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Dynamics of bubbles in general relativity

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This is a systematic study of the evolution of thin shell bubbles in general relativity. We develop the general thin-wall formalism first elaborated by Israel and apply it to the investigation of the motion of various bubbles arising in the course of phase transitions in the very early Universe including new phase bubbles, old phase remnants, and domains. We consider metric junction conditions and derive constraints both on the decay of metastable states and on the evolution of nonequilibrium scalar field configurations (fluctuations) following from the global geometry of spacetime.

I. INTRODUCTION

Great attention was paid quite recently (see, e.g., Refs. 1–14) to the study of the new and important subject of bubbles arising in the course of cosmological phase transitions.[15] These are both new phase bubbles in interiors of an old phase[1–3,5–12] and old phase remnants surrounded by the new phase.[4,5,8–10,13] It has become clear that an account of general-relativistic effects is necessary in investigations of these objects.[3–10,13,14]

A full account of gravity in cosmological phase-transition phenomena seems to be a rather complicated problem; to date this question has not yet been investigated properly despite its importance.

In this paper we shall investigate systematically the dynamics of space regions occupied by different phases in the framework of general relativity assuming that the transient layer is thin enough. We shall also consider the properties of boundaries of phase separation. The quite adequate formalism for investigation of such problems is that of metric junctions on thin shells.[16,17,5–10] In this approach the surface of phase separation is the three-dimensional hypersurface dividing all of four-dimensional spacetime into two parts. We shall carry out a detailed evaluation of spherically symmetric surfaces of phase separation, both timelike (TL) and spacelike (SL).

The shape of the TL surface determines, in particular, the motion of bubble walls. In this case one encounters the following three physically different problems: (i) the evolution of new phase bubbles surrounded by an old phase, (ii) the evolution of remnants of the old phase surrounded by the new phase, and (iii) the case of the domain structure.

(i) The decay of a metastable state in the Universe could proceed by means of new phase bubble nucleation in interiors of the old phase. Thus, the problem of investigation of phase transitions consists of two parts. First, one needs to know the probability of bubble nucleation. Second, it is necessary to study the subsequent evolution of newly formed bubbles. In a pure vacuum case both these problems reduce to one as follows: the motion of the nucleated bubble is given by analytic continuation into real time of the Euclidean configuration describing the process of subbarrier tunneling.[1–3] The probability of the thermodynamical formation of a bubble was found in Ref. 18. However, the problem of bubble growth requires, in this case, special consideration. Usually bubble evolution is treated similarly to detonation wave propagation[19] (or to the motion of the so-called "condensation discontinuities"[19]). In Refs. 11 and 12 a similar approach has been used for the investigation of phase transitions in the very early Universe. However, the detonation wave approximation is valid only for nonsingular surfaces of phase separation. (Recall that a surface of phase separation is called singular if the energy-momentum tensor surface density on the shell is not equal to zero identically.) In a number of cases the surface of phase separation may not be treated as nonsingular. For example, the shell separating two phases with pure vacuum equations of state is always singular.[1–3,5] Another example is given by the shell in the "vacuum-burning" phenomenon.[7] We shall investigate the equation of motion of singular shells describing, as special limiting cases,[5,7] the growth of pure vacuum bubbles, vacuum combustion, and detonation wave propagation.

(ii) The processes of bubble nucleation, expansion, and collision continue until the Universe is filled with the new phase. In principle, the phase transition may never become completed. Such a situation arises if the cosmological expansion rate exceeds the bubble nucleation rate.[20] Nevertheless, most phase transitions certainly have been completed in our Universe and percolation through the old phase ceased at a certain moment. Thus old-phase remnants are isolated from one another beginning from that moment and the sizes of the remnants do not exceed certain maximum size. Black holes in the Universe could then originate from such remnants.[4,5,21] In this paper we consider only spherically symmetric remnants of the old phase. We suppose that the spherical formation of a remnant may be due to its surface tension.

