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Fourth generation and nucleon decay in supersymmetric theories

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Analysis of nucleon decay in $N=1$ supergravity unified models including the effect of a fourth generation of matter is given. Experimental constraints from nucleon-lifetime limits on the Kobayashi-Maskawa (KM) matrix that enters nucleon decay are obtained. The decays $K_L \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\bar{\nu}$ are analyzed under these constraints, since the combination of the KM matrix that enters nucleon decay also enters these rare decays. The branching ratio $K^+ \to \pi^+\nu\bar{\nu}$ in four generations is shown to be considerably larger than for the three-generation case except for certain narrow domains of the KM matrix for two of the four branches of solutions. Bounds on V_{ub} and $V_{ub'}$ are also obtained.

I. INTRODUCTION

Proton decay provides a strong experimental test for any grand unified theory (GUT). Thus the current experimental bounds[1] on the decay $p \to e^+\pi^0$ clearly rule out the minimal SU(5) GUT model. A great deal of work exists in the literature on nucleon decay in supergravity unified models.[2–10] In supergravity models,[11] proton decay proceeds through the exchange of the superheavy Higgsino triplet. Since the Higgsino mass M_H is governed by physics at the GUT scale, it is not determined theoretically. The absolute decay rates thus are not predicted, though branching ratios into the various modes are. However, as pointed out by Enqvist, Masiero, and Nanopoulos,[9] GUT models which preserve the gauge hierarchy generally require

$$M_H \lesssim M_G \tag{1.1}$$

(and often M_H is considerably less than the GUT mass M_G). For the standard SU(5) supergravity model with two Higgs doublets one has

$$M_G \simeq (1.0\pm 0.6)\times 10^{16} \text{ GeV} , \tag{1.2}$$

and thus Eq. (1.1) puts a significant upper bound on the proton lifetime, which, when combined with the experimental lower bounds,[1] eliminates certain supergravity models.[9,8,12]

Since supergravity nucleon decay proceeds through Higgsino interactions, the decay amplitudes depend explicitly on Kobayashi-Maskawa (KM) matrix elements V_{ij}. Thus the decay rates are sensitive to the values of V_{ij} and the number of generations. Most significant is the fact that the *same* combinations of KM matrix elements also appear in the rare K-meson decay modes $K_L \to \mu\bar{\mu}$ and $K^+ \to \pi^+\nu\bar{\nu}$. Thus in supergravity models it is possible to correlate the proton lifetime with the rare K decay rates, and in this fashion probe for the existence of a fourth generation of quarks and leptons. We will see below that for the standard SU(5) supergravity models, the existing data are consistent with the existence of a fourth generation, but that strong restrictions can be placed on various KM matrix elements, which in fact eliminate some conjectured four-generation KM matrices. Furthermore, the decay rate for $K^+ \to \pi^+\nu\bar{\nu}$ is predicted to be generally larger for four generations than for three generations, a result that is experimentally testable.[13] Thus when proton decay is combined with the rare K decays, supergravity models make experimental predictions which allow one to distinguish the number of generations which have light neutrinos.

In Sec. II we review the supergravity proton-decay results for two and three generations. Section III then extends this analysis to four generations and examines the correlations with the rare K decay modes. Section IV discusses the constraints proton decay imposes on the KM matrix elements V_{ub} and $V_{ub'}$. Section V gives a summary of the results and conclusions. The Appendix lists the main proton-decay formulas.

II. PROTON-DECAY AMPLITUDES

The effective dimension-six nucleon-decay amplitudes in supergravity GUT's arise from Higgsino-triplet exchange followed by gaugino "dressing." The characteristic diagrams for W-ino dressing are shown in Fig. 1 where KM factors arise at each vertex. The full result involves, in addition, gluino and Z-ino dressing as well as $RRRR$ dimension-five operators[7,8] and is quite complicated. It is given, generalized for an arbitrary number of generations, in the Appendix for the decay modes $N \to \bar{\nu}K$ and $N \to \bar{\nu}\pi$. We restrict our discussion in this paper to the supergravity models with large D terms [e.g., renormalization-group (RG) models] where the Higgs mixing angle α_H is small, i.e., $\alpha_H \approx 10°–25°$.

