Absence of spontaneous parity violations in three-dimensional QED induced by infrared effects

H. T. Cho*
Department of Physics, Brown University, Providence, Rhode Island 02912
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The possibility of dynamical parity violations in three-dimensional QED (QED_3) by infrared effects is examined using a previously defined infrared approximation. $\langle \bar{\psi}\psi \rangle$ and the photon propagator $\tilde{D}_{\mu\nu}(p)$ are calculated in this approximation for the mass of the fermion, $m \neq 0$. Then, as $m \to 0$, $\langle \bar{\psi}\psi \rangle$ is found to be zero and $\tilde{D}_{\mu\nu}(p)$ to have a pole corresponding to zero mass. These results imply that there are no dynamical violations of parity induced by infrared effects in QED_3.

I. INTRODUCTION

In $(2+1)$-dimensional gauge theories, a parity-violating but gauge-invariant mass term can be given to the vector boson.[1] In fact, the fermion mass term is also parity violating. The theories with these mass terms are superrenormalizable and infrared (IR) finite.

It has been shown that even when the vector-boson-mass term is absent from the Lagrangian, it can be induced by 1-loop radiative corrections.[2] In addition, higher-loop corrections will not contribute.[3] However, the form of the induced mass term depends on the choice of the regularization one uses to remove the ultraviolet (UV) divergences in the Feynman-diagram calculations. This term may or may not vanish as the mass of the fermion goes to zero depending on the regularization.[4] Therefore, there is an ambiguity as to whether there are dynamical generations of mass by radiative corrections,[5] which in turn violate parity spontaneously.

In the case of QED, this problem of spontaneous parity violation has been studied nonperturbatively in the large-N expansion[6] (where N is the number of fermion flavors), using the Schwinger-Dyson equations and the effective-potential approach. For odd N, dynamical generation of mass does not occur. For even N, this may occur, but it happens in such a way that parity is not violated. This case is actually equivalent to the spontaneous breaking of chiral symmetry when one uses the 4×4 Dirac matrices instead of the 2×2 ones.[7]

In the following we are concerned with another aspect of three-dimensional (QED_3): Without mass terms this theory is IR divergent perturbatively.[8] Although these divergences may just be artifacts of the perturbation series,[9] they may still lead to dynamical generation of fermion mass and spontaneous parity violation.[10] Here we shall investigate this possibility by explicitly extracting the IR structure of QED_3 for $N=1$. We shall use 2×2 Dirac matrices, and thus dynamical generation of mass will imply spontaneous violation of parity.

Two quantities will be calculated: the order parameter $\langle \bar{\psi}(x)\psi(x) \rangle$ and the photon propagator $\tilde{D}_{\mu\nu}(p)$. If the vacuum is invariant under parity transformation, i.e., $\mathcal{P}\,|\,0\rangle = |\,0\rangle$, then the order parameter is

$$\begin{aligned}\langle \bar{\psi}(x)\psi(x) \rangle &= \langle \mathcal{P}^{-1}\bar{\psi}(x)\psi(x)\mathcal{P} \rangle \\ &= \langle \mathcal{P}^{-1}\psi^{\dagger}(x)\mathcal{P}\gamma_3\mathcal{P}^{-1}\psi(x)\mathcal{P} \rangle \\ &= \langle \psi^{\dagger}(x')\gamma_1\gamma_3\gamma_1\psi(x') \rangle \\ &= -\langle \bar{\psi}(x')\psi(x') \rangle \\ &= -\langle \bar{\psi}(x)\psi(x) \rangle \;, \end{aligned} \tag{1.1}$$

where the parity transformation is defined as

$$\mathcal{P}^{-1}\psi(x)\mathcal{P} = \gamma_1\psi(x') \;, \tag{1.2}$$

with $x=(x_1,x_2,x_3)$ and $x'=(-x_1,x_2,x_3)$. Therefore, a nonzero value of $\langle \bar{\psi}(x)\psi(x) \rangle$ will indicate parity violation by the vacuum. Another way to investigate this invariance is by examining directly the structure of the photon propagator $\tilde{D}_{\mu\nu}(p)$, to see whether a pole corresponding to a nonzero mass occurs.

To extract the most IR part of the two quantities mentioned above, we adopt the nonperturbative IR method developed by Fried,[11,12] in which virtual photons are separated into their low-momentum (soft) and high-momentum (hard) parts with respect to some intrinsic mass scale in the theory. The number of hard photons can then be taken as the appropriate expansion parameter in a strong-coupling limit. Of course, this expansion is only good for phenomena dominated by IR effects. Nevertheless, when this approach was first applied to the calculation of the renormalization-group β functions of ϕ^4 theory[11] and QED[12] in four dimensions, it was found to be of relevance to the strong-coupling regime of these theories. This IR approach has also been found useful in the investigation of the spontaneous chiral-symmetry breaking in two-dimensional gauge theories,[13,14] which is supposed to be an IR and strong-coupling phenomenon; there the coupling constant has the dimension of mass, and hence the chiral limit is also the strong-coupling limit.

