Unpolarized and polarized structure functions and spin tests at supercollider energies

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We present a simple analytic parametrization for quark and gluon distributions inside a polarized or unpolarized proton. It is valid for small enough x and for very large Q^2, the kinematic domain relevant for supercollider energies. We calculate some helicity hadron asymmetries in polarized-hadron collisions in the multi-TeV energy range and we discuss further possibilities of significant spin effects.

I. INTRODUCTION

Experiments with the next generation of hadron-hadron colliders, the so-called high-energy and high-luminosity supercolliders, will allow us to set limitations of the standard model. In this framework the fundamental constituents of matter are quarks, which carry color and fractional charge, and leptons, all being pointlike structureless and spin-$\frac{1}{2}$ objects which are arranged into three known generations. The dynamics of these fermions is described by a gauge field theory where their interactions are mediated by spin-1 gauge bosons. In spite of the present success of this simple and beautiful picture, several questions are still unanswered and many more detailed tests remain to be investigated. In particular we expect a multi-TeV hadron collider to illuminate the physics of the electroweak symmetry breaking with the discovery of a Higgs boson whose mass is still unknown. Another fundamental issue is the question of parity violation, which might be a low-energy property if nature becomes left-right symmetric at high energies. We also hope to understand the dynamical origin of *CP* violation in weak interactions and why a particular number of generations should exist. For understanding many important related issues at a lower-energy hadron collider, spin has been proven to be very useful. Therefore, at this level a natural question to ask, since all the basic fields except the Higgs field carry a nonzero spin, is the following: what is the importance of spin effects in the multi-TeV energy range and does physics require the polarization of the proton beams in a supercollider?[1]

One of the prerequisites for answering these questions is a reliable determination of the parton distributions, since hadron collisions must be interpreted in terms of quark and gluon interactions which cannot be studied directly, unlike lepton interactions. The x and Q^2 behavior of unpolarized and polarized nucleon structure functions has been carefully examined in the literature[2,3] in view of testing scaling violations predicted by perturbative QCD which turn out to be a slowly varying effect for Q^2 above 100 GeV^2. The scarcity of data for polarized structure functions in the low-Q^2 region implies a larger theoretical uncertainty which can be reduced by using some analytic requirements and various sum rules.[4]

At supercollider energies the kinematic region of interest is $10^2 < Q^2 < 10^8$ GeV^2 and $10^{-5} < x < 10^{-1}$ but these small values of x, even at low Q^2, are not probed by current experiments. The small-x region is indeed crucial because as Q^2 increases since there are more and more partons produced with a low momentum, the structure functions become more important at small x (Ref. 5). However, our present limited knowledge on the experimental side at low x and Q^2 is not so critical because uncertainties are reduced for higher Q^2 and the dominant contribution of the structure functions turns out to be rather simple.

The aim of this paper is to provide a set of structure functions in terms of a simple parametrization allowing a reliable evaluation of high-energy cross sections. In Sec. II we will give analytic expressions for unpolarized and polarized parton distributions and we will calculate the parton differential luminosities which are very useful for a fast estimate of the production rates. Then in Sec. III we will present some asymmetries which are relevant QCD spin tests at future colliders. Further spin effects of potential interest for new physics will be discussed in Sec. IV where we will give our concluding remarks.

II. UNPOLARIZED-POLARIZED PARTON DISTRIBUTIONS AND PARTON LUMINOSITIES

For a given parton (quark, antiquark, or gluon) we denote by $f_{\pm}(x,Q^2)$ the parton distributions in a polarized nucleon either with helicity parallel (+) or antiparallel (−) to the parent nucleon helicity. We define as usual the *unpolarized distribution* $f = f_+ + f_-$ and the *parton helicity asymmetry* $\Delta f = f_+ - f_-$. Both f and Δf obey the coupled Lipatov-Altarelli-Parisi equations[6] whose resolution under some approximations leads to analytic parametrizations of their x and Q^2 dependences. For small enough x the behavior of these distributions is known and a simple expression for the sea u quark u_s and for the gluon G unpolarized distributions was proposed in Ref. 7. A better agreement with the numerical solutions provided by Eichten, Hinchliffe, Lane, and Quigg[2] (EHLQ) can be obtained up to $x = 0.1$ and above by including a factor related to the large-x behavior of the distribution. By taking $t = \ln(Q^2/\Lambda^2)$ with $\Lambda = 200$