(Analyses may also be carried out for the case $\alpha_H \simeq 45°$.) In this case, if the gravitino mass $m_{3/2}$ is not too small, i.e., $m_{3/2} \gtrsim 150$ GeV, the W-ino dressing dominates, and the effective dimension-six Lagrangian for $N \to \bar{\nu}_i K$ (where ν_i is the ith-generation neutrino) reduces to

$$\mathcal{L}_0(N \to \bar{\nu}_i K) = (\alpha_2{}^2/M_H)(m_i^d V_{iu}^\dagger)(2M_W{}^2 \sin 2\alpha_H)^{-1} \times \left[\left[\sum_{j=2} P_j A_j^* m_i^u F_{ij} \right] (\alpha_i^L + \beta_i^L) + \Delta_i \right] , \tag{2.1}$$

where V_{ij} are the KM matrix elements (with phases so that $V_{id} =$ real),

$$A_i = V_{id} V_{is}^* , \tag{2.2}$$

F_{ij} is the form factor resulting from the W-ino triangle loop integral of Fig. 1, m_i^d and m_i^u are the u- and d-quark masses, P_i is the additional (PC-violating) phases of nucleon decay, α_i^L and β_i^L are the $LLLL$ four-field quark-lepton interactions,[8] and Δ_i are additional, generally small, contributions from other gaugino clothings. F_{ij}, α_i^L, and Δ_i are given in the Appendix.

The KM factors governing the $N \to \bar{\nu}_i K$ mode are V_{ui} and the combination A_i of Eq. (2.2). The A_i also enter into the rare K decay modes. Thus for the branching ratio of $K^+ \to \pi^+ \nu\bar{\nu}$ one has[14–16]

$$B(K^+ \to \pi^+ \nu\bar{\nu}) \simeq 1.5 N_\nu \times 10^{-5} \left| \sum A_i D(x_i) \right|^2 , \tag{2.3}$$

where N_ν is the number of generations (i.e., light neutrinos) $x_i = (m_i^u/M_W)^2$ and

$$D(x) = \frac{1}{4}x + \frac{3}{4}\frac{x}{x-1} + \frac{3}{4}\frac{(x-2)x}{(x-1)^2}\ln x . \tag{2.4}$$

Also, from the bound on $K_L \to \mu\bar{\mu}$, one has[14,15]

$$Q(K_L) \simeq \left| \mathrm{Re} \sum A_i C(x_i) \right| \lesssim 2 \times 10^{-3} , \tag{2.5}$$

where

$$C(x) = \frac{1}{4}x - \frac{3}{4}\frac{x}{x-1} + \frac{3}{4}\left[\frac{x}{x-1} \right]^2 \ln x . \tag{2.6}$$

In addition, the A_i obey the unitarity relation

$$\sum A_i = 0 . \tag{2.7}$$

From Eqs. (2.1), (2.3), and (2.5), one sees it is possible to relate proton decay and the rare K decay modes.

The proton-decay rates depend sensitively on the number of generations. Thus if one considers only the first two generations, one finds that $N \to \bar{\nu} K$ is the dominant mode.[2,3] From the experimental upper bound on these modes[1] and Eq. (2.1), one can obtain a lower bound on M_H (Refs. 7 and 8). One finds, for a squark mass $m_{\tilde{q}} \simeq 180$ GeV, $M_H \gtrsim (10\text{–}70) \times 10^{16}$ seriously violating the theoretical constraint of Eq. (1.1). (A similar result occurs for no-scale models.[9]) One may, of course, reduce the decay rate by increasing the squark mass, which enters in the triangle loop of Fig. 1. However, the inconsistency remains for squarks with mass $m_{\tilde{q}} \lesssim 350$ GeV. Hence models where the second generation dominates are excluded for squarks which are in the mass range where they would be detectable at the Fermilab Tevatron and/or the Superconducting Super Collider.

When three generations are considered, the possibility of suppressing the $\bar{\nu}_i K$ modes arises via an approximate cancellation between the second and third generations.[7,8] From Eq. (2.1) this can occur if

$$A_2 m_c P_2 F_{ic} + A_3 m_t P_3 F_{it} \approx 0 . \tag{2.8}$$

The form factors F_{ij} are approximately independent of the generation index i and so the suppression occurs universally for all modes $\bar{\nu}_i K$. As discussed in Refs. 7 and 8, Eq. (2.8) can be satisfied for $m_t \gtrsim 50$ GeV and provided the PC-violating phase δ is $\approx 180°$, P_3/P_2 is approximately real and A_P, the Polonyi constant, is not small, i.e., $A_P \approx 1$ (Ref. 17). Simultaneously, the $\bar{\nu}_i \pi$ and $\bar{\nu}_i \rho$ modes are enhanced, making them comparable to or larger than the $\bar{\nu}_i K$ modes. To satisfy Eq. (1.1), the cancellation in Eq. (2.8) need not be precise, i.e.,

$$| A_2 m_c P_2 F_{ic} + A_3 m_t P_3 F_{it} | \lesssim 0.2 \, | A_2 m_c P_2 F_{ic} | . \tag{2.9}$$

