Disentanglement of quantum wave functions: Answer to "Comment on 'Unified dynamics for microscopic and macroscopic systems' "

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It is shown that the assumption of a stochastic localization process for the quantum wave function is essentially different from the suppression of coherence over macroscopic distances arising from the interaction with the environment and allows for a conceptually complete derivation of the classical behavior of macroscopic bodies.

Before answering to the specific criticisms contained in the preceding Comment by Joos,[1] we want to draw attention to the fact that, besides deep differences, there are common points in the motivations at the basis of the paper by Joos and Zeh[2] (JZ) quoted by Joos and of our paper[3] (GRW).

The first common point is the conviction that the problems of interpretation of quantum mechanics cannot be solved by sticking to the strict application of the N-body Schrödinger equation to the local system, this expression meaning, typically, the system micro-object plus apparatus or, more generally, a macroscopic system. The second common point stays in the central role ascribed in both papers to the derivation from quantum mechanics of the classical behavior of macroscopic systems and to the related problem of removing coherence between macroscopically different positions of a macroscopic object as a whole.

The deep conceptual difference between the two treatments is that JZ escape the strict application of the N-body Schrödinger equation to the local system alone by considering its unavoidable coupling to the environment, while GRW stick to the local system accepting a (stochastic) modification of the dynamical behavior of the elementary constituents of a system. This modification induces a disentanglement of the wave functions when the macroscopic level is attained and this makes it consistent to consider the local system alone. The difference between the two attitudes is, as we shall see, very significant, mainly because the GRW approach allows for the interpretation of the wave function as an objective property of the local system while the approach of JZ does not.

We come now to the discussion of the specific criticisms of Joos, starting from the second one. This is based on the consideration that, since the correspondence between non-pure-case density operators and statistical ensembles is not one to one but one to many, the association to a given density operator of a specific statistical ensemble is arbitrary. In particular, this situation should not allow us to extract particle trajectories from the suppression of nondiagonal terms in the density matrix $\langle x \mid \rho \mid x' \rangle$ of the center-of-mass coordinate of a particle. We are grateful to Joos for giving us the opportunity of clarifying this point which is, or may be, not completely clear in the GRW paper. We will show here that no unjustified conclusion was drawn by GRW.

In fact, for a single microscopic or macroscopic particle, the considered process (which, in the latter case, is derived from the underlying microscopic description and is a consequence of the occurring localizations of the constituents) is actually a stochastic process for the wave function, so that the statistical operator $T[\rho]$ describes an ensemble which is the union of subensembles each being a pure case with a well-localized wave function. This should have been understood from the fact that we describe the localization process not simply by the map (operation) $\rho \rightarrow T[\rho]$ containing all possible localizations of the particle, but by the whole family (operation-valued measure) $T_I[\rho]$, where each $T_I[\rho]$ represents the statistical operator corresponding to localizations in the space interval I. Inspection of the expression for $T_I[\rho]$ shows that, when I shrinks to a point, $T_I[\rho]$ becomes a pure case when ρ is pure. Therefore the family $T_I[\rho]$ associates to each localization a definite wave function. In connection with the above remarks, we mention that Bell, in a recent work,[4] has paraphrased the essential content of the GRW paper in terms of wave functions. In the case of a macroscopic body, the treatment allows a consistent description of its evolution in terms of trajectories because the equations realize a particular issue and select it out of the many represented in the resulting mixture. Actually, GRW derive classical trajectories either by using the family $T_I[\rho]$ (Sec. V) or dealing directly with wave functions and pure state statistical operators (Sec. VIII). The derivation of classical trajectories is therefore quite legitimate.

One thing more, however, has to be discussed in connection with the second criticism. In Sec. VI of the GRW paper, as mentioned above, we give to the localization process a universal character by assuming that, for a system of N constituents, it occurs individually for each constituent. Then we show (this is a key point) that, for a system having a well-localized internal structure, a localization of a single constituent is essentially equivalent to a localization of the center of mass, i.e., $T^i[\rho]=T^Q[\rho]$, where T^i describes the localization of the single constituent i and T^Q the localization of the center of mass. In light of our previous discussion, equality of the two families T_I^i and T_J^Q is required. Referring for brevity to a system of two particles of equal masses, let us denote by Q and q the c.m. and relative coordinates, respectively. Then we assume $\rho=\rho_Q\rho_q =(\,|\phi\rangle\langle\phi|\,)(\,|\chi\rangle\langle\chi|\,)$, where the c.m. wave function $\phi(Q)$ is arbitrary and the internal wave function $\chi(q)$ can be taken to be $\delta(q-a)$ since the internal structure is well localized. One easily finds $T_I^i[\rho_Q\rho_q]=T_J^Q[\rho_Q]\rho_q$, where J is obtained by displacing I by $a/2$. Therefore the localization of the center of mass can be described in terms of wave functions when it is derived from the localization of single components just as it can be when it is assumed for a single particle. In the GRW paper, the discussion contained in the three paragraphs following Eq. (6.15), though less formal, is equivalent to this result.