MeV and $S(Q^2)=\ln(t/t_0)$ where t_0 corresponds to the initial momentum value $Q_0{}^2=5$ GeV^2, we therefore propose to use

$$xG(x,S)=K(S)\exp\left[12\left[\frac{S}{b}\ln(1/x)\right]^{1/2}\right](1-x)^6 , \quad (1)$$

$$xu_s(x,S)=\frac{1}{2\sqrt{3}}\left[\frac{S}{b\ln(1/x)}\right]^{1/2} xG(x,S) , \quad (2)$$

with

$$K(S)=50.36[\exp(S)-0.957]\exp(-7.597\sqrt{S}) ,$$

as determined in Ref. 7 and where $b=33-2N_f$, N_f being the number of quark flavors. Similarly for the valence u quark, by using the results quoted in Ref. 4 we will take

$$xu_v(x,S)=K'(S)\exp\left[4\sqrt{2}\left[\frac{S}{b}\ln(1/x)\right]^{1/2}\right]x^{0.65}(1-x)^3 , \quad (3)$$

with $K'(S)=2\sqrt{S}\exp(-1.5S)$. For the valence d quark we take

$$xd_v(x,S)=0.5(1-x)xu_v(x,S) . \quad (4)$$

Like EHLQ, we assume that the sea is isosymmetric (i.e., $u_s=d_s$) and for the heavy-flavor components of the sea $q_s(x,S)$, since in this paper we are concerned with energies above their mass thresholds we will simply take

$$xq_{(x,S)}=q_s xu_s(x,S) , \quad (5)$$

with

$$u_s:s_s:c_s:b_s:t_s:\ \ 1:0.82:0.33:0.23:0.14 .$$

These ratios are obtained by comparing the corresponding luminosities given in EHLQ for $(\hat{s})^{1/2}$ around 1 TeV and $\sqrt{s}=40$ TeV. They slightly underestimate their values for $Q^2=100$ TeV^2.

Let us now turn to the polarized distributions and more precisely to the parton helicity asymmetries. For the gluon helicity asymmetry the small-x behavior has been derived in Ref. 4 and by fitting the numerical results of Ref. 3 we obtain

$$x\Delta G(x,S)=K''(S)x\exp\left[4(10+\sqrt{64-6N_f})^{1/2}\times\left[\frac{S}{b}\ln(1/x)\right]^{1/2}\right](1-x)^6 , \quad (6)$$

with $K''(S)=\exp(-4.5\sqrt{S})$.

For the valence-quark helicity asymmetries the low-energy data coming form SLAC-Yale experiment[8] is consistent with the original Carlitz-Kaur broken-SU(6) model.[9] We will prefer an improved version of this approach introduced in Ref. 3 as model I. So we will take

$$\Delta u_v(x,S)=\cos 2\theta_u u_v(x,S)-\tfrac{2}{3}\cos 2\theta_d d_v(x,S) ,$$
$$\Delta d_v(x,S)=-\tfrac{1}{3}\cos 2\theta_d d_v(x,S) , \quad (7)$$

which are directly expressed in terms of the unpolarized distributions by means of two dilution factors $\cos 2\theta_u$ and $\cos 2\theta_d$ given by the simple formulas

$$\cos 2\theta_u=[1+H_0^u(1-x)^2/\sqrt{x}\,]^{-1} \quad \text{with } H_0^u=0.09-0.04S ,$$
$$\cos 2\theta_d=[1+H_0^d(1-x)/\sqrt{x}\,]^{-1} \quad \text{with } H_0^d=0.03-0.01S . \quad (8)$$

In this model the sea-quark helicity asymmetry remains positive and it increases with larger x and higher Q^2. For the u-sea quark it can be parametrized as

$$\Delta u_s(x,S)=(0.7+0.5\ln S)xu_s(x,S) , \quad (9)$$

while the other flavors are obtained by using the same rules as for the unpolarized distributions.