Thus one finds Eq. (1.1) is obeyed for a wide range of parameters.[8,10] The three-generation supergravity models, then, are consistent with existing proton-decay data, and make interesting predictions which could be tested by the Kamiokande Collaboration with their planned "Super Kamiokande" detector.

As discussed in Ref. 18, existing data plus three-generation unitarity of the KM matrix puts an upper bound on A_3. We find

$$| A_3 | \leq 0.001\,51 . \tag{2.10}$$

Since the dominant contribution to $Q(K_L)$ comes from the third generation, the $K \to \mu\bar{\mu}$ constraint (2.5) also puts a bound on A_3. This bound dominates Eq. (2.10) when $m_t \gtrsim 130$ GeV. Bounds on A_3 then produce upper bounds on the $K^+ \to \pi^+ \nu\bar{\nu}$ rate from Eq. (2.3). As can be seen in Table I, the three-generation $B(K^+ \to \pi^+ \nu\bar{\nu})$ reaches a maximum of about 8×10^{-10} for $m_t = 140$ GeV, and is less at higher and lower values of m_t. These results will help distinguish three- and four-generation models.

TABLE I. Bounds on A_3, $B(K^+ \to \pi^+ \nu\bar{\nu})$, and ρ_3 in three-generation models, as a function of the t-quark mass m_t. (All energies are in GeV.)

m_t	$\vert A_3 \vert_{\max}$	$B(K^+ \to \pi^+ \nu\bar{\nu})_{\max}$	$(m_t \vert \rho_3 \vert)_{\min}$
50	0.001 51	1.67×10^{-10}	199
60	0.001 51	2.18×10^{-10}	199
100	0.001 51	4.89×10^{-10}	199
120	0.001 51	6.66×10^{-10}	199
130	0.001 51	7.66×10^{-10}	199
140	0.001 41	7.81×10^{-10}	213
150	0.001 27	7.43×10^{-10}	237
200	0.000 81	6.11×10^{-10}	371

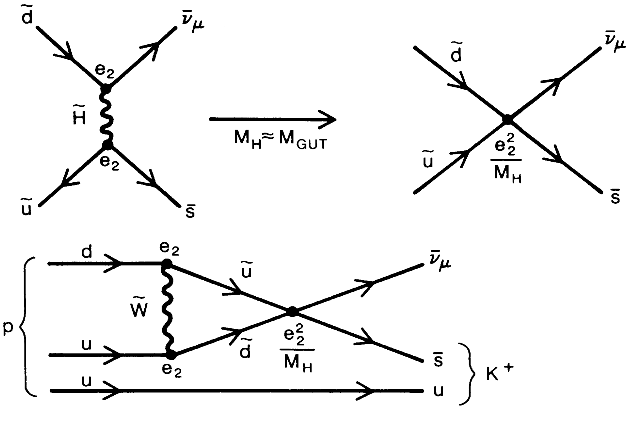

FIG. 1. Diagrams leading to proton decay with W-ino ($\tilde{W}$) dressing. $\tilde{u}$ and $\tilde{d}$ are the u and d squarks.

One may write the supersymmetry (SUSY) proton-decay constraint (2.8) as

$$m_t\,|\rho_3| \simeq m_c \left|\frac{A_2}{A_3}\right|, \quad \rho_3 = \frac{P_3}{P_2}\frac{F_{it}}{F_{ic}} \;. \tag{2.11}$$

Since $|A_2| \simeq 0.20$, Eq. (2.11) yields a lower bound on $|\rho_3|$, which is also shown in Table I. Of course in supergravity models, the value of ρ_3 is determined dynamically by the loop integrals of Fig. 1, and in general the lower bounds of Table I can be satisfied. Note that only the last column for $m_t\rho_3$ of Table I depends on the SUSY model, and the other columns hold equally well for the three-generation standard model.