For QED_3, the coupling is also dimensional, and the limit for the fermion mass $m\to 0$ is then a strong-coupling limit. In addition, we are interested in the IR properties of the theory, and thus we choose to follow the IR approach mentioned above. Here, we shall consider only the first term of the IR expansion, that is, we shall ignore all the hard photons. In principle, the correction terms can be calculated by including some hard-photon contributions, even though these calculations may be quite tedious. Since we are concerned only with the leading IR behavior, we shall assume that the first term of this IR expansion is already enough for our purpose. To further simplify our calculation we also use the quenched (1-fermion-loop) approximation. Then $\langle \bar{\psi}(x)\psi(x)\rangle$ and $\tilde{D}_{\mu\nu}(p)$ are calculated in these approximations for the fermion mass $m \neq 0$. Finally, the limit $m \to 0$ is taken to see if parity is violated spontaneously in the IR region of the theory.

In the next section we describe briefly the IR approximation we adopt. In Sec. III, $\langle \bar{\psi}(x)\psi(x)\rangle$ is calculated in the quenched IR approximation, and its behavior as $m \to 0$ examined. In Sec. IV the vacuum polarization $\Pi_{\mu\nu}(p)$ of the photon propagator is calculated in the same approximation. Using that, the photon propagator is constructed and analyzed. Section V contains conclusions and discussions. An appendix is used to derive (2.8) in Sec. II.

II. THE INFRARED APPROXIMATION SCHEME

In this section we describe briefly the IR approximation scheme which is used in the subsequent calculations (for details see Refs. 11–13).

The Lagrangian (in Euclidean space) we are dealing with is

$$\mathcal{L} = \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \bar{\psi}(\partial\!\!\!/ + ie A\!\!\!/ + m)\psi + \frac{1}{2\lambda}(\partial_\mu A_\mu)^2 \,, \quad (2.1)$$

where the last term is the gauge-fixing term, and $\gamma_\mu \equiv \sigma_\mu$ (Pauli matrices). We leave λ arbitrary so that we can keep track of the gauge-choice dependence of the quantities we calculate.

The generating functional can be written as

$$Z[j,\bar{\eta},\eta] = N\int [d\bar{\psi}][d\psi][dA]\exp\left[-\int \mathcal{L} + \int (jA + \bar{\psi}\eta + \bar{\eta}\psi)\right]$$

$$= N\int [dA]\exp\left[-\tfrac{1}{4}\int F_{\mu\nu}F_{\mu\nu} - \frac{1}{2\lambda}\int(\partial_\mu A_\mu)^2 + \int j_\mu A_\mu + \int \bar{\eta}(\partial\!\!\!/ + ieA\!\!\!/ + m)^{-1}\eta + L\right] , \quad (2.2)$$

where

$$L = \mathrm{Tr}\ln[1 + ieA\!\!\!/(\partial\!\!\!/ + m)^{-1}]$$

and $Z[j,\bar{\eta},\eta]$ is normalized in such a way that $Z[0]=1$.

The IR approximation will be implemented in L. Using the proper-time method,[15] one can write

$$L = \mathrm{Tr}\ln[1 + ieA\!\!\!/(\partial\!\!\!/ + m)^{-1}]$$

$$= (-1)\mathrm{Tr}\int_0^e de'(iA\!\!\!/)(\partial\!\!\!/ + ie'A\!\!\!/)\int_0^\infty d\tau\, e^{-\tau m^2} e^{\tau(\partial\!\!\!/ + ie'A\!\!\!/)^2} + (-1)\mathrm{Tr}\int_0^e de'(iA\!\!\!/)(-m)\int_0^\infty d\tau\, e^{-\tau m^2} e^{\tau(\partial\!\!\!/ + ie'A\!\!\!/)^2}$$

$$= \int d^3x \left[-\tfrac{1}{2}\mathrm{tr}\int_0^\infty \frac{d\tau}{\tau} e^{-\tau m^2}\langle x \,|\, e^{\tau(\partial\!\!\!/ + ie'A\!\!\!/)^2} - e^{\tau\partial\!\!\!/\partial\!\!\!/} \,|\, x\rangle + (m)\mathrm{tr}\int_0^\infty d\tau\, e^{-\tau m^2}\int_0^e de'[iA\!\!\!/(x)]\langle x \,|\, e^{\tau(\partial\!\!\!/ + ie'A\!\!\!/)^2} \,|\, x\rangle\right] , \quad (2.3)$$

where in the last lines the trace is only over γ matrices. Note that the second term is a trace over odd number of γ matrices. It is nonzero in the 2×2 representation of γ matrices we are using. This term will nevertheless vanish for even-dimensional cases.

