitarized amplitudes $A_0^{\pi\pi, U}$ and $A_0^{K\bar{K}, U}$ are then generated from (6).

In our model we have used only one resonance: namely, $f_0(1300)$. The justification for neglecting $f_0(975)$ and $f_0(1590)$ is that the former has a small width⁴ (≈ 33 MeV) while the latter appears to decay⁴ into $\eta\eta'$ and $\eta\eta$ channels only. Models using two resonances have been used in the past by one of the authors.⁷

The same method is used in solving the coupled-channel problem in the $I=1$ state, where we assume that the $K\bar{K}$ state couples with $\pi\eta$ final state. We ignore the possible coupling to the $\pi\eta'$ channel. We use a \underline{K} matrix with an $a_0(980)$ pole. We use⁴ $\Gamma(a_0) = 54$ MeV and

$$B(a_0 \rightarrow K\bar{K}) / B(a_0 \rightarrow \pi\eta) = 1.$$

These assumptions together with the ‘‘factorization’’ approximation, $\det \underline{\Gamma} = 0$, determine the matrix $\underline{\Gamma}$. We find $\Gamma_{11} = -0.23$ GeV and $\Gamma_{22} = -0.17$ GeV, where the subscripts 11 and 22 represent $K\bar{K}$ and $\pi\eta$ channels, respectively.

Though we have used

$$B(a_0 \rightarrow K\bar{K}) / B(a_0 \rightarrow \pi\eta) = 1,$$

the experimental situation regarding the branching fractions of a_0 is far from clear. If we assume that $f_1(1285)$ [old $D(1285)$] decays into $K\bar{K}\pi$ and $\eta\pi\pi$ via $a_0\pi$, then a_0 has a larger branching fraction into $\eta\pi$ than into $K\bar{K}$. On the other hand, if the decays of $\eta(1440)$ [old $\iota(1440)$] into $K\bar{K}\pi$ and $\eta\pi\pi$ is interpreted as going via $a_0\pi$, then one is led to a highly suppressed $a_0 \rightarrow \eta\pi$ branching fraction compared to $a_0 \rightarrow K\bar{K}$. The reader is referred to Ref. 10 for a detailed discussion of this problem.

In the following we summarize the result of such a coupled-channel calculation:

$$\begin{aligned} I=0: \quad A_0^{\pi\pi, U} &= 0.966 A_0^{\pi\pi} \exp(166^\circ), \\ A_0^{K\bar{K}, U} &= 0.975 A_0^{K\bar{K}} \exp(173^\circ); \\ I=1: \quad A_1^{K\bar{K}, U} &= 0.992 A_1^{K\bar{K}} \exp(172^\circ), \\ A_1^{\pi\eta, U} &= 0.992 A_1^{\pi\eta} \exp(175^\circ). \end{aligned} \quad (12)$$

Note that because of the resonant nature of the scattering matrix, a choice of the phase in the second quadrant is made. Note also that the magnitude of the nonunitarized amplitudes has hardly been affected. This is due to the fact that the resonance activity has occurred well below the D -meson mass. A broad resonance straddling the D -meson mass region would have affected the magnitude much more. Thus, effectively, the amplitudes are simply rotated. This implies that the decay amplitudes for processes such as $D^0 \rightarrow \eta\eta, \eta\eta', \pi^0\eta'$ and $D^+ \rightarrow \eta\pi^+, \eta'\pi^+$, which involve a single isospin in the final state, are simply rotated as a result of final-state interactions, which leaves the rates unaffected.

D_s^+ decays. We now discuss the unitarization of the D_s^+ decay amplitudes. The amplitudes for $D_s^+ \rightarrow K^0 \pi^+$, $K^+ \pi^0$, and $K^+ \eta$ involve an annihilation term, while $D_s^+ \rightarrow K^+ \eta'$ is a pure spectator process in the limit of zero η - η' mixing. Moreover, as⁹ in the Cabibbo-angle-favored $D \rightarrow PP$ decays, the annihilation term contributes more significantly to $D_s^+ \rightarrow \pi^+ \eta$ than to $D_s^+ \rightarrow K^0 \pi^+$ or $K^+ \pi^0$. As⁹ in $D \rightarrow K\pi$ decays, the annihilation channel contains $K_0^*(1350)$. We, therefore, expect the isospin- $\frac{1}{2}$ amplitude, whether it is generated by the spectator process or the annihilation process, to be driven through this resonance.