III. FOUR-GENERATION MODEL

For four generations, the situation is more complicated as less is known about the four-generation KM matrix. The condition that the $\bar{\nu}_i K$ proton-decay modes be suppressed so that Eq. (1.1) remains valid, therefore, is a useful constraint. From Eq. (2.1) we write this in the form

$$A_3 m_t \rho_3 + A_4 m_{t'} \rho_4 \simeq -m_c A_2 \;, \tag{3.1}$$

where

$$\rho_3 \simeq \frac{P_3}{P_2}\frac{F_{it}}{F_{ic}}, \quad \rho_4 \simeq \frac{P_4}{P_2}\frac{F_{it'}}{F_{ic}} \;, \tag{3.2}$$

and $m_{t'}$ is the t'-quark mass. Equation (3.1) and the unitarity condition (2.7) allow one to solve for A_3 and A_4:

$$A_3 = [m_{t'}(A_1 + A_2)\rho_4 - m_c A_2]/(m_t\rho_3 - m_{t'}\rho_4) \;, \tag{3.3}$$

$$A_4 = [-m_t(A_1 + A_2)\rho_3 + m_c A_2]/(m_t\rho_3 - m_{t'}\rho_4) \;. \tag{3.4}$$

The results depend sensitively on the combination $A_1 + A_2$, which unfortunately is not well determined experimentally. Using four-channel unitarity and experiment, one may derive $V_{cs} = 0.9171 \pm 0.1085$ which is slightly better than $V_{cs}^{\text{expt}} = 0.95 \pm 0.14$. This yields $A_2 = -0.1898 \pm 0.0314$ and

$$A_1 + A_2 = 0.0246 \pm 0.0315 \;. \tag{3.5}$$

In analyzing this case we will make use of the UA1 experimental lower bounds on the t- and t'-quark masses and the τ'-lepton mass:[19]

$$m_t, m_{t'} \gtrsim 40 \text{ GeV} \;, \tag{3.6a}$$

$$m_{\tau'} > 41 \text{ GeV} \;. \tag{3.6b}$$

Furthermore, in the renormalization-group analysis of the supergravity models, the requirement that $SU(2)\times U(1)$ breaking correctly occurs at the W mass scale gives upper bounds on the t', b', and τ' masses:[20,21]

$$m_{t'} < 140 \text{ GeV}, \quad m_{b'} < 135 \text{ GeV} \;, \tag{3.7a}$$

$$m_{\tau'} < 70 \text{ GeV} \;. \tag{3.7b}$$

In general, $SU(2)\times U(1)$ breaking in RG models will not occur at W mass scale unless at least one quark mass is large (i.e., $\gtrsim M_W$) while if the quark masses are too heavy, the breaking will occur at too high a mass scale. Since $m_t < m_{t'}$ we will assume in the following that $m_t \lesssim M_W$ and $m_{t'} \gtrsim M_W$ as well as the constraints (3.6) and (3.7). We will also assume P_3/P_2 are relatively real so that Eq. (3.1) can be approximately satisfied.

In three generations, the condition (2.8) which suppresses the $\bar{\nu}_i K$ modes, required that $|\rho_3| \approx 2-4$ as can be seen from Table I. For four generations, the corresponding condition (3.1) does not require ρ_3 and ρ_4 to be large, as the two terms on the left-hand side can add coherently. Thus one may consider two possible cases.

(i) Models with no L-R mixing. Here we assume $A_P \simeq 0$ and hence there is no L-R mixing in the squark mass matrices (such a situation is realized in certain sectors of the superstring-inspired models[22]). For this case the squarks in different generations are approximately degenerate so that $F_{ic} \simeq F_{it} \simeq F_{it'}$. Then $\rho_3 \simeq \pm 1$ and $\rho_4 \simeq \pm 1$ where the signs of ρ_3 and ρ_4 are determined by the phases P_3/P_2 and P_4/P_2.

(ii) Models with large L-R mixing. Here one characteristically expects $A_P \approx 1$ so that there is large L-R mixing in the squark mass matrices for the heavy-quark generations. Then one expects $|\rho_3| \neq |\rho_4|$ and both can be large, particularly when m_t and $m_{t'}$ are large. In the following analysis, we will consider only case (i).