We have checked that the above set of parton distributions [Eqs. (1)–(5)] follows the numerical solutions provided by EHLQ within 10–15%, except for Q^2 as low as 10 GeV^2 so it leads to a reliable determination of hadron cross sections. One way to check that is to compute the differential parton luminosity, a convenient quantity introduced by EHLQ, defined as

$$\tau\frac{d\mathcal{L}_{ij}}{d\tau}=\frac{\tau}{1+\delta_{ij}}\int_\tau^1\frac{dx}{x}[f_i^{(a)}(x)f_j^{(b)}(\tau/x)+f_j^{(a)}(x)f_i^{(b)}(\tau/x)] , \quad (10)$$

where $f_i^{(a)}(x)$ is the distribution of partons of type i carrying the fraction momentum x of hadron a. It represents the number of parton-i–parton-j collisions per unit τ with subprocess energy square $\hat{s}=\tau s$, s being the total energy square of the hadron-a–hadron-b collision. Thus the differential cross section with respect to the scaling variable τ for the hadronic reaction $a+b\to c+X$ is given by

$$\frac{d\sigma}{d\tau}=\sum_{ij}\tau\frac{d\mathcal{L}_{ij}}{d\tau}\hat{\sigma}_{ij} . \quad (11)$$

Here $\hat{\sigma}_{ij}$ is the subprocess cross section and by dimensional analysis one has $\hat{\sigma}_{ij}=k/\hat{s}$ with coupling constant k. Therefore, the quantity $(\tau/\hat{s})d\mathcal{L}_{ij}/d\tau$ which has the dimension of a cross section can be used to make a fast estimate of the production rate for a given process. This notion of parton luminosity can be generalized to the case of hadronic reaction with *one or two* polarized beams by replacing in Eq. (10) *one or two* unpolarized distributions f by the corresponding parton helicity asymmetries Δf. Therefore, we define the two following useful quantities: namely, the single-polarized luminosity

$$\tau \frac{d\mathcal{L}_{\Delta ij}}{d\tau} = \tau \int_\tau^1 \frac{dx}{x} \Delta f_i^{(a)}(x) f_j^{(b)}(\tau/x) \tag{12}$$

and the double-polarized luminosity

$$\tau \frac{d\mathcal{L}_{\Delta i \Delta j}}{d\tau} = \frac{\tau}{1+\delta_{ij}} \int_\tau^1 \frac{dx}{x} [\Delta f_i^{(a)}(x) \Delta f_j^{(b)}(\tau/x) + \Delta f_j^{(a)}(x) \Delta f_i^{(b)}(\tau/x)] \, . \tag{13}$$

We show all these parton luminosities in Fig. 1 for gluon-gluon interactions and in Fig. 2 for u-quark-gluon interactions. By making ratios of the (single- or double-) polarized luminosity to the unpolarized luminosity, one can estimate how much a (single or double) subprocess helicity asymmetry will contribute to the corresponding helicity asymmetry in hadron-hadron collisions due to the dilution effect of the parton distributions. For example, in a reaction with one beam longitudinally polarized whose cross sections are $d\sigma^{(\pm)}/d\tau$ [see Eq. (11)], the single-helicity hadron asymmetry defined as

$$A_L = \frac{d\sigma^{(-)}/d\tau - d\sigma^{(+)}/d\tau}{d\sigma^{(-)}/d\tau + d\sigma^{(+)}/d\tau} \, , \tag{14}$$

can be roughly evaluated by means of these ratios. If $\hat{a}_L^{ij}$, the subprocess helicity asymmetry, is constant one has

$$A_L = \sum_{ij} \frac{\tau(d\mathcal{L}_{\Delta ij}/d\tau)}{\tau(d\mathcal{L}_{ij}/d\tau)} \hat{a}_L^{ij} \, . \tag{15}$$

