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Comments

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Comment on "Unified dynamics for microscopic and macroscopic systems"

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A nonunitary dynamics for a macroscopic object arises as a natural consequence of a universally valid Schrödinger equation and thus need not be postulated *ad hoc*. The density-matrix formalism does not, however, allow one to deduce classical states (particle trajectories) without invoking additional principles.

In a recent publication[1] Ghirardi, Rimini, and Weber proposed a "modification of standard quantum mechanics, which leaves things unchanged for microscopic objects, while, for macroscopic objects, transforms quantum mechanics into a stochastic mechanics in phase space exhibiting the classical features." Attempts of this kind have a long history and are founded on various motivations. For example, measurementlike processes necessarily produce superpositions of macroscopically different states,[2] which are hard to interpret. The authors propose to alter the quantum-mechanical law of evolution so that localized states for macro-objects naturally emerge. We would like to comment on this paper as follows.

(1) Localized states obviously would be prepared by position measurements. The authors therefore look for a dynamics which describes macro-objects *as if* their position were continuously measured. They arrive at (or rather, *postulate*) a modified evolution equation for the density matrix of a macroscopic mass point. The Schrödinger equation, e.g., for, the center-of-mass coordinate of a free particle,

$$i\partial_t \,|\,\psi\rangle = \frac{\hat{p}^2}{2m}\,|\,\psi\rangle\ , \tag{1}$$

is then replaced by a density-matrix equation, which in its simplest form reads

$$i\frac{\partial\rho(x,x',t)}{\partial t} = \frac{1}{2m}\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2}\right]\rho - i\Lambda(x-x')^2\rho\ . \tag{2}$$

The first term corresponds to Eq. (1) (unitary evolution), while the second term leads to destruction of coherence between different positions (nondiagonal elements in the position representation of the density matrix). This equation, however, at least in this simplified form [Eq. (3.5) of Ref. 1] *automatically results* from a global Schrödinger equation if the interaction of a macroscopic object with its natural environment is properly taken into account, as was shown, for example, in Ref. 3 [see Eq. (3.75)]. As a technical remark, there seems to be no reason to invoke sophisticated formalisms as "effect-valued measures" to successfully describe incomplete measurements in quantum mechanics. This, however, is only a minor point. We want to stress here that a *realistic* application of quantum theory already yields an irreversible nonunitary dynamics for macroscopic objects. Hence there is no freedom for a "possible numerical choice of the parameters" in the suggested equation, since these are fixed by the properties of matter (and interactions) we find in nature. Admittedly, Ghirardi, Rimini, and Weber intend to *postulate* a new *fundamental* dynamics for the density matrix. However, as long as their main objective—suppression of coherence over macroscopic distances—can be achieved by applying standard quantum mechanics to realistic situations, there seems to be little motivation for this particular attempt.

(2) From a process acting like a position measurement one intuitively expects the production of an ensemble of small wave packets when starting with a broad one, finally leading to something like "particle trajectories." Indeed, Eq. (2) contains a characteristic length scale

$$\lambda = (\Lambda m)^{1/4} \tag{3}$$

as well as a time scale

$$\tau = \left[\frac{m}{\Lambda}\right]^{1/2} \tag{4}$$

and it is tempting to assume that after a "relaxation time" τ the initial wave packet is split into an ensemble of wave packets of width λ. For $t \gg \tau$ one would expect a stochastic movement of such "particles."[1,4,5] However, by using a density matrix there is no unambiguous

way to achieve such a goal since the decomposition of a (nonpure) density matrix into an ensemble of states is not unique. The reason behind that is of course that a density matrix does *not* by itself describe a probability distribution of states (as would a classical phase-space distribution) but is only a calculational tool to evaluate probabilities for the occurrence of certain states in a certain measurement. Therefore suppression of nondiagonal terms in the density matrix $\rho(x,x')$ of the center-of-mass coordinate of a mass point is not sufficient to extract particle trajectories. Quite generally, any division of a nonpure density matrix into a certain ensemble of *quantum* states is arbitrary and there seems to be no way, neither theoretically nor *experimentally,* to distinguish between different representations of one and the same density matrix. As an illustration consider a Gaussian density matrix (we use the notation of Ref. 3)

$$\rho(x,x')=2\left[\frac{C}{\pi}\right]^{1/2}\exp\{-[A(x-x')^2+iB(x-x')(x+x')+C(x+x')^2]\}\ . \tag{5}$$

If A,B,C are assumed time dependent, such a density matrix is a solution of Eq. (2) with A,B,C obeying ordinary differential equations [see Eq. (3.84) of Ref. 3]. This density matrix can be rewritten as an ensemble

$$\rho=\int dp\ P(p)\phi_p^*(x')\phi_p(x) \tag{6}$$

of wave packets

$$\phi_p(x)=\left[\frac{4C}{\pi}\right]^{1/4}\exp\{-[(2C+iB)x^2+ipx]\} \tag{7}$$

with the probability distribution

$$P(p)=[4\pi(A-C)]^{-1/2}\exp\left[-\frac{p^2}{4(A-C)}\right] \tag{8}$$

or as an ensemble

$$\rho=\sum_n p_n\psi_n^*(x')\psi_n(x) \tag{9}$$

of diagonalizing orthogonal states (oscillator wave functions)

$$\psi_n(x)=\frac{(AC)^{1/4}}{2^{n-1}n!\pi^{1/2}}H_n(2(AC)^{1/4}x)\times\exp\{-x^2[2(AC)^{1/2}+iB]\} \tag{10}$$

with

$$p_n=\frac{2C^{1/2}}{A^{1/2}+C^{1/2}}\left[\frac{A^{1/2}-C^{1/2}}{A^{1/2}+C^{1/2}}\right]^n \tag{11}$$

or in many other ways. The coherence length of the full density matrix is $l=(8A)^{-1/2}$, whereas the fictitious ensemble members have various widths, which need not be small even if l is small. Despite this mathematical fact, one may still hope to reconstruct some ensemble of "classical states," but this seems only possible by invoking additional principles which go beyond the density-matrix formalism. For example, it has been suggested[6] that in Everett-type interpretations of quantum mechanics the eigenstates of the density matrices of certain subsystems (connected with observers) define the "branching" of the wave function (corresponding to the usual collapse). Then the difference between the eigenstates in the continuous case, which do not correspond to classical states [the states (10) are generally broad wave packets], and those for discrete variables (e.g., information storage in the brain) is of importance.[3,7]

In view of the fact that such attempts so far seem unconvincing, it may well turn out necessary to find a new description for a physical "state" encompassing classical (local) as well as quantum (nonlocal) state concepts.[8]

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