Coupled-channel effects in the variational calculation of the glueball mass in lattice Hamiltonian SU(N) gauge theory

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A coupled-channel calculation of the variational eigenenergies of the SU(N) Hamiltonian on a lattice is presented. A trial physical vacuum state and one and two 0^{++} glueball states are the basis states of a Hamiltonian matrix whose lowest eigenenergies are the SU(N) ground-state energy to be minimized and the glueball mass. We show that the coupled-channel effects do not disappear in the infinite-volume limit and can, in principle, have a significant effect on the glueball mass. In practice, however, in the present context the effects turn out to be small.

I. INTRODUCTION

The Hamiltonian formulation of SU(N) gauge theory on a three-dimensional lattice has been presented by Chin, Long, and Robson.[1] They calculate vacuum and 0^{++} glueball energies in a variational calculation using a one-parameter Gaussian functional of the one-loop Wilson action as a trial ground state. Minimizing the vacuum energy fixes the variational parameter as an implicit function of the running coupling strength g^2. A glueball state orthogonal to the vacuum state is then constructed, and a value derived for the glueball mass M. The test for success of the lattice calculation is that, for some range of the lattice scale a, the product Ma should scale with the running coupling $g^2(a)$ in the way predicted for the continuum by two-loop renormalization-group-improved perturbation theory with the induced characteristic energy Λ, which is

$$Ma \sim \Lambda a = \left[\frac{48\pi^2}{11Ng^2}\right]^{51/121} \exp\left[\frac{-24\pi^2}{11Ng^2}\right] \times \left[1+\frac{51g^2}{88\pi^2}\right]^{(51/121)}, \quad (1)$$

for SU(N) with N^2-1 colored gluons. Chin, Long, and Robson achieve scaling on a 6^3 lattice for $N=5$ and 6, and more recently for 4 but not yet for 3 (Ref 2).

In an attempt to improve this situation, we have investigated the effect of coupling zero-, one-, and two-glueball channels, diagonalizing the resulting 3×3 matrix Hamiltonian and then minimizing the ground-state energy. The calculation uses only the original Monte Carlo computer data[3] for the Wilson action.[4] The result of the coupled-channel calculation is the appearance of glueball mass corrections which do *not* disappear in the infinite-volume limit and can in principle be large. In the present context, however, these corrections do not improve scaling in SU(3).

For two channels, the original variational minimum of the ground-state energy can become inverted to a local maximum. The two-channel ground-state-energy minimum occurs nearby at a slightly shifted value of the variational parameter. At the same time, the two-channel glueball mass can be substantially increased over its one-channel value. The shift in the ground-state-energy minimum is a form of spontaneous symmetry breaking (SSB) related to the Jahn-Teller effect. It occurs for all values of the variational parameter in SU(3) and for weak-coupling values in SU(4,5,6). A careful inspection shows that these conclusions are independent of the number of plaquettes in the lattice, and persist in the infinite-volume limit.

These conclusions do depend, however, on the number of channels included in the calculation. A full three-channel calculation is very complicated but one can analyze simple models which incorporate the main features of the Monte Carlo data. In the simplest such model, the criterion for SSB is not achieved. The three-channel variational minimum seems to remain at the same place as the one-channel minimum, which would result in a miniscule shift of the three-channel glueball mass. A more realistic model calculation using asymmetries characteristic of the Monte Carlo data shows that shifts in the location of the ground-state-energy minimum can occur with resulting, but small, shifts in the glueball mass. Similar results occur when the full smoothed and interpolated Monte Carlo data are used.

The final result turns out to be a remarkable stability of the ground-state energy and the glueball mass at very near their one-channel values. The particular form of the coupling terms of the matrix Hamiltonian responsible for this stability is discussed. The three-channel case is conjectured to be indicative of exact results of an infinite-channel calculation.

II. CALCULATION

A. One-channel case

We refer to the original work of Chin, Long, and Robson[1] for definitions and details of calculation. The basic operator is the one-loop Wilson action S summed over $N_p=6^3\times3$ spacelike plaquettes

$$S=\frac{1}{2NN_p}\sum_p \mathrm{tr}(U_p+U_p^\dagger), \quad (2)$$

where $U_p = U_1 U_2 U_3^\dagger U_4^\dagger$ on the plaquette p and

$$U_l = \exp\left[ia\frac{\lambda^\alpha}{2} A_l^\alpha(p)\right]$$

is the connection on the links around p. Also a is the lattice size, λ^α the generator of SU(N), and $A_l^\alpha(p)$ the gauge field component l. The physical-vacuum trial functional[5,6]

$$|\psi_0\rangle = C_0 \exp(2N^2 N_p AS/2)\,|0\rangle\ , \tag{3}$$

normalized to unity through the constant C_0, is used to calculate an expectation value $P(A)$ of S