Although gluon-gluon interactions are more efficient than u-quark–gluon interactions to build up the cross sections up to $(\hat{s})^{1/2} = 1$ TeV or so, they lead to smaller helicity hadron asymmetries. Clearly single-helicity hadron asymmetries are expected to be larger than double-helicity asymmetries. For *single*-particle (or jet) inclusive production A_L vanishes unless there is a parity-violating interaction at the level of the subprocess. We will come back to this important question later and we will also mention some interesting cases of parity-conserving helicity (or transverse spin) asymmetries, but before that we will calculate some double-helicity hadron asymmetries as an illustration of QCD spin tests in the multi-TeV energy range.

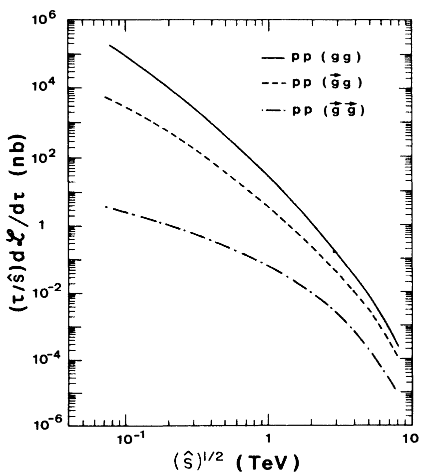

FIG. 1. The quantity $(\tau/\hat{s})d\mathcal{L}/d\tau$ for gg interactions in proton-proton collisions at $\sqrt{s} = 40$ TeV. Solid curve, unpolarized gluon distributions; dashed curve, one polarized gluon distribution; dashed-dotted curve, two polarized gluon distributions.

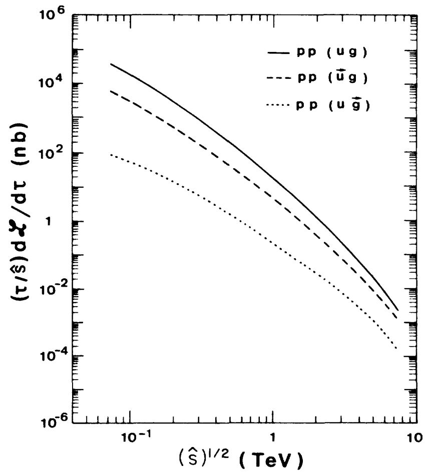

FIG. 2. The quantity $(\tau/\hat{s})d\mathcal{L}/d\tau$ for ug interactions in proton-proton collisions at $\sqrt{s} = 40$ TeV. Solid curve, unpolarized distributions; dashed curve, polarized u-quark distribution; dashed-dotted curve, polarized gluon distribution.

III. QCD SPIN TESTS AND HELICITY HADRON ASYMMETRIES

All predictions for hadron cross sections are provided by means of the QCD parton model which is the basis of all the estimates of backgrounds of the standard model and signals of new physics relevant to future hadron supercolliders. Since the very small-x region is so critical, it is legitimate to ask the following: how well do we understand the QCD parton model and how reliable are its predictions involving partons at very small x? The answer to these questions will be given by measuring various kinds of inclusive cross sections, for example, in jet production, which will test the small-x behavior of the parton distributions involved in the dominant subprocesses. Similarly the observation of the corresponding helicity hadron asymmetries will be a testing ground of the parton helicity asymmetries. Let us recall that in any inclusive reaction of the type $a+b \to c$ (or jet) $+X$ with both initial hadrons longitudinally polarized, the double-helicity hadron asymmetries A_{LL} defined as

$$A_{LL} = \frac{d\sigma_{a(+)b(+)} - d\sigma_{a(+)b(-)}}{d\sigma_{a(+)b(+)} + d\sigma_{a(+)b(-)}} \, , \tag{16}$$