The solutions for A_3 and A_4 of Eqs. (3.3) and (3.4) can be used to calculate $B(K^+ \to \pi^+ \nu\bar{\nu})$ and $Q(K_L)$ of Eqs. (2.3) and (2.5). However, these quantities will depend on $A_1 + A_2$ which, as seen in Eq. (3.5), is poorly determined experimentally. It is therefore better to think of $A_1 + A_2$ as a free parameter [within the allowed ranges of Eq. (3.5)] and see what constraints can be put on it.

We first ask what is the allowed ranges of $A_1 + A_2$ which do not violate the $K \to \mu\bar{\mu}$ constraint Eq. (2.5) when m_t and $m_{t'}$ are in the above ranges 40 GeV $\lesssim m_t \lesssim M_W$, and $M_W \lesssim m_t' \lesssim 140$ GeV. For the four cases of $\rho_3 = \pm 1$ and $\rho_4 = \pm 1$ (no L-R mixing) results are given in Table II. We see that even when all four cases are taken together, the range allowed for $A_1 + A_2$ by

TABLE II. Minimum and maximum values for A_1+A_2 allowed by the $K\rightarrow\mu\bar{\mu}$ constraint of Eq. (2.5).

ρ_3	ρ_4	$(A_1+A_2)_{\min}$	$(A_1+A_2)_{\max}$
+1	+1	−0.0257	−0.0049
−1	−1	0.0047	0.0223
+1	−1	−0.0098	0.0032
−1	+1	−0.0032	0.0095

Eq. (2.5) is much narrower than even the direct experimental 2σ bounds from Eq. (3.5):

$$-0.0384 < A_1+A_2 < 0.0876 \ . \tag{3.8}$$

We next ask under what circumstances does the four-generation $K^+\rightarrow\pi^+\nu\bar{\nu}$ rate fall *below* the three-generation rate of Table I while not violating the $K\rightarrow\mu\bar{\mu}$ constraint of Eq. (2.5). We find that for the entire mass ranges of m_t and $m_{t'}$ this can never happen when $\rho_3=\rho_4=\pm1$. For $\rho_3=-\rho_4=\pm1$ and fixed m_t and $m_{t'}$ there is only a narrow band in the values of A_1+A_2 about 0.002 wide where this occurs around A_1+A_2 close to zero. For any m_t and $m_{t'}$ the four-generation $K^+\rightarrow\pi^+\nu\bar{\nu}$ rate lies below the three-generation one only when

$$-0.006 \lesssim A_1+A_2 \lesssim 0.002 \ . \tag{3.9}$$

Thus almost always the $K^+\rightarrow\pi^+\nu\bar{\nu}$ rate for supergravity models with four generations is larger than that expected from the standard model with three generations, and so this decay is a good indicator of new physics.

When A_1+A_2 lies in the ranges of Table II allowed by the $K\rightarrow\mu\bar{\mu}$ constraint, the $B(K^+\rightarrow\pi^+\nu\bar{\nu})$ is usually considerably larger than the three-generation limits of Table I [except in the small domain of Eq. (3.9) where there is an accidental cancellation between the third and fourth generations]. Some examples are given in Table III where it is also shown that when the limits of A_1+A_2 of Table II are exceeded, the bound Eq. (2.5) on $Q(K_L)$ can be badly violated.

TABLE III. Examples of $Q(K_L)$ and $B(K^+\rightarrow\pi^+\nu\bar{\nu})$ for various t and t' masses (in GeV) for the case $\rho_3=-1=\rho_4$. Large values of A_1+A_2 [still consistent with the experimental bounds of Eq. (3.8)] cause $Q(K_L)$ to exceed the experimental limit of 2×10^{-3}.

m_t	$m_{t'}$	A_1+A_2	$Q(K_L)$	$B(K^+)$
40	140	0.010	0.8×10^{-3}	15.7×10^{-10}
40	100	0.010	0.9×10^{-3}	15.8×10^{-10}
80	140	0.010	0.9×10^{-3}	15×10^{-10}
50	100	0.015	0.9×10^{-3}	11.2×10^{-10}
100	140	−0.018	16.6×10^{-3}	67×10^{-10}
40	140	0.056	14.4×10^{-3}	4.1×10^{-10}
40	60	0.056	8.0×10^{-3}	3.7×10^{-10}