$$P(A) = \langle\psi_0|S|\psi_0\rangle\ . \tag{4}$$

The lattice calculation is characterized by the Monte Carlo numerical data for P as a function of the variational parameter A, and its derivatives. The data for SU(3) (Ref. 3) are reproduced in Fig. 1. The operator S is used to generate a one-glueball state

$$|\psi_1\rangle = C_1(S-P)\,|\psi_0\rangle \tag{5}$$

orthogonal to $|\psi_0\rangle$ and normalized to unity. In the calculation of Chin, Long, and Robson the expectation values of the Kogut-Susskind SU(N) lattice Hamiltonian[7,8]

$$H = \frac{g^2}{a}\left[\sum_l \tfrac{1}{2} E_l^\alpha E_l^\alpha + \frac{2N}{g^4}\sum_p\left[1-\frac{1}{2N}\mathrm{tr}(U_p+U_p^\dagger)\right]\right] \tag{6}$$

are required. The color-electric field E_l^α is the generalized momentum canonically conjugate to the potential A_l^α on the link l.

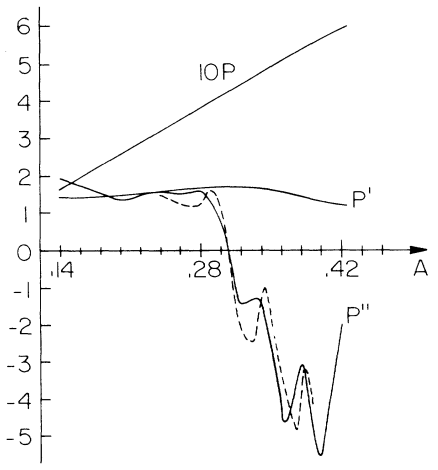


FIG. 1. The data $P(A)$, $P'(A)$, and $P''(A)$ for SU(3) from Ref. 3. The dashed line on $P''(A)$ shows the effect of local cubic smoothing of the data used in Sec. III.

The trial ground-state energy is

$$E_0 = \langle\psi_0|H|\psi_0\rangle = \left[\frac{AP}{2}+y(1-P)\right]\left[\frac{N_p(N^2-1)}{\xi a}\right]\ . \tag{7}$$

The one-channel glueball mass

$$M_1 \equiv \epsilon = \langle\psi_1|H|\psi_1\rangle - E_0 = f\left[1+\frac{P}{AP'}+\left[\frac{A}{2}-y\right]\frac{P''}{P'}\right]\left[\frac{N_p(N^2-1)}{\xi a}\right]\ , \tag{8}$$

where the lattice factor $f = 1/2N^2N_p$. Here $y = \xi^2 2N^2/(N^2-1)$, $\xi = 1/Ng^2$, and $P' = dP/dA$, etc. These are to be evaluated at the minimum of E_0 which occurs at A_0 where

$$y = \frac{A_0(y)}{2} + \frac{P(A_0(y))}{2P'(A_0(y))} \tag{9}$$

determines the variational parameter A_0 for a given value of y, the running coupling constant, or equivalently of the lattice scale a through Eq. (1).

B. Two-channel case

If one calculates the coupling between the zero- and one-glueball channels, the off-diagonal matrix element of the Hamiltonian

$$\langle\psi_1|H|\psi_0\rangle = \Delta = (f/P')^{1/2}\left[\left[\frac{A}{2}-y\right]P'+\frac{P}{2}\right]\times\left[\frac{N_p(N^2-1)}{\xi a}\right] \tag{10}$$

turns out to be proportional to (dE_0/dA) which is zero at A_0 where E_0 is a minimum. At first glance one would conclude that there is no coupling between channels. Closer inspection shows that this conclusion is not always correct. The two-channel Hamiltonian

$$H = E_0 + \begin{bmatrix} 0 & \Delta \\ \Delta & \epsilon \end{bmatrix} \tag{11}$$

has a ground-state energy

$$E_2 = E_0 + \tfrac{1}{2}[\epsilon - (\epsilon^2+4\Delta^2)^{1/2}]\ , \tag{12}$$

and a glueball mass

$$M_2 = (\epsilon^2+4\Delta^2)^{1/2}\ . \tag{13}$$

These are to be evaluated at the minimum of E_2. Surprisingly, the minimum of E_0 is not necessarily the minimum of E_2 but can be, and for SU(3) is, a local maximum. In this case, the minimum of E_2 occurs nearby where Δ is not zero and where the glueball mass shift is significant. To see this result, expand around A_0 where E_0 is a minimum:

$$E_0(A)=E_0(A_0)+\alpha x^2+O(x^3)\ , \tag{14}$$

with $x=A-A_0$. The quadratic coefficient α is positive and given by

$$\alpha=\frac{1}{2}\frac{d^2E_0}{dA^2}=\frac{1}{2}\left[P'-\frac{PP''}{2P'}\right]\left[\frac{N_p(N^2-1)}{\xi a}\right] \tag{15}$$

evaluated at A_0. Also, expanding Δ and ϵ about $A=A_0$, one obtains

$$\Delta=(f/P')^{1/2}2\alpha x+O(x^2) \tag{16}$$

and

$$\epsilon=f\left[1+\frac{P}{AP'}-\frac{P''P}{2P'^2}\right]\left[\frac{N_p(N^2-1)}{\xi a}\right]+O(x)\ . \tag{17}$$

Expanding E_2 to order x^2 gives

$$E_2\simeq E_0(A_0)+\alpha x^2\left[1-\frac{4\alpha f}{P'\epsilon}\right]+O(x^3)\ . \tag{18}$$

If the coefficient of x^2 is positive, the minimum of E_2 is the minimum of E_0 at $A=A_0$ where $\Delta=0$. There is no coupling and the glueball mass is ϵ as before. However, if

$$Z=\frac{4\alpha f}{P'\epsilon}-1=\frac{1-P/AP'-PP''/2P'^2}{1+P/AP'-PP''/2P'^2}>0\ , \tag{19}$$

then the sign of the x^2 term in E_2 is opposite to that in E_0. The minimum of E_0 becomes a local maximum of E_2. The minimum of E_2 is nearby at $x_{\min}=(A_{\min}-A_0)$. This shift in the minimum is a form of spontaneous symmetry breaking reminiscent[9] of the Jahn-Teller effect in crystals where the off-diagonal matrix element coupling the lattice to the electrons is linear in a lattice deformation parameter analagous to the x of Eq. (14).

For SU(3) and $A_0=0.36$, Chin's results give $Z=0.25$. Actual calculation shows that SSB occurs in SU(3) for all A_0 as shown in Fig. 2 and can make corrections to the glueball mass as large as 30%. The minimum in E_2 occurs for $Z>0$ at $x_{\min}$ given by

$$x_{\min}^2=[(1+Z)^2-1]\frac{\epsilon}{4\alpha(1+Z)}\ . \tag{20}$$

In the harmonic approximation there are equal minima at $\pm x_{\min}$. The absolute minimum will depend on anharmonic terms in the Hamiltonian matrix. Anharmonic terms have been calculated, and also exact calculations using interpolated Monte Carlo data have been done which substantiate these conclusions based on the harmonic approximation. At the minimum of E_2, the glueball mass is

$$M_2=\epsilon(1+Z)=\frac{N^2-1}{2N^2}\frac{2}{\xi a}\left[1-\frac{PP''}{2P'^2}\right]\ , \tag{21}$$

and the shift in the ground-state energy is

$$E_2(A_{\min})-E_0(A_0)=-\tfrac{1}{4}\epsilon\frac{Z^2}{1+Z}\ . \tag{22}$$

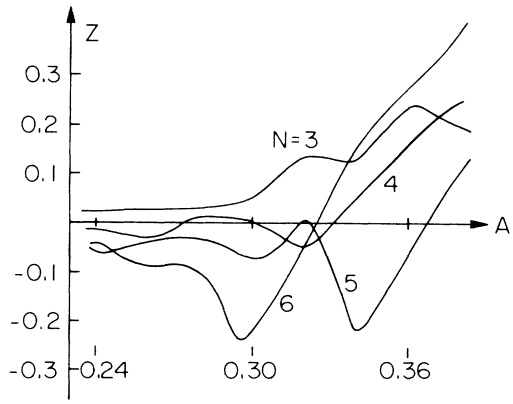


FIG. 2. The parameter $Z(A)$ defined in Eq. (19) for SU(3,4,5,6) evaluated from the data of Ref. 3. $Z(A)>0$ corresponds to a shift of the two-channel minimum of E_2 from the minimum of E_0.

The volume, that is the N_p, dependence of the various quantities involved is interesting. The criterion for SSB is in fact independent of N_p as can be seen from Eq. (19), so SSB will *not* go away as $N_p\to\infty$. The location of the minimum of E_2 is very close to that of E_0, and

$$x_{\min}=A_{\min}-A_0\sim\frac{1}{\sqrt{N_p}}\ , \tag{23}$$

from the N_p dependence of α. A typical value is $x_{\min}=0.004\,96$ at $A_0=0.36$, $N_p=3\times6^3$, with $f=1/11\,664$ for SU(3). This is to be compared with the interval $\Delta A=0.01$ used in the Monte Carlo calculations. The coupling matrix element

$$\begin{aligned}\Delta&\sim\sqrt{N_p}\ \text{nominally}\\&=0\ \text{at}\ x=0\\&\sim(\sqrt{N_p})(1/\sqrt{N_p})\sim1\ \text{at}\ x_{\min}\ ,\end{aligned} \tag{24}$$

and is comparable to ϵ.