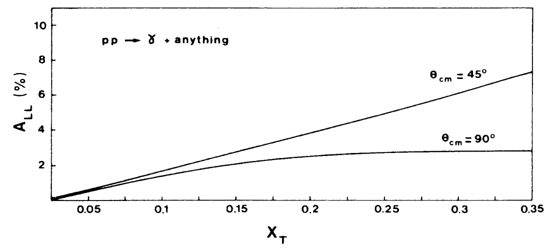

FIG. 3. Predictions for the double-helicity hadron asymmetry A_{LL} in $pp \to \gamma X$ vs x_T, at very high energies ($\sqrt{s} \geq 10$ TeV), for two different values of the production c.m. angle $\theta_{\text{c.m.}} = 45°$ and 90°

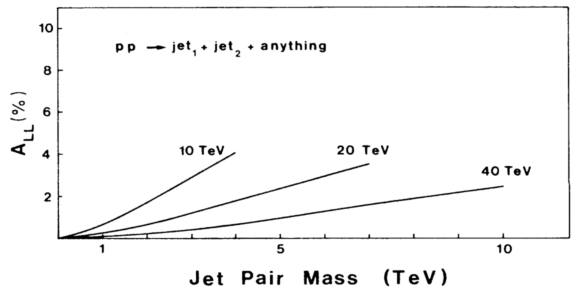

FIG. 5. Predictions for the double-helicity hadron asymmetry A_{LL} vs the jet-pair mass for two-jet events in pp collisions at three different energies.

is given by

$$A_{LL} d\sigma = \sum_{ij} \int dx_a dx_b [\Delta f_i^{(a)}(x_a, Q^2) \Delta f_j^{(b)}(x_b, Q^2) + (i \to j)] \hat{a}_{LL}^{ij} \hat{\sigma}_{ij} \ , \quad (17)$$

where $d\sigma$ denotes the unpolarized hadronic cross section and $\hat{a}_{LL}^{ij}$ the subprocess double-helicity asymmetry for initial partons i and j. The explicit expressions of these quantities for various subprocesses are given in Ref. 10.

First we consider the production of direct photon at large transverse momentum p_T. Since the photon originates in the hard-scattering subprocess and does not fragment, it gives a further insight in the underlying interaction. In the QCD parton model direct photons are produced via the annihilation subprocess $\bar{q}q \to \gamma g$ and the Compton subprocess $gq \to \gamma q$. The Compton subprocess has a positive $\hat{a}_{LL}$ whereas for the annihilation one has $\hat{a}_{LL} = -1$. We have calculated the unpolarized cross section and we found it in good agreement with the results of Ref. 11 at 90° and for different values of the production center-of-mass angle $\theta_{\text{c.m.}}$. As it is well known, away from 90° the Compton subprocess dominates largely over annihilation except at very large p_T. The results we obtained for the double-helicity asymmetry are shown in Fig. 3 for $\theta_{\text{c.m.}} = 45°$ and 90°. In both cases A_{LL} is positive which reflects the dominance of the Compton subprocess. We also see that A_{LL} scales with $x_T = 2p_T\sqrt{s}$, but since the cross section does not, the measurement of A_{LL} at fixed x_T will be easier for $\sqrt{s} = 10$ TeV than for $\sqrt{s} = 40$ TeV. At 45° A_{LL} rises with x_T, whereas it is flatter at 90°. This is due to the fact that at 90° the positive Compton asymmetry is reduced by the contribution of the negative annihilation asymmetry which becomes more and more important for increasing p_T.

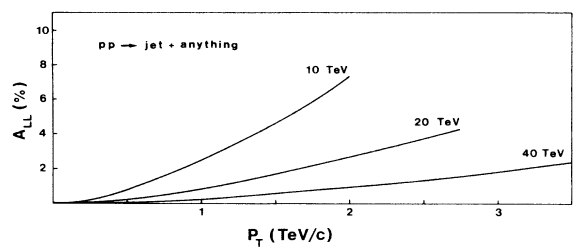

FIG. 4. Predictions for the double-helicity hadron asymmetry A_{LL} in $pp \to \text{jet} + X$ at $y = 0$ vs p_T for three different energies.