Now we split $A_\mu(x)$ into its soft and hard parts, e.g.,

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3} e^{ipx}\tilde{A}_\mu(p)[e^{-p^2/\mu^2} + (1 - e^{-p^2/\mu^2})]$$

$$\equiv A_\mu^S(x) + A_\mu^H(x) \,, \quad (2.4)$$

where $\mu = 1/\sqrt{\tau}$. Therefore, $A_\mu^S(x)$ is highly damped for momenta

$$p \geq \frac{1}{\sqrt{\tau}} \Longrightarrow \tau \geq \frac{1}{p^2} \,. \quad (2.5)$$

In (2.3), because of the presence of $e^{-\tau m^2}$, the magnitude of τ is restricted to be of the order of

$$\tau \leq \frac{1}{m^2} \,. \quad (2.6)$$

Combining (2.5) and (2.6) we see that $A_\mu^S(x)$ is highly damped for momenta

$$p^2 \geq m^2 \,. \quad (2.7)$$

Here, m is the only dimensional parameter in $\mathcal{L}$ and it is therefore used to define the IR approximation.

Retaining only $A_\mu^S(x)$ in $A_\mu(x)$ as the first approximation, and performing a related "multipole expansion" (see the Appendix), we find

$$\langle x\,|\,e^{\tau(\partial+ieA)^2}\,|\,x\rangle\to\langle x\,|\,e^{\tau(\partial+ieA)^2}\,|\,x\rangle_{\rm IR}=\frac{1}{8\pi^{3/2}\tau^{3/2}}\int d^3a\;\delta(\mathbf{a}-e\tau{}^*\mathbf{F}^S(x))\left[\frac{a}{\sinh a}\right]^{1/2}(a\coth a-\boldsymbol{\gamma}\cdot\mathbf{a})\;, \tag{2.8}$$

where

$${}^*F_\mu(x)=\tfrac{1}{2}\epsilon_{\mu\nu\alpha}F_{\nu\alpha}(x),\quad {}^*F^S_\mu(x)=\tfrac{1}{2}\epsilon_{\mu\nu\alpha}[\partial_\nu A^S_\alpha(x)-\partial_\alpha A^S_\nu(x)]\;.$$

Using (2.8), L can be written under this approximation as

$$\begin{aligned}L_{\rm IR}&=\int d^3x\left[-\tfrac{1}{2}{\rm tr}\int_0^\infty\frac{d\tau}{\tau}e^{-\tau m^2}\langle x\,|\,e^{\tau(\partial+ie'A)^2}-e^{\tau\partial\partial}\,|\,x\rangle_S+(m){\rm tr}\int_0^\infty d\tau\,e^{-\tau m^2}\int_0^e de'[iA^S(x)]\langle x\,|\,e^{\tau(\partial+ie'A)^2}\,|\,x\rangle_S\right]\\&=\left[-\frac{1}{8\pi^{3/2}}\right]\int d^3x\int_0^\infty\frac{d\tau}{\tau^{5/2}}e^{-\tau m^2}\left\{\left[\int d^3a\;\delta(\mathbf{a}-e\tau{}^*\mathbf{F}^S(x))\left[\frac{a}{\sinh a}\right]^{1/2}a\coth a\right]-1\right\}\\&\quad+\left[\frac{-im}{4\pi^{3/2}}\right]\int d^3x\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{-\tau m^2}\int_0^e de'\int d^3a\;\delta(\mathbf{a}-e\tau{}^*\mathbf{F}^S(x))\left[\frac{a}{\sinh a}\right]^{1/2}\mathbf{a}\cdot\mathbf{A}^S(x)\;.\end{aligned}\tag{2.9}$$

This expression is the main quantity to be used in the calculations under this IR approximation in the subsequent sections.

III. ORDER PARAMETER $\langle\bar\psi(x)\psi(x)\rangle$

In this section we evaluate $\langle\bar\psi(x)\psi(x)\rangle$ in the IR approximation set up in Sec. II. Then we take the fermion mass $m\to 0$, and examine whether $\langle\bar\psi(x)\psi(x)\rangle$ vanishes.

By the conventional action principle, one can write

$$\langle\bar\psi(x)\psi(x)\rangle=\frac{-1}{\int d^3x}\frac{\partial}{\partial m}\ln Z\bigg|_{j=\bar\eta=\eta=0}\;, \tag{3.1}$$

where Z is the generating functional in (2.2).

In the quenched approximation, we retain only the first nontrivial term in the expansion of e^L, i.e.,

$$\langle\bar\psi(x)\psi(x)\rangle\to\frac{-1}{\int d^3x}\frac{\partial}{\partial m}N\int[dA]L\,\exp\left[-\tfrac{1}{4}\int F_{\mu\nu}F_{\mu\nu}-\frac{1}{2\lambda}\int(\partial_\mu A_\mu)^2\right]\;, \tag{3.2}$$

where

$$N^{-1}=\int[dA]\exp\left[-\tfrac{1}{4}\int F_{\mu\nu}F_{\mu\nu}-\frac{1}{2\lambda}\int(\partial_\mu A_\mu)^2\right]\;.$$

Here, we implement the IR approximation in L,

$$\langle\bar\psi(x)\psi(x)\rangle\to\langle\bar\psi(x)\psi(x)\rangle_{\rm IR}=\frac{(-1)}{\int d^3x}\frac{\partial}{\partial m}N\int[dA]L_{\rm IR}\exp\left[-\tfrac{1}{4}\int F_{\mu\nu}F_{\mu\nu}-\frac{1}{2\lambda}\int(\partial_\mu A_\mu)^2\right] \tag{3.3}$$

in this quenched IR approximation, with $L_{\rm IR}$ given by (2.9).