The shift in the ground-state energy

$$\begin{aligned}E_2-E_0&\approx\alpha x^2\\&\sim N_p\ \text{nominally}\\&=0\ \text{at}\ x=0\\&\sim N_p(1/\sqrt{N_p})^2\sim1\ \text{at}\ x_{\min}\ ,\end{aligned} \tag{25}$$

which is ignorable if we are just interested in the ground-state energy, but important in the calculation of the glueball mass. The shift in the glueball mass is

$$\begin{aligned}M_2-\epsilon&\sim\frac{\Delta^2}{\epsilon}\sim N_p\ \text{nominally}\\&=0\ \text{at}\ x=0\\&\sim1\ \text{at}\ x=x_{\min}\ .\end{aligned} \tag{26}$$

Thus, there is a shift in the glueball mass which survives in the $N_p\to\infty$ limit.

One key question is whether or not SSB helps scaling for SU(3). It does not. In our efforts to exploit the one-loop Wilson action data to the fullest, we have explored the two-channel calculation further, including terms anharmonic in x in the expansion around A_0 of E_0, Δ, and ϵ. We discuss these results later, together with the results of the three-channel model.

In the next section we calculate the effect of a third channel, with two glueballs, coupled to the above two channels. The coupling of the third channel seems at first sight to eliminate the possibility of SSB and render the above discussion of pedagogic interest only. A closer examination shows that the variational minimum in E_3 is very flat and the location of the variational minimum can shift and the glueball mass M_3 will depend on the detailed position of the minimum. Following a three-channel model calculation we finally discuss the one-, two-, and three-channel results for the actual Monte Carlo data on $P(A)$ (Ref. 10).

C. Three-channel case

The basis states for the three-channel calculation are $|\psi_0\rangle$ and $|\psi_1\rangle$ as before and also a two-glueball state

$$|\psi_2\rangle = C_2[(S^2 - \langle\psi_0|S^2|\psi_0\rangle)|\psi_0\rangle - \langle\psi_1|S^2|\psi_0\rangle|\psi_1\rangle], \tag{27}$$

which is orthogonal to the other two and normalized to unity. The Hamiltonian matrix

$$H = E_0 + \begin{pmatrix} 0 & E_{01} & E_{02} \\ E_{01} & E_{11} & E_{12} \\ E_{02} & E_{12} & E_{22} \end{pmatrix} \tag{28}$$

has matrix elements $E_{11}=\epsilon$, $E_{01}=\Delta$ as before, and

$$E_{12} = \sqrt{2}\Delta , \tag{29}$$

$$E_{22} = 2\epsilon , \tag{30}$$

$$E_{02} = \frac{f}{\sqrt{2}}(1 - P/AP' - PP''/2P'^2)\left[\frac{N_p(N^2-1)}{\xi a}\right] . \tag{31}$$

These expressions are valid for arbitrary values of the variational parameter A, not just $A_0(y)$ determined by the minimum of E_0. The matrix elements E_{02}, E_{12}, and E_{22} are leading terms in expansions in the lattice parameter $f = 1/(2N^2N_p)$ and neglect terms of relative order f.

The matrix elements E_{02}, E_{12}, and E_{22}, of Eqs. (29), (30), and (31) are deceptively simple but their calculation is nontrivial and involves considerable cancellation. The problem is that, without orthogonalization, the operator S^2 creates to leading order in N_p the same one-glueball state as does S. Without orthogonalization to $|\psi_1\rangle$ and to leading order in N_p, a two-channel calculation using $|\psi_0\rangle$ and $|\psi_2\rangle$ would have the Hamiltonian of Eq. (11). Orthogonalization with respect to $|\psi_1\rangle$ reduces the length of $|\psi_2\rangle$ by two orders of N_p. The matrix element E_{02} has an exact cancellation of the two leading orders of N_p leaving the result Eq. (31). The cancellations are exact for any (A,y) not just at $A_0(y)$ of Eq. (9). Similar cancellations are required in E_{12} and E_{22} to have the correct N_p dependence.