The first criticism of Joos can be summarized as follows. If one takes into account the unavoidable interactions of a macroscopic system with its natural environment and considers the pure Schrödinger evolution of the global system (local system plus environment), then, when the degrees of freedom of the environment are eliminated, one remains, for the local system alone, with a reduced dynamics which, as shown by JZ, suppresses long-distance coherence in the sense that such a coherence cannot be detected as long as the environment is not inspected. Then, there would be no reason to postulate a modification of the basic dynamics as in the GRW approach.

To make clear that this criticism is not relevant, we shall point out the limitations which affect the JZ approach, in our opinion. If one keeps the pure Schrödinger dynamics, linear superpositions of macroscopically different states can always occur. In the situation considered by JZ, the local system plus its environment happen to be in states of such a kind. This is the usual puzzling problem of quantum mechanics. Resorting to the celebrated example of Schrödinger's cat, in the approach of JZ one is faced with states of the type $\psi=\phi_{\text{alive}}\Phi_1+\phi_{\text{dead}}\Phi_2$, Φ_1 and Φ_2 being states of the environment. Everybody agrees on the fact that the two states Φ_1 and Φ_2 are practically orthogonal, either because a single particle in the environment is in orthogonal states in Φ_1 and Φ_2 or because many particles are in slightly different states. Then the coherence between the two terms cannot be detected in measurements which do not also involve the environment. This fact, however, does not eliminate our uneasiness (as well as that of many others) with the state ψ. Joos seems to share, with respect to this problem, the attitude of contenting himself with the following proposition: intriguing linear superpositions occur, but for the limited class of experiments one is able to perform, everything goes as if the corresponding, nonintriguing statistical mixture would be there. The limitations of this point of view in the framework of JZ are similar to its limitations in other approaches to quantum measurement and are clearly stated also by JZ. First, the local description is "assumed"; i.e., the neglecting of the environment cannot be derived in principle—"perhaps (it is argued by JZ) it can be justified by a fundamental (underivable) assumption about the local nature of the observer." Second, the probabilistic interpretation leading to the collapse of the wave function is "presupposed"—"no unitary treatment of the time dependence can explain why only one of these dynamically independent components (the single terms in ψ, in our example) is experienced." The latter difficulty is strictly related to the problem, appropriately discussed by Shimony,[5] of justifying the agreement among different observers about the results of one experiment singled out from the statistical ensemble. The conclusion of JZ is that "the difficulty in giving a *complete* derivation of classical concepts may as well signal the need for entirely novel concepts." We further note that the loss of coherence is obtained in the JZ approach by considering a kind of process which enlarges to the environment (or to the rest of the world) the entanglement of quantum wave functions and then neglecting the environment. We think that the processes considered by JZ actually take place. But we think also that enlarging quantum entanglement is not a solution and that, on the contrary, the solution (at least the type of solution we like) must come from reducing it. In our opinion, the proposal contained in the GRW paper provides a way out of the problems discussed above. The GRW process produces a strong disentanglement (also with respect to the entangling processes considered by JZ) in such a way that the local system has a definite wave function at all times and this wave function has no strange feature except for a very short transient situation, at most. The price to be paid is the introduction of a stochasticity at the level of the basic evolution of the wave function. The merit of the GRW approach is, we think, to show that this can be done in the framework of a consistent theoretical scheme in such a way that the considered process is practically ineffective for micro-objects and does not disturb appreciably the motion of macro-objects, the only effect of it being that of forbidding the unacceptable superpositions.

The differences between the two treatments can be summarized by saying that the process considered by JZ gives rise, for the local physical system, to improper mixtures. This is the reason why any interpretation of ρ in terms of wave functions is arbitrary and in fact it is not attempted there. On the contrary, the localization process considered by GRW gives rise to proper mixtures, whether or not the coupling to the environment is considered, and therefore it allows one to assign a definite (acceptable) wave function to the local system.

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[4]J. S. Bell, in *Schrödinger-Centenary Celebration of a Polymath,* edited by C. W. Kilmister (Cambridge University Press, Cambridge, England, 1987), p. 41.
[5]A. Shimony, Am. J. Phys. **31**, 755 (1963).