The Gaussian functional integration in (3.3) can be carried out giving

$$\langle\bar\psi(x)\psi(x)\rangle_{\rm IR}=\left[\frac{-m}{4\pi^{3/2}}\right]\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{-\tau m^2}\left\{\frac{1}{2\pi^{1/2}}\left[\frac{1}{e^3\tau^3K^{3/2}}\right]\left[\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2}a^3\coth a\,\exp\left[-\frac{a^2}{4(e\tau)^2K}\right]\right]-1\right\}\;, \tag{3.4}$$

where

$$K=\frac{1}{48\sqrt{2}\pi^{3/2}\tau^{3/2}}\;.$$

Note that the expression (3.4) is independent of λ, the gauge-choice parameter.

Since $\langle\bar\psi(x)\psi(x)\rangle$ is proportional to m, it would not vanish as $m\to 0$ only when the integral is diverging ($\sim 1/m$ as $m\to 0$). Therefore, we concentrate on the integral to see whether it diverges as $m\to 0$.

Note, firstly that (3.4) contains no UV divergence (near $\tau \to 0$), but it will be convenient for computation to insert an UV cutoff Λ, i.e.,

$$\int_0^\infty d\tau \cdots \to \int_{1/\Lambda^2}^\infty d\tau \cdots \ . \tag{3.5}$$

Any IR divergences will not be affected by this replacement.

Secondly, since the function

$$\left[\frac{a}{\sinh a}\right]^{1/2} a \coth a \to \begin{cases} 1, & a \to 0, \\ \sqrt{2} a^{3/2} e^{-a/2}, & a \to \infty \end{cases} \tag{3.6}$$

is everywhere well behaved, the approximation

$$\int_0^\infty da \left[\frac{a}{\sinh a}\right]^{1/2} a \coth a \cdots \to \int_0^\pi da \cdots + \int_\pi^\infty da \sqrt{2} a^{3/2} e^{-a/2} \cdots \tag{3.7}$$

will not affect the singularities of the integral.

Under these replacements, one can calculate explicitly the integral in (3.4) as $m \to 0$:

$$\begin{aligned} &\int_0^\infty \frac{d\tau}{\tau^{3/2}} \left\{ \left[\frac{1}{2\pi^{1/2}} \left[\frac{1}{e^3 \tau^3 K^{3/2}} \right] \int_0^\infty da \left[\frac{a}{\sinh a} \right]^{1/2} a^3 \coth a \exp\left[-\frac{a^2}{4(e\tau)^2 K} \right] \right] - 1 \right\} \\ &\to \int_{1/\Lambda^2}^\infty \frac{d\tau}{\tau^{3/2}} \left\{ \frac{1}{2\pi^{1/2}} \left[\frac{1}{e^3\tau^3 K^{3/2}} \right] \left[\int_0^\pi da\, a^2 \exp\left[-\frac{a^2}{4(e\tau)^2 K} \right] + \int_\pi^\infty da \sqrt{2} a^{7/2} e^{-a/2} \exp\left[-\frac{a^2}{4(e\tau)^2 K} \right] \right] - 1 \right\} \\ &= \frac{e^2}{2\sqrt{2}\pi^{1/2}} \Gamma\left[-\frac{1}{2}, \frac{\pi}{2} \right] - \frac{e^2}{4\sqrt{2}\pi^{7/2}} \ , \end{aligned} \tag{3.8}$$

where

$$\Gamma(\nu, x) = \int_x^\infty dy\, y^{\nu-1} e^{-y}$$

is the incomplete Γ function. Equation (3.8) is finite and independent of Λ, the UV cutoff.

Using this result in (3.4) we have

$$\lim_{m \to 0} \langle \bar{\psi}(x) \psi(x) \rangle_{\text{IR}} = \lim_{m \to 0} \left[\frac{-m}{4\pi^{3/2}} \right] \times \text{finite contribution} = 0 \ . \tag{3.9}$$

Therefore, at least in this 1-fermion-loop level, there are no signs of spontaneous violation of parity by the vacuum due to IR effects.

IV. THE PHOTON PROPAGATOR

A more direct way to see whether the photon develops a mass term dynamically is to examine the mass pole of the photon propagator. In this section the photon propagator will be calculated explicitly in the quenched IR approximation. Then the $m \to 0$ limit of this propagator will be examined to see whether there is a pole corresponding to nonzero mass. This will give further indication of whether parity is violated dynamically in this model.