For a first understanding of the effect of three channels, we make a harmonic expansion of Δ as in Eq. (16), evaluate ϵ and E_{02} at A_0, and keep terms of order x^2 in the perturbation expansion of E_3:

$$E_3 \simeq E_0(A_0) + \alpha x^2 + \frac{\langle\psi_0|H|\psi_1\rangle^2}{-E_{11}} + \frac{\langle\psi_0|H|\psi_2\rangle^2}{-E_{22}} + 2\frac{\langle\psi_0|H|\psi_1\rangle\langle\psi_1|H|\psi_2\rangle\langle\psi_2|H|\psi_0\rangle}{(-E_{11})(-E_{22})} . \tag{32}$$

The result is

$$E_3 \simeq E_0(A_0) - \frac{E_{02}^2}{2\epsilon} + \alpha x^2\left[1 - \frac{4\alpha f}{P'\epsilon}\left[1 - \frac{\sqrt{2}E_{02}}{\epsilon}\right]\right] . \tag{33}$$

Comparison with the corresponding two-channel result of Eq. (18) shows that the effect of coupling to the two-glueball channel is to replace $4\alpha f/P'\epsilon$ by

$$\frac{4\alpha f}{P'\epsilon}\left[1 - \frac{\sqrt{2}E_{02}}{\epsilon}\right] ,$$

in the criterion for SSB, Eq. (19). From Eqs. (31), (16), and (19) one can show that this is just $(1+Z)(1-Z) = 1-Z^2$ and is always less than one, so no SSB can occur for any value of the parameters, at least in perturbation theory.

However, this result means that now the parabolic minimum in the ground-state energy is flatter by a factor Z^2 which can be small. The next question is whether or not the location of the minimum of this very flat function is sensitive to nonperturbative effects and can be shifted from $x=0$ by some small amount of order $\sqrt{f}$ with a resulting significant change in the glueball mass. To answer this question we solve the eigenvalue equation directly for a model of the full matrix Hamiltonian in which ϵ and E_{02} are taken to be constant at their value at A_0, Δ is a linear function of $x = A - A_0$, and E_0 is quadratic in x, as in the two-channel case of Eqs. (11)–(13). The Hamiltonian can be best expressed in terms of the two variables Z and w with

$$Z = \sqrt{2}\frac{E_{02}}{E_{11}} \tag{34}$$

as before, and w defined by

$$E_{01}^2/E_{11}^2 = (1+Z)\frac{\alpha x^2}{E_{11}} = (1+Z)w^2 . \tag{35}$$

The eigenvalues in units of E_{11} are the three roots ϵ_3 of

$$(\epsilon_3 - 1) - [Z^2/2 + 3(1+Z)w^2 + 1](\epsilon_3 - 1) - (1+2Z)(1+Z)w^2 = 0 . \tag{36}$$

If $\epsilon_3(1,2,3)$ are labeled in order of increasing energy, then the three-channel model ground-state energy is

$$E_3 = w^2 + \epsilon_3(1) , \tag{37}$$

and the three-channel model glueball mass is

$$M_3 = \epsilon_3(2) - \epsilon_3(1) . \tag{38}$$

These are plotted in Fig. 3 for $Z = 0.1$ and $w = 0$–1. For comparison, the one- and two-channel results

$$E_1 = E_0 = w^2, \quad M_1 = 1 , \tag{39}$$

and

$$E_2 = w^2 + \tfrac{1}{2}[1 - \sqrt{1 + 4(1+Z)w^2}] , \tag{40}$$

$$M_2 = \sqrt{1 + 4(1+Z)w^2} , \tag{41}$$

are also plotted. For $Z = 0.1$, which is quite characteristic of the SU(3) data, there is interesting structure in the model calculation. The two-channel ground-state energy E_2 has its minimum shifted to $w \simeq 0.23$ away from the minimum of E_1 at $w = 0$. The Monte Carlo calculations are made at intervals of $\Delta A = 0.01$ and the minimum in E_2 lies about halfway between neighboring data points. The glueball mass M_2 is increased $\sim 12\%$. Note the split vertical scale so that the negative excursion of E_2 is magnified 100 times. The actual minimum is very shallow. For the three-channel case there is an absolute minimum at $w = 0$ with $E_3 = -0.2497 \times 10^{-2}$ and a local minimum at $w \simeq 0.55$ with $E_3 = -0.2461 \times 10^{-2}$. The three-channel glueball mass shifts by only two percent at the local minimum. In all the model calculations for $Z < 0.5$ (which range includes all the Monte Carlo data) there is an absolute minimum of E_3 at $w = 0$ and a local minimum at $w^2 < 1$.

Next we look in the Monte Carlo data of SU(3) for any of the structure indicated in the model calculations. The Monte Carlo sweeps have been made at intervals of $\Delta A = 0.01$ for $A = 0.13$–0.42. Strong coupling corresponds to small A. Scaling was originally observed in SU(5) and SU(6) near $A = 0.35$. The coupled-channel structure is on a scale somewhat smaller than the spacing of the Monte Carlo data so we have manufactured preliminary input by a linear interpolation of the data on P, P', P''.