One (or some) phase transition may never become complete, and then the entire visible part of the Universe should lie inside one bubble[22] of a corresponding new phase (such a situation may be realized[22] in the framework of a phase transition with inflation[23]). In such a case we have no remnants of old phase to investigate. The new phase bubble (where we live) expands forever and has a tremendous size but it is clear that it is nevertheless nothing but a bubble, so one might study it as such. Moreover, in the chaotic inflationary scenario[24] there is no phase transition at all. However, the inflating scalar field fluctuation possesses a greater energy density than the outlying Universe, so we may treat it formally as a remnant of the old phase as well.[10]

(iii) The investigation of the domain structure in the Universe,[25–27] in particular, of domains with different gauge symmetries of the ground state[27] and of *CP* domains[25,26] playing possibly an important role in the production of the baryon asymmetry of the Universe,[26] is also of great interest. The plane domain walls were considered in Ref. 25. However, domain boundaries would form closed surfaces when abundances of the phases reach the value given by the percolation theory. Such an isolated domain can be again considered approximately as a spherically symmetric bubble and treated in the same terms as the old phase remnants.[5]

As far as the physical meaning of a spacelike hypersurface of phase separation is concerned, we would like to emphasize that it differs essentially from that of a timelike hypersurface, the latter describing real motion of bubble walls, while the SL junction describes processes of the fast creation of a new phase. Investigating such a junction one can connect parameters of a new and old phase.[6]

The paper is constituted as follows. In Sec. II we present some elements of the general formalism; in Sec. III we consider the decay of a metastable state and derive constraints on parameters of the decaying vacuum; in Sec. IV the growth of new-phase bubbles is studied; in Sec. V old-phase remnants and domains with nonzero outer mass are investigated. Section VI is devoted to the investigation of lightlike shells.

II. EVOLUTION OF THIN SHELLS. GENERAL FORMALISM

In this section we shall obtain equations describing a shell separating two media with different properties. Since we are interested mainly in surfaces of phase separation arising at first-order phase transitions in the framework of modern unified theories of fundamental interactions (which presumably took place in the very early Universe), we shall derive the equations of interest taking into account general-relativistic effects.

Let us imagine that spacetime is divided into three regions: the first one is occupied by the phase I, the second region is occupied by the phase II, while the transient layer represents the third region. In the limit of vanishing thickness of the transient layer we obtain an infinitely thin wall with the energy-momentum tensor $T_\mu{}^\nu$ having, in general, singularities on it.

Let us enumerate somehow all points on the given spacetime manifold and let the equation of the phase separation hypersurface Σ, in the chosen coordinate y^μ, have the form

$$F(y^\mu)=0 , \tag{2.1}$$

F being a certain unknown function. We introduce the function $\tilde{n}(y^\mu)$ as

$$\tilde{n}(y^\mu)=\frac{F(y^\mu)}{|\partial_\nu F \partial^\nu F|^{1/2}} . \tag{2.2}$$

If a displacement vector dy^μ lies on the hypersurface $\tilde{n}=$const, then

$$d\tilde{n}=\tilde{n}_{,\mu}dy^\mu=0 , \tag{2.3}$$

$\tilde{n}_{,\mu}\equiv\partial_\mu\tilde{n}$ being the usual derivative. Therefore, a vector with covariant components $\tilde{n}_{,\mu}$ is just the normal vector to the surface $\tilde{n}=$const. Because of the normalization factor in Eq. (2.2) the vector

$$N_\mu\equiv\tilde{n}_{,\mu}|_\Sigma \tag{2.4}$$

is the unit normal vector to the hypersurface Σ,

$$N_\mu N^\mu=\epsilon , \tag{2.5}$$

where $\epsilon=-1$ for a timelike hypersurface and $\epsilon=+1$ for a spacelike hypersurface, respectively. The case of the null hypersurface ($\epsilon=0$) will be considered separately in Sec. VI.

According to the assumptions, $T_\mu{}^\nu$ has singularities on the hypersurface $\tilde{n}=0$ only and therefore it has to be written in the covariant form

$$T_\mu{}^\nu(y)=\tilde{T}_\mu{}^\nu(y)+\mathcal{D}_\mu^{+\nu}(y)\theta(\tilde{n})+\mathcal{D}_\mu^{-\nu}(y)\theta(-\tilde{n}) +S_\mu{}^\nu(y)\delta(\tilde{n})+\cdots , \tag{2.6}$$

where $\delta(\tilde{n})$ and $\theta(\tilde{n})$ are δ function and θ function, respectively, $S_\mu{}^\nu$ is a surface density of the $T_\mu{}^\nu$ on the shell and $\tilde{T}_\mu{}^\nu$ is the regular part of $T_\mu{}^\nu$. Then, from the continuity equation