Define

$$I_{\mu\nu}(x-y) \equiv \frac{\delta}{\delta j_\mu(x)} \frac{\delta}{\delta j_\nu(y)} N \int [dA](1 + L_{\text{IR}}) \exp\left[-\tfrac{1}{4} \int F_{\mu\nu} F_{\mu\nu} - \frac{1}{2\lambda} \int (\partial_\mu A_\mu)^2 + \int j_\mu A_\mu \right] \Bigg|_{j=0} \tag{4.1}$$

and $\tilde{I}_{\mu\nu}(p)$ its Fourier transform. Again, L_{IR} is the expression (2.9) obtained in Sec. II. $\tilde{I}_{\mu\nu}(p)$ is effectively the photon propagator in the quenched IR approximation. If we take just the connected diagrams of $\tilde{I}_{\mu\nu}(p)$, they turn out to be 1-particle irreducible (1PI) also.

In (4.1), the integration over $A_\mu(x)$ is just Gaussian and can be carried through giving

$$\tilde{I}_{\mu\nu}(p)_{\text{1PI}} = \tilde{\Delta}_{\mu\nu}(p) + P_{\mu\nu} M(p) + \epsilon_{\mu\nu\alpha} \frac{p_\alpha}{p} Q(p) \ , \tag{4.2}$$

where

$$\tilde{\Delta}_{\mu\nu}(p) \doteq \frac{1}{p^2}\left[\delta_{\mu\nu}-(1-\lambda)\frac{p_\mu p_\nu}{p^2}\right], \quad P_{\mu\nu}=\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2} ,$$

$$M(p)=-\frac{1}{32\pi^2 e^3\alpha^{5/2}p^2}\int_0^\infty \frac{d\tau}{\tau^{7/4}}e^{-\tau m^2}e^{-2\tau p^2}\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2} a^3\coth a\exp\left[-\frac{a^2}{4\alpha e^2\tau^{1/2}}\right]\left[-1+\frac{a^2}{6\alpha e^2\tau^{1/2}}\right], \tag{4.3}$$

$$Q(p)=-\frac{m}{24\pi^2\alpha^{5/2}p^3}\int_0^\infty \frac{d\tau}{\tau^{7/4}}e^{-\tau m^2}e^{-2\tau p^2}\int_0^e \frac{de'}{e'^4}\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2} a^4\exp\left[-\frac{a^2}{4\alpha e'^2\tau^{1/2}}\right], \tag{4.4}$$

$$\alpha=\frac{1}{48\sqrt{2}\pi^{3/2}} ,$$

and $\tilde{I}_{\mu\nu}(p)_{\rm 1PI}$ is the 1-particle irreducible part of $\tilde{I}_{\mu\nu}(p)$. Note that $\epsilon_{\mu\nu\alpha}(p_\alpha/p)Q(p)$ is the induced mass term for the photon mentioned in Sec. I.

From $\tilde{I}_{\mu\nu}(p)_{\rm 1PI}$ one can obtain the vacuum polarization $\Pi_{\mu\nu}(p)$ of the photon in this quenched IR approximation:

$$\Pi_{\mu\nu}(p)=\tilde{\Delta}^{-1}{}_{\mu\rho}(p)[\tilde{I}_{\rho\sigma}(p)_{\rm 1PI}-\tilde{\Delta}_{\rho\sigma}(p)]\tilde{\Delta}^{-1}{}_{\sigma\nu}(p)=p^4P_{\mu\nu}M(p)+\epsilon_{\mu\nu\alpha}\frac{p_\alpha}{p}p^4Q(p) . \tag{4.5}$$

Note that $\Pi_{\mu\nu}(p)$ here is gauge invariant,

$$p_\mu\Pi_{\mu\nu}(p)=0 , \tag{4.6}$$

and it is independent of λ, the gauge parameter.

Using $\Pi_{\mu\nu}(p)$, we can construct the photon propagator, i.e.,

$$\begin{aligned}\tilde{D}_{\mu\nu}(p)&=\left[\frac{1}{\tilde{\Delta}^{-1}(p)-\Pi(p)}\right]_{\mu\nu}\\ &=\frac{1-p^2M(p)}{p^2[1-p^2M(p)]^2+[p^3Q(p)]^2}P_{\mu\nu}+\lambda\frac{p_\mu p_\nu}{p^4}+\epsilon_{\mu\nu\alpha}\frac{p_\alpha}{p}\left[\frac{p^2Q(p)}{p^2[1-p^2M(p)]^2+[p^3Q(p)]^2}\right] .\end{aligned} \tag{4.7}$$

We shall analyze $p^2M(p)$ and $p^3Q(p)$ in the same way as in Sec. III by replacing (note again that they contain no UV divergence)

$$\int_0^\infty d\tau\cdots\to\int_{1/\Lambda^2}^\infty d\tau\cdots , \tag{4.8}$$

$$\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2} a\coth a\cdots\to\int_0^\pi da\cdots+\int_\pi^\infty da\sqrt{2}a^{3/2}e^{-a/2}\cdots , \tag{4.9}$$

$$\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2}\cdots\to\int_0^\pi da\cdots+\int_\pi^\infty da\sqrt{2}a^{1/2}e^{-a/2}\cdots . \tag{4.10}$$