The procedure for unitarization of these amplitudes has been discussed earlier⁹ in the context of the Cabibbo-angle-favored $D \rightarrow PP$ decays. We shall be content with writing the final result.

Since there is no resonance activity in the πK system with isospin $\frac{3}{2}$, the unitarized amplitude $A_{3/2}^U$ will be taken to be simply the nonunitarized amplitude $A_{3/2}$ rotated by an angle $\delta_{3/2}$. The unitarized amplitude for decay leading to the πK system with isospin $\frac{1}{2}$ is given by⁹

$$A_{1/2}^U = A_{1/2} \frac{s - m^{*2}}{s - m^{*2} + i\gamma k}, \quad (13)$$

where m^* is $K_0^*(1350)$ mass, the πK center-of-mass momentum, and γ the reduced width. With $\Gamma_{K_0^*} \simeq 500$ MeV (corresponding to $\gamma \simeq 1.2$ GeV) we obtain, with

$$B(D^0 \rightarrow \bar{K}^0 K^0)/B(D^0 \rightarrow K^+ K^-) = 1.17 \times 10^{-4} \quad [\text{experiment (Ref. 5): } < 0.73], \quad (16)$$

$$B(D^0 \rightarrow K^+ K^-)/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.45_{-0.07}^{+0.08} \quad [\text{experiment (Ref. 5): } 0.50 \pm 0.21], \quad (17)$$

$$B(D^+ \rightarrow \eta' \pi^+)/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.11. \quad (18)$$

The ratio in (16) rises to 1.12×10^{-3} for

$$B(a_0 \rightarrow K\bar{K})/B(a_0 \rightarrow \pi\eta) = 0.3;$$

other predictions are insensitive to this ratio.

The best measure of the ratio C_1/C_2 appears to be

$$B(D^+ \rightarrow \bar{K}^0 K^+)/B(D^+ \rightarrow \bar{K}^0 \pi^+).$$

For the three values of $C_1/C_2 = (-4.9, -4, -3)$ we find (0.154, 0.186, 0.273) for this ratio. The Mark III value¹ for this ratio is $0.317 \pm 0.086 \pm 0.048$. Thus, a value of -4 or -3 for C_1/C_2 appears to be favored. In fact, Mark III data¹ imply a constraint, $-4.1 < C_1/C_2$

$$s = m_{D_s^+}^2,$$

$$A_{1/2}^U = 0.89 A_{1/2} \exp(i150.5^\circ). \quad (14)$$

We do not know δ_3 ; however, in our calculations we have used $\delta_{1/2} - \delta_{3/2} = 120^\circ$. In this choice we are guided by our experience⁹ with $D \rightarrow K\pi$ decays.

The annihilation form factor $f_0(s)$, in absence of final-state interactions, is parametrized as

$$f_0(m_{D_s^+}^2) = \lambda m_{D_s^+}^2 / (m_{D_s^+}^2 - m^{*2}). \quad (15)$$

In the choice of λ we are again guided by our experience with $D \rightarrow K\pi$ decays.

We now discuss our results. Our parameters are the following: $D^0 \rightarrow \pi\pi$, $K\bar{K}$, and $\pi\eta$ parameters as defined in (12); $\delta_0^{\pi\pi} - \delta_2^{\pi\pi} = 146^\circ$, $\delta_0^{K\bar{K}} - \delta_1^{K\bar{K}} \simeq 0$, $\delta_{1/2}^{K\pi} - \delta_{3/2}^{K\pi} = 120^\circ$;

$$\Gamma(f_0(1300)) = 250 \text{ MeV},$$

$$B(f_0 \rightarrow \pi\pi)/B(f_0 \rightarrow K\bar{K}) = 9;$$

$$\Gamma(a_0(980)) = 54 \text{ MeV},$$

$$B(a_0 \rightarrow \eta\pi^0)/B(a_0 \rightarrow K\bar{K}) = 1,$$

$\tan\theta_C = 0.23$, $f_\pi = 93$ MeV, $f_K = 120$ MeV, $f_\eta = 112$ MeV, τ_{D^0} , τ_{D^+} , and $\tau_{D_s^+}$ from Ref. 5.

Independent of the ratio C_1/C_2 , we predict (errors come from τ_{D^0} and τ_{D^+})

< -2.4 .