$$T_\mu{}^\nu{}_{;\nu}=0 , \tag{2.7}$$

one obtains, equating the coefficients at the corresponding singular functions,

$$S_\mu{}^\nu\partial_\nu\tilde{n}=0 , \tag{2.8a}$$

$$S_\mu{}^\nu{}_{;\nu}+(\mathcal{D}_\mu^{+\nu}-\mathcal{D}_\mu^{-\nu})\partial_\nu\tilde{n}=0 , \tag{2.8b}$$

$$\mathcal{D}_\mu^{-\nu}{}_{;\nu}\theta(-\tilde{n})+\mathcal{D}_\mu^{+\nu}{}_{;\nu}\theta(\tilde{n})+\tilde{T}_\mu{}^\nu{}_{;\nu}=0, \ldots . \tag{2.8c}$$

Here a semicolon denotes a covariant derivative.

If in the problem under consideration there are in addition to the energy-momentum conservation any other conservation laws (for example, those of charge, entropy, etc.), then it is convenient to write in the form (2.8) all the continuity equations also, corresponding to each conserved current.

In the case when one considers the spacetime manifold as fixed, i.e., if one does not take into account the back reaction of the moving matter on the spacetime metric, the obtained equations determine completely the phase separation surface. This is true, in particular, if one neglects gravitation, i.e., in the case of flat Minkowski spacetime of special relativity. We would note that even in the case of the flat geometry of spacetime, the covariant derivatives in Eqs. (2.8) have to be preserved nevertheless, since the phase separation surface may be curved in general.

The expansion (2.6) for the energy-momentum tensor (as well as similar expansions for all other conserved currents) is in fact not the most general one. Generally speaking, $T_\mu{}^\nu$ could contain terms proportional to derivatives of the δ function, etc. One can find the form of Eqs. (2.8) in every such case. However, we have preserved here, in the expansion of $T_\mu{}^\nu$ terms proportional to the δ function only. We shall dwell upon the same problem in the next section as well.

In order to include the back reaction of moving matter in the manifold, we need additional equations for dynamical variables describing the spacetime. In the next section we shall consider only the Einstein equations for the metric tensor. Equations (2.8) are obviously valid for an arbitrary theory of the spacetime manifold.

A. The Einstein equations for thin shells

To begin with, let us write the Einstein equations for the metric tensor $g_{\mu\nu}$ of spacetime [we use the signature $(+\,-\,-\,-)$ and units $\hbar=c=1$; the gravitational constant is $\kappa=M_{\rm Pl}{}^{-2}$, where $M_{\rm Pl}=1.2\times10^{19}$ GeV is the Planck mass]:

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=8\pi T_{\mu\nu}/M_{\rm Pl}{}^2 \ . \tag{2.9}$$

Here $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the energy-momentum tensor of matter fields, $R_{\mu\nu}$ being expressed in terms of the Christoffel connections $\Gamma_{\mu\nu}{}^\lambda$ as

$$R_{\mu\nu}=\partial_{[\rho}\Gamma_{\mu]\nu}^{\rho}+\Gamma_{\rho[\sigma}^{\sigma}\Gamma_{\mu]\nu}^{\rho} \tag{2.10}$$

and

$$\Gamma_{\mu\nu}{}^\lambda=\tfrac{1}{2}g^{\lambda\sigma}(g_{\sigma\mu,\nu}+g_{\sigma\nu,\mu}-g_{\mu\nu,\sigma}) \ . \tag{2.11}$$

We intend to apply the Einstein equations to thin boundaries of phase separation using the thin-wall approximation. One must realize that when speaking about a thin shell we always keep in mind that the thickness of the shell we are dealing with must not be smaller than $M_{\rm Pl}{}^{-1}$, provided gravity is described just by the classical Einstein equations. Working in the framework of the Einstein equations only, one can treat the problem in various ways.

(1) Let the geometry of spacetime be given, including the form of the phase separation surface. One can then find from the Einstein equations the corresponding energy-momentum distribution in the space; in particular, find the quantity $S_i{}^j$.

(2) On the other hand, one can consider that the structure of tensors $T_\mu{}^\nu$ and $S_i{}^j$ is given (in other words, both the equation of state of matter and the symmetry inherent to the problem are given). One can thus find from the Einstein equations (using in addition initial conditions) the geometry of spacetime.

Here we shall deal just with this latter treatment of the problem. We shall imply that the structure of the energy-momentum tensor, in the case of the cosmological phase transitions we are interested in, can be somehow found from a given field theory.