Then as $m\to 0$ and $p^2\to 0$, we have

$$\begin{aligned}\lim_{m\to 0,p^2\to 0}p^2M(p)\to -\frac{1}{32\pi^2e^3\alpha^{5/2}}\int_{1/\Lambda^2}^\infty\frac{d\tau}{\tau^{7/4}}\Bigg[&\int_0^\pi da\, a^2\exp\left[-\frac{a^2}{4\alpha e^2\tau^{1/2}}\right]\left[-1+\frac{a^2}{6\alpha e^2\tau^{1/2}}\right]\\ &+\sqrt{2}\int_\pi^\infty da\, a^{7/2}e^{-a/2}\left[-1+\frac{a^2}{6\alpha e^2\tau^{1/2}}\right]\Bigg]=\frac{1}{12\pi^{3/2}\alpha}\end{aligned} \tag{4.11}$$

which is finite and independent of Λ.

And for $p^3Q(p)/m$, this quantity diverges as $m\to 0$, $p^2\to 0$. To examine this divergence, we write

$$\begin{aligned}\lim_{m\to 0,p^2\to 0}\frac{p^3Q(p)}{m}&=\lim_{p^2\to 0}-\frac{1}{24\pi^2\alpha^{5/2}}\int_0^{1/(2p^2)^{1/2}}\frac{d\tau}{\tau^{7/4}}\int_0^e\frac{de'}{e'^4}\int_0^\infty da\left[\frac{a}{\sinh a}\right]^{1/2}a^4\exp\left[\frac{a^2}{4\alpha e'^2\tau^{1/2}}\right]\\ &=\lim_{p^2\to 0}\frac{1}{12\pi^{3/2}\alpha}\int_0^\infty da\, a\left[\frac{a}{\sinh a}\right]^{1/2}\left[4\ln a+2C-4+\ln\frac{\sqrt{2p^2}}{(e^2\alpha)^2}\right] ,\end{aligned} \tag{4.12}$$

where C is the Euler constant. Equation (4.12) diverges as $\ln p$ as $p^2\to 0$. To further analyze this quantity we use the replacements (4.8)–(4.10):

$$\lim_{m\to 0,p^2\to 0}\frac{p^3Q(p)}{m}\to\lim_{p^2\to 0}\frac{1}{12\pi^{3/2}\alpha}\left[\left[\int_0^{\pi}da\,a+\int_{\pi}^{\infty}da\sqrt{2}a^{3/2}e^{-a/2}\right]\left[4\ln a+2C-4+\ln\frac{\sqrt{2p^2}}{(e^2\alpha)^2}\right]\right]$$

$$=\lim_{p^2\to 0}\frac{1}{12\pi^{3/2}\alpha}\left\{\left[2C-4+\ln\frac{\sqrt{2p^2}}{e^4\alpha^2}\right]\left[\frac{\pi^2}{2}+8\Gamma\left[\frac{5}{2},\frac{\pi}{2}\right]\right]\right.$$
$$+\left[\pi^2(\ln\pi^2-1)+4\sqrt{2}\ln\pi^2e^{-\pi/2}(\pi^{3/2}+3\pi^{1/2}+3\pi^{-1/2})\right.$$
$$+32\Gamma\left[\frac{3}{2},\frac{\pi}{2}\right]+48\Gamma\left[\frac{1}{2},\frac{\pi}{2}\right]+8\Gamma\left[-\frac{1}{2},\frac{\pi}{2}\right]$$
$$\left.\left.-12\sqrt{2}\int_{\pi}^{\infty}da\,a^{-3/2}(\ln a)e^{-a/2}\right]\right\}, \tag{4.13}$$

where the integral in the last term is finite because

$$\int_{\pi}^{\infty}da\,a^{-3/2}(\ln a)e^{-a/2}<\int_{\pi}^{\infty}da\,a^{-3/2}\ln a=\frac{1}{\sqrt{\pi}}(\ln\pi^2+4)\,.$$

Hence, the only divergent term in (4.13) is $\ln\sqrt{2p^2}$ (as $p^2\to 0$). Therefore, if we keep p to be nonzero and let $m\to 0$ first, we have

$$\lim_{m\to 0}\frac{p^3Q(p)}{m}\text{ is finite}\Longrightarrow\lim_{m\to 0}p^3Q(p)=0\,. \tag{4.14}$$

Using this result in (4.7), we have

$$\lim_{m\to 0}\tilde{D}_{\mu\nu}(p)=\frac{1}{p^2[1-p^2\overline{M}(p)]}\Pi_{\mu\nu}(p)+\lambda\frac{p_\mu p_\nu}{p^4}\,, \tag{4.15}$$

where

$$\overline{M}(p)=\lim_{m\to 0}M(p)\,. \tag{4.16}$$

From (4.11) we have

$$\lim_{p^2\to 0}\overline{M}(p)\text{is finite}\,. \tag{4.17}$$

Therefore, in (4.15) $p^2=0$ is a pole for $\tilde{D}_{\mu\nu}(p)$, i.e., the photon remains massless, and no spontaneous generations of mass occur.