There are two features of the data which seem to be significant and which can be directly incorporated into an improved model calculation. For a substantial portion of the data including $A_0 = 0.30$, 0.31, and 0.32 the matrix elements E_{11} and E_{02} have a linear dependence on w. Returning to the model calculation, we write as a better representation of the data

$$\begin{aligned} E_{11}(w) &= E_{11}(0)\left[1 - \frac{w}{15}\right] , \\ E_{02}(w) &= E_{11}(0)\frac{Z}{\sqrt{2}}\left[1 + \frac{w}{3}\right] , \\ E_{01}{}^2(w) &= (1+Z)w^2 E_{11}{}^2(0) , \\ E_0 &= w^2 E_{11}(0) . \end{aligned} \tag{42}$$

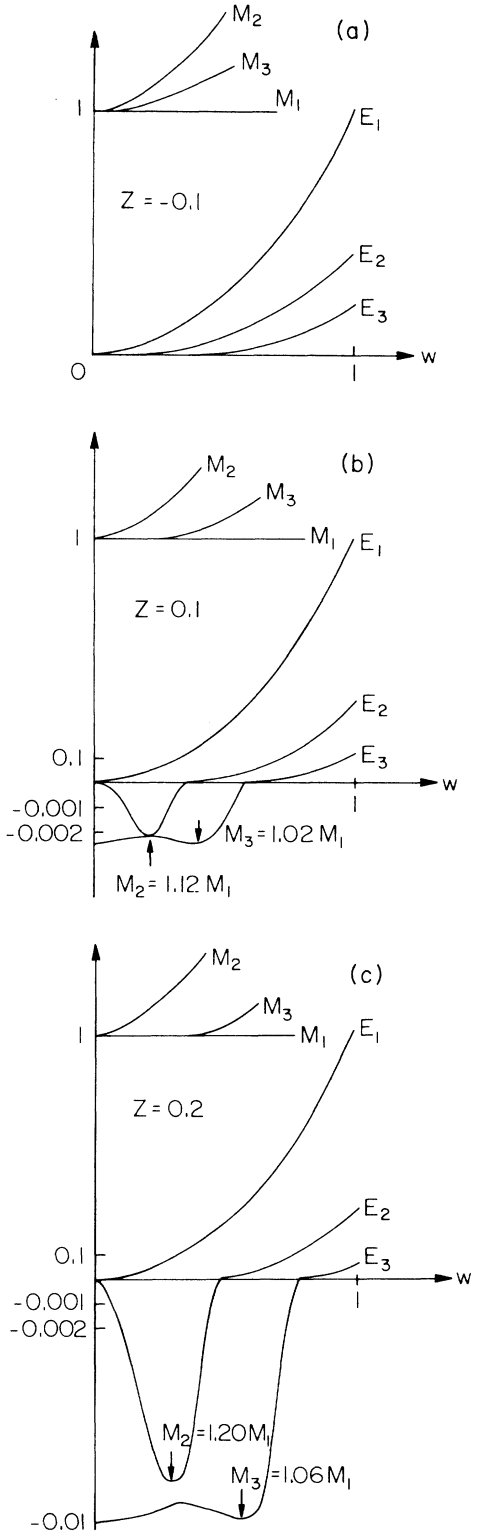


FIG. 3. Ground-state energies $E_{1,2,3}$ and glueball masses $M_{1,2,3}$ as a function of w defined in Eq. (35) for (a) $Z = -0.1$, (b) $Z = +0.1$, (c) $Z = +0.2$ of Eq. (34) for the symmetric one-, two-, and three-channel model of Eqs. (34)–(41). All energies are in units of the one-channel glueball mass M_1. Note the vertical scale change for negative energies.

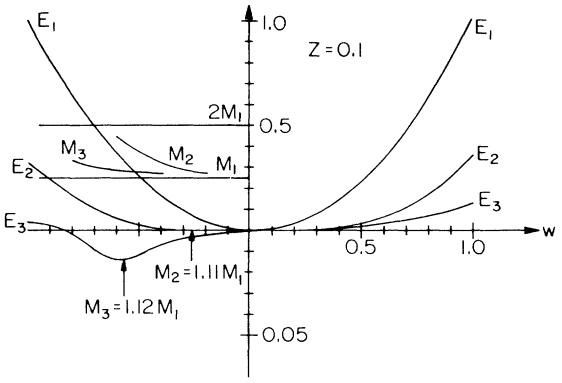


FIG. 4. As in Fig. 3(b), but for the asymmetric model of Eq. (42).