If $T_\mu{}^\nu$ entering the right-hand sides of the Einstein equations (2.9) contains first derivatives of the δ function, then connection coefficients (the derivatives of which enter the curvature tensor) should contain the δ function. Furthermore, if the spacetime manifold has the usual Riemannian structure [i.e., if the connections are related to metric coefficients in the usual manner (2.11)] then the metric coefficients should be discontinuous on the surface of phase separation. Therefore, Einstein gravity becomes inapplicable in this case. Such singularities in $T_\mu{}^\nu$ may arise, for example, in supergravity in eleven dimensions if one tries to construct a surface separating two phases with different values of parameters characterizing any given compactification down to $M^4\times S^7$. However, the terms $\sim R^2$ in the action lead to higher derivatives of the metric in the equations of motion and one can compensate δ' in $T_\mu{}^\nu$ with a continuous metric. Though consideration of a situation with derivative of the δ function in $T_\mu{}^\nu$ is of indubitable interest (it might even happen that in a number of theories one will have to consider such hypersurfaces), we shall nevertheless restrict ourselves in this paper to the study of the problem with the energy-momentum tensor having singularities no stronger than those given by δ functions.

Thus, let there be given a (pseudo-)Riemannian manifold ${}^{(4)}V$ with a three-dimensional hypersurface ${}^{(3)}\Sigma\subset{}^{(4)}V$ dividing it into two parts: V^+ and V^-. If all components of the tensor $S_\mu{}^\nu$ [see (2.6)] are equal to zero, then we shall call the corresponding hypersurface of phase separation a nonsingular shell. First derivatives of the metric $g_{\mu\nu}$ are in this case continuous on Σ, though second derivatives could be discontinuous. If on the contrary some of the quantities $S_\mu{}^\nu$ are not equal to zero, then we shall call the corresponding hypersurface a singular shell. In this case, first derivatives of $g_{\mu\nu}$ are discontinuous on Σ while the metric itself is still continuous on Σ.

1. The first junction

Let $\{y^\mu\}^+$ ($\{y^\mu\}^-$) be an arbitrary coordinate system in the V_+ (V_-) region (the greek indices take four values and numerate components of four-dimensional tensors while latin indices take three values and numerate components of the three-dimensional tensors of the hyperspace Σ), and $\{x^i\}$ be an arbitrary coordinate system on Σ. The metric $g^{+}_{\mu\nu}$ ($g^{-}_{\mu\nu}$) determines the geometry on the V_+ (V_-) region:

$$ds^2 = g_{\mu\nu} dy^\mu dy^\nu \; . \tag{2.12}$$

Before proceeding to the Einstein equations on the shell, one has to match continuously $g^{+}_{\mu\nu}$ and $g^{-}_{\mu\nu}$ on Σ. We shall call such a procedure the first junction. Let the equation of the hypersurface found in the coordinates y^+ be $F^+(y^+)=0$, while in the coordinates y^- let it be $F^-(y^-)=0$. Introduce new coordinates $(n,x^i)^{\pm}$ in such a way that the surfaces $n^{\pm}=0$ coincide with the surfaces $F^{\pm}(y^{\pm})=0$, respectively. Since we have four arbitrary functions of coordinate transformations in each region, we may reduce the metric to the form

$$\begin{aligned} ds^2 &= \epsilon\, dn^{+2} + \gamma^{+}_{ij}(x^+, n^+) dx^{+i} dx^{+j} \; , \\ ds^2 &= \epsilon\, dn^{-2} + \gamma^{-}_{ij}(x^-, n^-) dx^{-i} dx^{-j} \; , \end{aligned} \tag{2.13}$$

where $\gamma^{\pm}_{ij}$ are functions of $F^{\pm}$. The condition of the first junction will be satisfied if there exists on the junction surface $n=0$ a transformation of coordinates $x^+ = x^+(x^-)$ such that

$$\gamma^{+}_{ij}(x^+,0) = \gamma^{-}_{kl}(x^-,0) \frac{\partial x^{-k}}{\partial x^{+i}} \frac{\partial x^{-l}}{\partial x^{+j}} \; . \tag{2.14}$$

Given the two metrics $g^{+}_{\mu\nu}$ and $g^{-}_{\mu\nu}$, can we find such a hypersurface Σ where the first junction is possible?