V. CONCLUSION AND DISCUSSIONS

In Secs. III and IV we have calculated $\langle\bar{\psi}(x)\psi(x)\rangle$ and $\tilde{D}_{\mu\nu}(p)$ in the quenched IR approximation. There are no signs of spontaneous violations of parity due to IR effects. As $m\to 0$, $\langle\bar{\psi}(x)\psi(x)\rangle$ is zero and $\tilde{D}_{\mu\nu}(p)$ has a pole corresponding to mass zero. Since the IR approximation extracts the most IR part of these quantities, we can conclude that in the IR sector of QED_3 there are no spontaneous generations of mass, and hence no spontaneous violations of parity. It is interesting to note that this result agrees with that of the large-N (fixed e^2N) expansion,[6] where for odd N spontaneous generation of fermion mass does not occur; here, we work with a single fermion flavor, that is, $N=1$.

Our result can be compared with those on spontaneous chiral-symmetry breaking (SCSB) of gauge theories in $1+1$ dimensions using the same IR approximation[13,14] where SCSB is found to occur for general SU(N) case. Here, in $2+1$ dimensions, chiral symmetry cannot be defined because one cannot construct γ_5 in the 2×2 representation of γ matrices. Nevertheless, parity in $2+1$ dimensions and chiral symmetry in $1+1$ dimensions are closely related, and that is why the ordered parameter $\langle\bar{\psi}(x)\psi(x)\rangle$ is used in both analyses. However, here we have a negative result: parity seems not to be violated by IR effects, in contrast with the positive result in $1+1$ dimensions.

Nonquenched effects have been estimated[16] in investigating the SCSB in QED_2. The result in the quenched approximation is found to be unchanged qualitatively. Hence, we expect here also the result will be qualitatively the same even when nonquenched corrections are included.

Lastly the IR approximation we have been using is not manifestly gauge invariance. Gauge transformation is a local (in coordinate space) operation, while the IR approximation is a nonlocal one.[17] To remedy this, we have deliberately kept λ, the gauge-choice parameter, throughout the calculation. Since our final result is dependent of λ, it is invariant under change of gauge.

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APPENDIX

In this appendix we derive (2.8) in Sec. II. Consider for $D_\mu=\partial\!\!\!/+ie A\!\!\!/$,

$$D\!\!\!\!/\,D\!\!\!\!/=\tfrac{1}{2}\{\gamma_\mu,\gamma_\nu\}D_\mu D_\nu+\tfrac{1}{4}[\gamma_\mu,\gamma_\nu][D_\mu,D_\nu]$$
$$=D_\mu D_\mu+ie\sigma_{\mu\nu}F_{\mu\nu}\,, \tag{A1}$$

where

$$\sigma_{\mu\nu}=\tfrac{1}{4}[\gamma_\mu,\gamma_\nu]=\tfrac{1}{2}i\epsilon_{\mu\nu\alpha}\gamma_\alpha \tag{A2}$$

in our 2×2 matrix representation.

Using the Fradkin representation,[18]

$$\begin{aligned}\langle x \mid e^{\tau(\partial+ieA)^2} \mid x\rangle &= \langle x \mid e^{\tau(D_\mu D_\mu + ie\sigma_{\mu\nu}F_{\mu\nu})} \mid x\rangle \\ &= N(\tau)\int [d\phi]\exp\left[-\tfrac{1}{4}\int_0^\tau d\tau'\phi^2(\tau')\right]\delta\left[\int_0^\tau d\tau'\phi(\tau')\right] \\ &\quad\times\left[\exp\left\{ie\int_0^\tau d\tau'\left[\phi_\mu(\tau')A_\mu\left[x-\int_0^{\tau'}d\tau''\phi(\tau'')\right]+\sigma_{\mu\nu}F_{\mu\nu}\left[x-\int_0^{\tau'}d\tau''\phi(\tau'')\right]\right]\right\}\right]_+ ,\end{aligned} \tag{A3}$$

where

$$[N(\tau)]^{-1}=\int[d\phi]\exp\left[-\tfrac{1}{4}\int_0^\tau d\tau'\phi^2(\tau')\right]$$

and the exponential is ordered with respect to τ.

In the first IR approximation, we simply replace $A_\mu(x)$ by $A_\mu^S(x)$ given in (2.4), i.e.,

$$\begin{aligned}A_\mu&\left[x-\int_0^{\tau'}d\tau''\phi(\tau'')\right]\\ &\to A_\mu^S\left[x-\int_0^{\tau'}d\tau''\phi(\tau'')\right]\\ &=\int\frac{d^3k}{(2\pi)^3}e^{ikx}\exp\left[-ik\int_0^{\tau'}d\tau''\phi(\tau'')\right]\tilde{A}^S(k) .\end{aligned} \tag{A4}$$

in $\tilde{A}^S(k)$ the momentum is restricted to be

$$k\le\frac{1}{\sqrt{\tau}} . \tag{A5}$$

Then, because of the presence of the Gaussian term in (A3),

$$\tau\phi^2\le 1 . \tag{A6}$$

Combining (A5) and (A6), we have

$$\left|k\int_0^{\tau'}d\tau''\phi(\tau'')\right|\le\frac{1}{\sqrt{\tau}}\sqrt{\tau}=1 . \tag{A7}$$