Second we turn to single-jet and two-jet production. The cross sections, which are several orders of magnitude larger than for direct photon production, are dominated by gluon-gluon and quark-gluon interactions[2] whose corresponding asymmetries $\hat{a}_{LL}$ are positive with a similar magnitude near 90°. A_{LL} for single-jet production at zero rapidity y vs p_T is shown in Fig. 4 at three different energies. It rises with p_T but the curves were not extended above p_T values for which the cross section goes below 10^{-40} cm^2/GeV which does not seem to be measurable. Indeed for p_T above 5 TeV, quark-quark interactions are expected to dominate and would lead to a larger asymmetry. In the physics of hadronic jets one can also consider final states consisting of two jets with equal and opposite p_T. The cross section is then expressed in terms of p_T and the rapidities y_1 and y_2 of the two jets. A larger cross section, which can be obtained after integration over the jet rapidities, is the invariant jet-pair mass distribution $d\sigma/dM$. These rapidities must lie in a limited range to prevent the opening angle of the dijet to be too small in particular for $y_1 = -y_2$. The double asymmetry A_{LL} versus the jet pair mass is shown in Fig. 5 at three different energies. The modest values of A_{LL} are again related to the dominance of gluon-gluon interactions except for very low jet-pair mass where there is no asymmetry anyway. Larger asymmetries due to quark-quark interactions are also expected for higher jet pair mass but they are hardly accessible experimentally.

IV. FURTHER POLARIZATION ISSUES AND CONCLUDING REMARKS

Although we are mainly concerned in this paper with the multi-TeV energy range, we would like to recall that unexpected large effects have been discovered in transverse single spin and spin-spin observables at much lower energies, in particular at large p_T for pp elastic scattering and hyperon inclusive production.[12] These data whose interpretation is not obvious have stimulated many interesting theoretical ideas.[10,13] However, the important question whether transverse spin effects will persist at ultrahigh energies remains open. For inclusive processes, if one observes at least two particles (or two jets) one can also consider a parity-conserving single-helicity hadron asymmetry. This asymmetry involves a correlation between the transverse momentum and angular distribution of the two objects produced in the final state. In the case of Drell-Yan dimuon production this correlation has been explicitly calculated and shown to be related to Sudakov corrections.[14]

In the multi-TeV energy range our simple prescriptions to generate a set of polarized distribution functions are fairly reliable and we believe they provide reasonable guesses for spin effects. Experimental observations which would depart, in sign and magnitude, from the above expectations for A_{LL} would cause a serious problem to present theoretical ideas on short-distance dynamics. In particular, all these double-helicity hadron asymmetries which are driven by the gluon helicity asymmetry, would be pretty much close to zero if gluons were not polarized. This gluon polarization can also be a useful tool to reveal the existence of a gluon-gluon fusion mechanism for the production of new strongly interacting particles. Moreover in electroweak interactions mediated by neutral and charged heavy bosons, the sign of the single asymmetry A_L [see Eq. (14)] allows us to decide unambiguously on the handedness of the currents. For single W^+ production $\hat{a}_L = +100\%$ and we anticipate $A_L = +13\%$ at 40 TeV and $+15\%$ at 20 TeV. Conversely the measurement of A_L giving the size of the effect, will provide a consistency check on the determination of the quark helicity asymmetries Δu and Δd and will give us an insight on the couplings of new gauge bosons if they exist. For physics beyond the standard model, direct photon production is claimed to be a clean signal to probe compositeness.[11] The observation of a nonzero A_L in this reaction can help to disentangle the structure of a parity-violation interaction due to the compositeness of quarks at a scale of a few TeV. Let us finally recall that the double asymmetries $\hat{a}^{ij}_{LL}$ for the production of supersymmetric partons are all -100%, so it leads to large and negative A_{LL} in high-p_T jets and missing energy.[15]

The usefulness of polarized protons beams for testing the standard model and in search for nonstandard physics including supersymmetry, technicolor, new gauge bosons, compositeness, and other more or less theoretical speculations will be extensively discussed in a forthcoming paper.[16]

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