IV. CONSTRAINTS ON V_{ub} AND $V_{ub'}$

Condition (3.1) is sufficient to suppress the decay $N\rightarrow\bar{\nu}_\mu K$. From Eq. (2.1) one sees that it will also suppress the $N\rightarrow\bar{\nu}_\tau K$ and $N\rightarrow\nu_{\tau'}K$ [and hence guarantee that Eq. (1.1) is obeyed] provided the front coefficients $m_b V_{ub}$ and $m_{b'}V_{ub'}$ are the same size or smaller than the second-generation factor $m_s V_{us}$. Thus one has approximately

$$|V_{ub}| \lesssim m_s |V_{us}|/m_b \simeq 6\times10^{-3} \ , \tag{4.1}$$

$$|V_{ub'}| \lesssim m_s |V_{us}|/m_{b'} \lesssim m_s |V_{us}|/(2m_{\tau'}) \lesssim 4\times10^{-4} \ , \tag{4.2}$$

where we have used the experimental bound Eq. (3.6b) and the theoretical estimate[19] $m_{b'}\simeq 2m_{\tau'}$. Equation (4.1) is not much stronger than the experimental bound $|V_{ub}|\le0.009$ [based $\Gamma(b\rightarrow u)/\Gamma(b\rightarrow c)\le0.008$ and a B lifetime $\tau_b=1.1$ ps]. Equation (4.2), however, is quite limiting.

Parametrizing the KM matrix elements in terms of the usual four-generation KM angles $\theta_1\cdots\theta_6$,

$$V_{ud}=c_1,\quad V_{us}=s_1c_3,\quad V_{ub}=s_1s_3c_5,\quad V_{ub'}=s_1s_3s_5 \ , \tag{4.3}$$

where $c_1\equiv\cos\theta_1$, $s_1\equiv\sin\theta_1$, etc., one can convert Eqs. (4.1) and (4.2) to constraints on θ_3 and θ_5. One finds

$$|\sin\theta_3| \lesssim 3\times10^{-2},\quad |\sin\theta_5| \lesssim 6\times10^{-2} \ . \tag{4.4}$$

In deducing Eq. (4.2), we have, of course, assumed that the fourth-generation neutrino $\nu_{\tau'}$ is massless so that the decay $N\rightarrow\bar{\nu}_{\tau'}K$ is energetically possible. A sufficiently heavy $\nu_{\tau'}$ could prevent this decay from occurring, eliminating the constraint (4.2)

V. CONCLUSIONS

The above discussion has shown that the fourth generation can effect nucleon decay in supergravity models in a significant way both through additional contributions to the dressing loop integrals of Fig. 1, as well as through the new decay modes involving the fourth sequential neutrino. The condition Eq. (1.1) that $M_H\lesssim M_{\rm GUT}$ plays a role similar to the requirement $M_X=M_{\rm GUT}$ in nonsupersymmetric SU(5) GUT models. This, combined with the fact that the same combination of KM matrix elements which enter in proton decay, i.e., A_i of Eq. (2.2), also enter into the rare K decays, $K_L\rightarrow\mu\bar{\mu}$ and $K^+\rightarrow\pi^+\nu\bar{\nu}$, allows one to determine information both on the values of KM matrix elements and the number of generations. Thus, aside from a small range Eq. (3.8), the four-generation $K^+\rightarrow\pi^+\nu\bar{\nu}$ rate will exceed the three-generation rate, and the branching ratio for the latter is bounded by 8×10^{-10} for any t-quark mass. Hence a precision measurement of this decay[13] will shed light on the existence of a fourth generation, and the determination of m_t will further greatly restrict supergravity models and the allowed range of A_1+A_2.

APPENDIX: NUCLEON-DECAY AMPLITUDES

The nucleon-decay amplitudes for three generations are given in Ref. 8. We generalize these results here for an arbitrary number of generations. In order to make contact with the notation of Ref. 8, we rewrite Eq. (2.1) factoring out the second-generation W-ino dressing contribution. For $N\to\bar{\nu}_i K$ dimension-six Lagrangian we have

$$\mathcal{L}_6(N\to\bar{\nu}_i K)=[(\alpha_2)^2(2M_H M_W{}^2\sin2\alpha_H)^{-1}P_2 m_c m_i^d V_{i1}^\dagger V_{21}V_{22}][F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})]$$
$$\times\left[\left[1+\sum_j y_{ij}^K+(y_{\tilde{g}}+y_{\tilde{z}})\delta_{i2}+\delta_i^K\right]\alpha_i^L+\left[1+\sum_j y_{ij}^K-(y_{\tilde{g}}-y_{\tilde{z}})\delta_{i2}\right]\beta_i^L+(y_{1i}^R\alpha_i^R+y_{2i}^R\beta_i^R)\sum_j\delta_{ij}\right], \tag{A1}$$