For a spacetime manifold of dimension N, Eqs. (2.14) give $N(N-1)/2$ conditions. On the other hand, there are only $N+1$ unknown functions: F^+, F^-, $x^{+i}(x^-)$. Thus, if the spacetime in question has the dimensionality $N=4$, then even the first junction is by no means always possible. Moreover, recall that one must also obey the Einstein equations. Therefore, we shall investigate the problem of interest in two following treatments: (1) Metrics $g^{-}_{\mu\nu}$ and $g^{+}_{\mu\nu}$ possessing a high symmetry are given which reduce the number of junction conditions. (2) The metric $g^{-}_{\mu\nu}$ is matched to the metric $g^{+}_{\mu\nu}$. In other words, the junction equations provide boundary conditions for the field equations in the V^- region.

After the first junction is carried out one can write the metric on the whole manifold in the form

$$ds^2 = \epsilon\, dn^2 + \gamma_{ij}(x,n) dx^i dx^j \; . \tag{2.15}$$

We shall call such coordinates the Gaussian normal coordinates. Since $n=0$ is the equation of the hypersurface Σ to be found, the interval

$$dl^2 = \gamma_{ij}(x,0) dx^i dx^j \tag{2.16}$$

determines the geometry on Σ. In what follows, the explicit expression for the coordinate $n = n(y^\mu)$ will not be necessary for us, though we shall use Gaussian coordinates. All we need is the unit vector of the outer normal to Σ given by Eqs. (2.2) and (2.4). (In the vicinity of Σ the coordinates $\tilde{n}$ and n coincide.)

A slicing of the manifold by the surfaces $n=$const leads to a corresponding decomposition of both vectors and tensors. So a vector A^μ is decomposed naturally giving the normal component A^n (A_n) and the tangential components A_i. At the given decomposition, A^n is a scalar while A_i is a three-dimensional vector with respect to transformations of coordinates x^k on the surfaces $n=$const. Similar decompositions take place for all tensors. For example, a second-rank tensor $Q_{\mu\nu}$ is decomposed as a scalar Q^{nn}, two three-vectors Q^n_i and Q^j_n, and a three-tensor Q_{ij}. If a vector A_μ is given in arbitrary coordinates $\{y^\mu\}$ then

$$\begin{aligned} A^n &= \frac{\partial n}{\partial y^\mu} A^\mu = n_{,\mu} A^\mu \; , \\ A_n &= \epsilon A^n, \quad A_i = \frac{\partial y^\mu}{\partial x^i} A_\mu \; . \end{aligned} \tag{2.17}$$

For a second-rank tensor $Q_{\alpha\beta}$ we obtain

$$\begin{aligned} & Q^{nn} = n_{,\alpha} n_{,\beta} Q^{\alpha\beta} \\ & Q^n_n = \epsilon Q^{nn}, \quad Q_{nn} = Q^{nn} \; , \\ & Q^n_i = n_{,\alpha} \frac{\partial y^\beta}{\partial x^i} Q^\alpha_\beta, \quad Q^j_n = n^\beta \frac{\partial x^j}{\partial y^\alpha} Q^\alpha_\beta \; , \\ & Q_{ij} = (\partial y^\alpha / \partial x^i)(\partial y^\beta / \partial x^j) Q_{\alpha\beta} \; . \end{aligned} \tag{2.18}$$

Note that the above-written expression for A^n allows one to choose the sign of the normal. We shall call the normal the outer one if the normal vector $n_{,\alpha}$ has the direction from V^- to V^+. Then, choosing the surfaces $n=$const in such a way that the values $n<0$ correspond to V^- ($n>0$ for V^+) one finds that the contravariant components of a unit outer normal vector should be

$$\mathcal{N}^n = +1, \quad \mathcal{N}^i = 0, \quad \mathcal{N}_i = 0, \quad \mathcal{N}_n = \epsilon \; . \tag{2.19}$$