Therefore, we can expand the exponential in (A4) and retain only the first nontrivial term in the expansion. Under this "multipole" approximation

$$\begin{aligned}\langle x \mid e^{\tau(\partial+ieA)^2} \mid x\rangle &\to \langle x \mid e^{\tau(\partial+ieA)^2} \mid x\rangle_{\mathrm{IR}} \\ &= N(\tau)\int\frac{d^3p}{(2\pi)^3}\int[d\phi]\exp\left[-\tfrac{1}{4}\int_0^\tau d\tau'\phi^2(\tau')+ip\int_0^\tau d\tau'\phi(\tau')\right] \\ &\quad\times\left[\exp\left[\frac{ie}{2}\int_0^\tau d\tau'\phi_\mu(\tau')\int_0^{\tau'}d\tau''\phi_\nu(\tau'')F_{\mu\nu}^S(x)+ie\tau\sigma_{\mu\nu}F_{\mu\nu}^S(x)\right]\right] .\end{aligned} \tag{A8}$$

Note that the ordered exponential has become an ordinary one. Now, the functional integral over ϕ is just Gaussian and can be evaluated with the help of the following equation:[13]

$$\begin{aligned}\int\frac{d^3p}{(2\pi)^3}N(\tau)\int[d\phi]&\exp\left[-\tfrac{1}{4}\int_0^\tau d\tau'\int_0^\tau d\tau''\phi_\mu(\tau')K_{\mu\nu}(\tau',\tau'')\phi_\nu(\tau'')+ip\int_0^\tau d\tau'\phi(\tau')\right]\\ &=\int\frac{d^3p}{(2\pi)^3}e^{-p_\mu p_\nu B_{\mu\nu}}(\det C)^{-1/2}=\frac{1}{(4\pi)^{3/2}}(\det B)^{-1/2}(\det C)^{-1/2} ,\end{aligned} \tag{A9}$$

where

$$K_{\mu\nu}(\tau',\tau'')=\delta_{\mu\nu}\delta(\tau'-\tau'')-\chi_{\mu\nu}[\theta(\tau'-\tau'')-\theta(\tau''-\tau')] ,$$

$$B_{\mu\nu}=\left[\frac{1}{\chi}\tanh\tau\chi\right]_{\mu\nu} ,\quad C_{\mu\nu}=(\cosh\tau\chi)_{\mu\nu} .$$

In (A8)

$$\chi_{\mu\nu}=ieF_{\mu\nu}^S(x)=ie\epsilon_{\mu\nu\alpha}{}^*F_\alpha^S(x) , \tag{A10}$$

then we have

$$\det B = \left[\frac{1}{\cosh[e\tau{}^*F^S(x)]}\right]^2 \left[\frac{\tau\sinh[e\tau{}^*F^S(x)]}{e\tau{}^*F^S(x)}\right]^3 , \tag{A11}$$

$$\det C = \{\cosh[e\tau{}^*F^S(x)]\}^2 , \tag{A12}$$

where ${}^*F^S(x)$ is the magnitude of ${}^*\mathbf{F}^S(x)$. In addition, using (A2) we can write

$$e^{ie\tau\sigma_{\mu\nu}F^S_{\mu\nu}(x)} = e^{ie\tau\gamma_\alpha{}^*F^S_\alpha(x)} = \cosh[e\tau{}^*F^S(x)] - \frac{\boldsymbol{\gamma}\cdot{}^*\mathbf{F}^S(x)}{{}^*F^S(x)}\sinh[e\tau{}^*F^S(x)] , \tag{A13}$$

because the γ matrices are just Pauli matrices. Putting all these together (A8) becomes

$$\begin{aligned}\langle x \,|\, e^{\tau(\not\partial+ie\not A)^2} \,|\, x\rangle_{\mathrm{IR}} &= \frac{1}{(4\pi)^{3/2}}\left[\frac{\tau\sinh[e\tau{}^*F^S(x)]}{e\tau{}^*F^S(x)}\right]^{-3/2}\left[\cosh[e\tau{}^*F^S(x)] - \frac{\boldsymbol{\gamma}\cdot{}^*\mathbf{F}^S(x)}{{}^*F^S}\sinh[e\tau{}^*F^S(x)]\right] \\ &= \frac{1}{8\pi^{3/2}\tau^{3/2}}\int d^3a\,\delta(\mathbf{a}-e\tau{}^*\mathbf{F}^S(x))\left[\frac{a}{\sinh a}\right]^{1/2}(a\coth a - \boldsymbol{\gamma}\cdot\mathbf{a}) ,\end{aligned} \tag{A14}$$

which is just (2.8) in Sec. II.

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*Present address: Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019.

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