where $j\geq3$. In Eq. (A1) one has, for the $t,t',\ldots$ squark contributions to the W-ino dressing diagrams,

$$y_{ij}^K=\frac{P_j}{P_2}\frac{m_j^u V_{j1}V_{j2}}{m_c V_{21}V_{22}}\frac{F(\tilde{u}_j;\tilde{d}_i;\tilde{W})+F(\tilde{u}_j;\tilde{e}_i;\tilde{W})}{F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})}, \tag{A2}$$

where the triangle loop form factors F are defined in Eq. (3.11) and the Appendix of Ref. 8. The gluino and Z-ino dressing contributions, $y_{\tilde{g}}$ and $y_{\tilde{z}}$, are defined in Eqs. (5.2) and (5.3) of Ref. 8. The contributions from the $RRRR$ dimension-five operator are given by

$$y_{1i}^R=\frac{P_1}{P_2}\frac{\sum_j m_d m_j^u V_{11}V_{j2}V_{ij}^\dagger Q(\tilde{e}_j;\tilde{u}_i;\tilde{W})}{m_c m_i^d V_{21}V_{22}V_{i1}^\dagger[F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})]}, \tag{A3}$$

$$y_{2i}^R=\frac{P_1}{P_2}\frac{\sum_j m_s m_j^u V_{12}V_{j1}^\dagger V_{ij}^\dagger Q(\tilde{e}_j;\tilde{u}_i;\tilde{W})}{m_c m_i^d V_{21}V_{22}V_{i1}^\dagger[F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})]}. \tag{A4}$$

The dimension-six quark-lepton operators are

$$\alpha_i^L=\epsilon_{abc}(d_{aL}\gamma^0 u_{bL})(s_{cL}\gamma^0\nu_{iL}), \tag{A5}$$

α_i^R is α_i^L with $(d_L,u_L\to d_R,u_R)$, and $\beta_i^{L,R}$ is $\alpha_i^{L,R}$ with $d\leftrightarrow s$. The quantity δ_i^K is the generalization of Δ_i^K and is generally small.

The dimension-six $N\to\nu_i\pi$ effective Lagrangian may be written as $(j=3,4,\ldots)$

$$\mathcal{L}_6(N\to\bar{\nu}_i\pi)=[(\alpha_2)^2(2M_H M_W{}^2\sin2\alpha_H)^{-1}P_2 m_c m_i^d(V_{21})^2V_{i1}^\dagger]$$
$$\times[F(\tilde{c},\tilde{d}_i)+F(\tilde{c},\tilde{e}_i)]\left[\left[1+\sum_j y_{ij}^\pi+y_i^{u\pi}+\delta_i^\pi\right]\gamma_i^L+y_{iR}\left[\sum_j\delta_{ij}\right]\gamma_i^R\right]. \tag{A6}$$

Here

$$\gamma_{ij}^\pi=\frac{P_j}{P_2}\left[\frac{m_j^u(V_{j1})^2}{m_c(V_{21})^2}\right]\left[\frac{F(\tilde{u}_j;\tilde{d}_i;\tilde{W})+F(\tilde{u}_j\tilde{e}_i;\tilde{W})}{F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})}\right]. \tag{A7}$$

The δ_i^π are generalizations of Δ_i^π of Ref. 8 and are generally small. $\gamma_i^{u\pi}$ is defined in Eq. (5.11) of Ref. 8. The y_{iR} arise from the $RRRR$ dimension-five couplings, and may be written as

$$y_{iR}=\frac{P_1}{P_2}\frac{V_{11}m_d\sum_j m_j^u V_{j1}V_{ij}^\dagger Q(\tilde{e}_j,\tilde{u}_i;\tilde{W})}{m_c m_i^d(V_{21})^2V_{i1}^\dagger\left[F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})\right]}, \tag{A8}$$

where the dimension-six quark-lepton operators $\gamma_i^{L,R}$ are defined in Eq. (5.9) of Ref. 8. The condition which suppresses the $N\to\bar{\nu}_i K$ modes

$$1+\sum_j y_{ij}^K\approx0 \tag{A9}$$

generally tends to enhance the $N\to\bar{\nu}_i\pi$ modes, as the corresponding structure in Eq. (A6), i.e., $1+\sum y_{ij}^\pi$, does not cancel.

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