The linear dependence on w of E_{11} and E_{02} introduces an asymmetry into the two- and three-channel model calculations. For the two-channel case, the minimum at $w<0$ becomes the absolute minimum of E_2. For the three-channel case, the local minimum at $w<0$ becomes an absolute minimum of E_3. The results of the asymmetric model are plotted in Fig. 4 for $Z=0.1$. For $Z=0.1$, the glueball masses at the minima of E_2 and E_3 are only increased by $\sim 10\%$ over the nominal one-channel glueball mass. For $Z=0.2$, the increase is $\sim 25\%$. $Z=0.1$ is in the region of hoped for scaling and $Z=0.2$ is probably into the weak-coupling region.

III. DISCUSSION OF DATA

The full interpolated data have been used in one-, two-, and three-channel calculations of the ground-state energy. Results are plotted in Figs. 5(a) and 5(b) for SU(3) $A_0=0.25$ and 0.31 where Chin finds the transition from strong to weak coupling.

Figure 5(a) shows an example where the data follow the expectation of the model calculation quite closely. The one-channel ground-state-energy shift in units of the glueball mass M_1,

$$E_1=[E_0(A,y(A_0))-E_0(A_0,y(A_0))]/E_{11}(A_0,y(A_0)) , \tag{43}$$

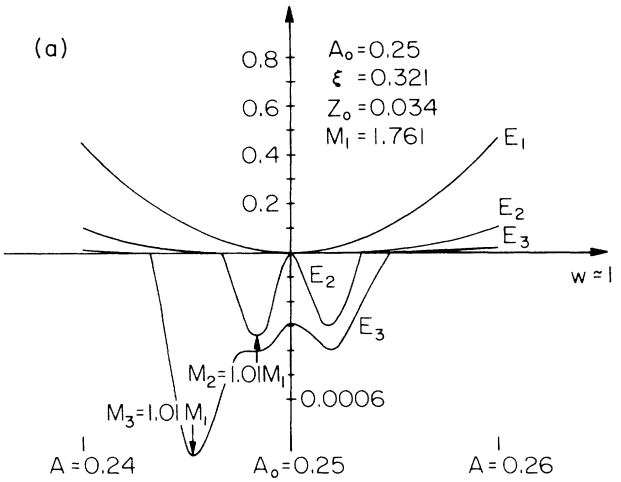
is plotted against the variable $A-A_0$, and w is defined as before as

$$(1+Z)w^2=E_{01}^2(A,y(A_0))/E_{11}^2(A_0,y(A_0)) , \tag{44}$$

where

$$Z=\sqrt{2}E_{02}(A_0,y(A_0))/E_{11}(A_0,y(A_0)) . \tag{45}$$

Z and w are the variables of our model calculation. The variables $A-A_0$ and w are almost linearly related. When w changes from -1 to 0 to $+1$, the parameter A typically changes by $A_0-0.015$ to A_0 to $A_0+0.015$. In the model, the energy shift E_1 is just the unit parabola w^2.



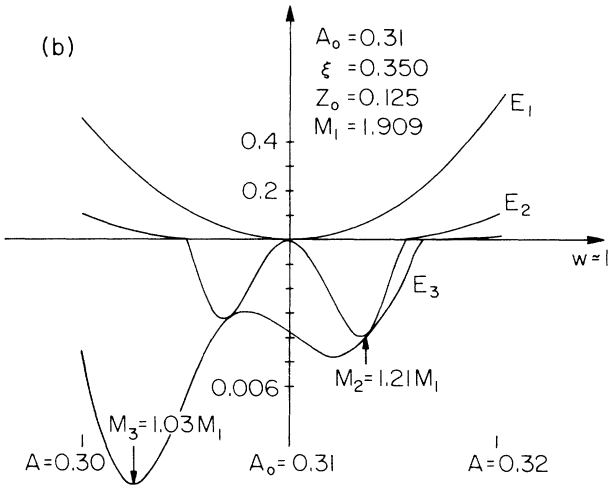


FIG. 5. Ground-state energies $E_{1,2,3}$ for one-, two-, and three-channel calculations using data of Ref. 3 smoothed and interpolated with a cubic function of A, for (a) $A_0=0.25$ and (b) $A_0=0.31$. Two- and three-channel glueball masses are indicated at the minima of E_2 and E_3. The results are plotted against $A-A_0$ which is approximately linear in w. All energies are in units of the one-channel glueball mass M_1. Note the vertical scale change for negative energies.

We have found it necessary, in order to stabilize the calculations for neighboring values of A_0, to artificially smooth the Monte Carlo data on a local scale. This has been done by fitting a cubic function of A to P on seven neighboring data points. The resulting fit to P is then used to evaluate P, P', and P'' on the interior four intervals of A. The data of Chin, Long, and Robson have already been smoothed so there is little change in P and P' ($\lesssim 0.1\%$ change in P, $\lesssim 2\%$ change in P') but there can be substantial changes in P'' as indicated in Fig. 1. It is not sufficient to smooth the second derivative only. Local correlations of P, P', and P'' must be retained in or-

der to reduce and to stabilize the local asymmetry of E_1 and to reduce the variability of the two- and three-channel calculations.