Normalizing to $B(D^+ \rightarrow \bar{K}^0 \pi^+)$, we predict, for $C_1/C_2 = (-4, 0, -3.0)$ (errors come from τ_{D^0} and τ_{D^+}),

$$B(D^0 \rightarrow K^+ K^-) = (0.35_{-0.07}^{+0.08}, 0.50_{-0.10}^{+0.11}), \quad (19)$$

$$B(D^0 \rightarrow \pi^+ \pi^-) = (0.074_{-0.015}^{+0.017}, 0.139_{-0.029}^{+0.033}). \quad (20)$$

Mark III data give⁵ $0.51 \pm 0.09 \pm 0.06$ for the branching ratio in (19) and $0.14 \pm 0.04 \pm 0.03$ for the branching ratio in (20). Confining ourselves to $C_1/C_2 = -3.0$, since it appears to fit the above data the best, we predict

$$B(D^0 \rightarrow \pi^0 \pi^0)/B(D^0 \rightarrow \pi^+ \pi^-) = 1.02 \quad [\text{experiment (Ref. 5): } < 1.43],$$

$$B(D^0 \rightarrow \eta\pi^0)/B(D^0 \rightarrow K^+ K^-) = 0.025 \quad [\text{experiment (Ref. 5): } < 1.36],$$

$$B(D^0 \rightarrow \eta' \pi^0)/B(D^0 \rightarrow K^+ K^-) = 0.003,$$

$$B(D^0 \rightarrow \eta\eta)/B(D^0 \rightarrow K^+ K^-) = 0.024 \quad [\text{experiment (Ref. 5): } < 1.82], \quad (21)$$

$$B(D^0 \rightarrow \eta\eta')/B(D^0 \rightarrow K^+ K^-) = 0.011,$$

$$B(D^+ \rightarrow \pi^+ \pi^0)/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.23 \quad [\text{experiment (Ref. 5): } < 0.32],$$

$$B(D^+ \rightarrow \eta\pi^+)/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.05.$$

Our predictions for $D_s^+ \rightarrow PP$ branching ratios are as follows. Independent of the ratio C_1/C_2 we find (errors come from⁵ τ_{D^+} and $\tau_{D_s^+}$)

$$B(D_s^+ \rightarrow K^+ \eta')/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.08 \pm 0.01. \quad (22)$$

Other branching ratios depend on the annihilation parameter λ . In Ref. 9 it was found that $D \rightarrow K\pi$ data were fit by $\lambda/f_+(0) \approx (3-4) \text{ GeV}^2$ for $C_1/C_2 = -3$. In the following, we quote the branching ratios normalized to $B(D^+ \rightarrow \bar{K}^0 K^+)$ for the two values $\lambda/f_+(0) = (3$ and $4) \text{ GeV}^2$. All the numbers should be read with errors of $\pm 10\%$ arising from τ_{D^+} and $\tau_{D_s^+}$:

$$\begin{aligned} B(D_s^+ \rightarrow K^0 \pi^+)/B(D^+ \rightarrow \bar{K}^0 K^+) &= (1.5, 2.4), \\ B(D_s^+ \rightarrow K^+ \pi^0)/B(D^+ \rightarrow \bar{K}^0 K^+) &= (1.29, 1.87), \quad (23) \\ B(D_s^+ \rightarrow \eta \pi^+)/B(D^+ \rightarrow \bar{K}^0 K^+) &= (0.73, 1.6). \end{aligned}$$

We remind the reader that $B(D^+ \rightarrow \bar{K}^0 K^+)$ is⁵ (1.01

$\pm 0.32 \pm 0.18\%$). Considerable attention has been paid to the Cabibbo-angle-suppressed charm \rightarrow two-body decays in the recent past.^{6,11-13} Reference 12 does not include final-state interactions while Ref. 6 uses SU(3) decay amplitudes. However, an explanation for the branching ratios $B(D^0 \rightarrow \pi^+ \pi^-)$ and $B(D^0 \rightarrow K^+ K^-)$ with an economy of parameters has so far been lacking. In Ref. 6 the decay amplitudes were assumed to be SU(3) symmetric and the phases of the amplitudes were treated as parameters. In the present work the decay amplitudes break SU(3) symmetry and the phases are generated by a coupled two-channel model.

In summary, using a coupled-channel final-state interaction formalism with $C_1/C_2 \approx -3.0$, we are able to fit all the measured $D \rightarrow PP$ Cabibbo-angle-suppressed rates. In particular, we can explain $D^0 \rightarrow \pi^+ \pi^-$ and $K^+ K^-$ rates. We make predictions on all the as yet unmeasured rates for $(D, D_s^+) \rightarrow PP$ decays. We emphasize that annihilation terms play no role in $(D^0, D^+) \rightarrow PP$ decays.

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