The glueball masses for the two- and three-channel calculations are stabilized at their one-channel values. This is especially true in the stronger-coupling cases ($A_0 = 0.20$ and 0.25) favored by Chin and Karliner for the scaling region of SU(3). For the weaker-coupling cases ($A_0 = 0.31$, 0.32, 0.36) the two-channel prediction M_2 is increased by approximately 20% and the three-channel prediction is increased by about 5%.

IV. DISCUSSION AND CONCLUSIONS

Our conclusion is a reaffirmation of faith in the potency of the original variational calculation. The ability of the one-channel variational calculation to predict the ground-state energy was not in question. The question was the validity of the glueball mass prediction. We investigated this problem within the context of the original one-parameter variational calculation by doing a coupled-channel calculation. The conclusion, after all is said and done, is a remarkable stability of the glueball mass prediction at very near its one-channel value.

The two-channel coupling vanishes at the minimum of the vacuum energy but not necessarily at the minimum of the two-channel ground-state energy, which can occur at a different value of the variational parameter. A form of spontaneous symmetry breaking can occur when a critical parameter Z of Eq. (19) is greater than zero. A model calculation predicts structure in the energy levels on a scale small compared to the intervals of A for which Monte Carlo data exist. The result is an increase in the predicted glueball mass of as much as twenty-five percent over the one-channel prediction. For positive Z, the excited state is pushed up by order Z, and the ground state is pushed down by order Z^2. Z is usually quite small so the ground-state energy is remarkably stable. Most of the two-channel shift of the glueball mass is a result of the upward shift of the first excited state. Positive Z and the resulting SSB in the two-channel case occurs for all SU(N). For SU(4,5,6), Z is greater than zero only near the onset of weak coupling where the lattice calculation fails. For SU(3), Z is greater than zero for all values of the variational parameter A so SSB occurs everywhere in the two-channel problem. For most of the scaling region of Chin and Karliner, Z is quite small and M_2 is within 10% of the one-channel value.

The three-channel calculation in which a two-glueball state is coupled to the vacuum and one-glueball states is also instructive. A perturbation calculation and a model calculation suggest as in Figs. 3(a)–3(c) that the SSB of the two-channel case no longer occurs. A more detailed model incorporating anharmonic terms characteristic of the data restores a form of SSB in the three-channel case as in Fig. 4. The full three-channel calculation using locally smoothed Monte Carlo data substantially restores M_3 to its single-channel value M_1.

The energy curve $E_3(A)$ is remarkably flat for the three-channel case. If multiple 0^{++} glueball excitations exhausted the spectrum of the Hamiltonian, the energy curves would become independent of A in the limit of an infinite number of channels. We would move from an approximate variational calculation to an exact solution in an arbitrary but complete basis. The three-channel calculation hopefully reflects this limit. Certainly adding more glueball channels is difficult and numerically impractical.

The coupled-channel calculation has returned essentially to the original one-channel result. The reason lies in the detailed structure of the Hamiltonian matrix. The one-parameter variational calculation can do only one thing and that is minimize the ground-state energy. It tries to do this by forcing the two-channel coupling to zero, so E_{01} should be zero at A_0. It is a surprise that E_{02} is not zero but that E_{12} is. The fact that E_{02} is small (like Z) means that the shift in the ground-state energy is small (like Z^2). Furthermore, the one-glueball state decouples at A_0 from both the zero- and two-glueball states so the glueball mass is shifted only by Z^2 at A_0. Since the three-channel energy curve is so flat we can suppose that it has converged to its infinite-channel value independent of A and equal for given y to the value at A_0, and thus differs only by $O(Z^2)$ from the variational calculation result. The dynamical effect of coupling could have been different. For example, Z is small because P is a rather featureless function interpolating known asymptotic values. If P were to have a cusp and a large second derivative, then Z would be large and a substantial shift in the glueball mass would result. Alternatively, if E_{02} had been zero at A_0 and E_{12} nonzero, then the ground state would have decoupled at A_0 and the glueball mass would have been significantly shifted.

Finally, one must acknowledge the power and the stability of the predictions of the original variational calculation. For significant improvements it is necessary to look beyond the one-parameter Gaussian functional of the one-loop Wilson action as a trial ground state, perhaps to the t expansion method of Horn and Weinstein[11] which shows promise of scaling for SU(3) (Refs. 2 and 12). Alternatively, the analysis of Hatfield[6] suggests that nonlocal correlations patterned after the electrodynamic vacuum functional should be used. This would require evaluating six- or eight-link loops rather than the basic four-link plaquette used here.

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