Hence

$$\mathcal{N}_\alpha = \epsilon n_{,\alpha} \; .$$

2. *The Einstein equations*

In the Gaussian system of coordinates (2.15) the components of the Christoffel symbols containing two or three indices n are equal to zero. Components not containing indices n at all are regular, since the three-dimensional geometry of the surface Σ is by assumption well defined. Thus, only those connection coefficients which contain just one index n, namely,

$$\Gamma^n_{ij} = -\tfrac{1}{2}\epsilon \gamma_{ij,n} \quad \text{and} \quad \Gamma^i_{jn} = \tfrac{1}{2}\gamma^{il}\gamma_{lj,n} \; , \tag{2.20}$$

are discontinuous at crossing Σ. One can write the expansions

$$\begin{aligned} \Gamma^n_{ij} &= \Gamma_{ij}^{+n} \theta(n) + \Gamma_{ij}^{-n} \theta(-n) + \tilde{\Gamma}^n_{ij} \; , \\ \Gamma^i_{jn} &= \Gamma^{i\;+}_{jn} \theta(n) + \Gamma^{i\;-}_{jn} \theta(-n) + \tilde{\Gamma}^i_{jn} \; , \end{aligned} \tag{2.21}$$

where $\tilde{\Gamma}$ are the corresponding regular parts. Substituting these expansions into the Einstein equations and using the expansion (2.6) for $T_\mu{}^\nu$ written in Gaussian coordinates, one easily finds that

$$S_\mu{}^n = 0 \tag{2.22}$$

for any μ [note that this is just Eq. (2.8a)] and

$$(\Gamma_{ij}^{n}-\gamma_{ij}\Gamma_{kl}^{n}\gamma^{kl})^{+}-(\ )^{-}=8\pi\kappa S_{ij}\ . \tag{2.23}$$

These are just the Einstein equations on the shell. They can be written in the more convenient form provided one takes into account that the covariant derivative of the unit normal vector to Σ is, in accordance with Eq. (2.19), given by

$$\mathcal{N}_{i;j}=-\epsilon\Gamma_{ij}^{n}\ . \tag{2.24}$$

The quantity $K_{ij}\equiv-\mathcal{N}_{i;j}=\epsilon\Gamma_{ij}^{n}$ is called the outer curvature tensor of the surface Σ. Since the quantity $\mathcal{N}_{\mu;\nu}$ is a tensor, one can find it in an arbitrary convenient system of coordinates $\{y^{\mu}\}$; then K_{ij} can be found using the formula

$$K_{ij}=-\frac{\partial y^{\alpha}}{\partial x^{i}}\frac{\partial y^{\beta}}{\partial x^{j}}\mathcal{N}_{\alpha;\beta}\ . \tag{2.25}$$

In this notation Eqs. (2.23) take the form

$$\epsilon([K_i{}^j]-\delta_i{}^j[K_l{}^l])=8\pi\kappa S_i{}^j\ , \tag{2.26}$$

where

$$[K_i{}^j]\equiv K_i{}^{j+}-K_i{}^{j-}$$

is the discontinuity of the outer curvature tensor. Noting also that $K_i{}^j=-\Gamma_{ni}^{j}$ one can rewrite $\binom{i}{n}$ and $\binom{n}{n}$ components of the Einstein equation as

$$-\epsilon(K_i{}^j{}_{|j}-K_l{}^l{}_{|i})=8\pi\kappa T_i{}^n\ , \tag{2.27}$$

$$-\tfrac{1}{2}{}^{(3)}R-\tfrac{1}{2}K_j{}^i\epsilon(K_i{}^j-\delta_i{}^jK_l{}^l)=8\pi\kappa T_n{}^n\ , \tag{2.28}$$

where ${}^{(3)}R$ is the three-curvature of the hypersurface Σ and the vertical bar denotes covariant differentiation with respect to the metric on Σ. Let us write each of these equations in the regions V^+ and V^- and then subtract the corresponding equations for the V^+ region from those for the V^- region. Using Eq. (2.26) we can write as a result

$$S_i{}^j{}_{|j}+[T_i{}^n]=0\ , \tag{2.29}$$

$$\{K_j{}^i\}S_i{}^j+[T_n{}^n]=0\ , \tag{2.30}$$

where

$$\{K_j{}^i\}=\tfrac{1}{2}(K_j{}^{i+}+K_j{}^{i-})\ .$$

But just the same form is taken by the i and n components of Eq. (2.8b) provided they are written in Gaussian coordinates. Thus, these equations are consequences of the Einstein equations for V^+, V^- regions and on the shell.

Now we see the way of solving various problems.

(1) If the metrics in V^+ and in V^- and the functions $S_i{}^j(x)$ are known, then the only nontrivial equations of the second junction are the Einstein equations on the shell (2.26).

(2) A calculation of $S_i{}^j(x)$ is a complicated field-theoretical problem. However, in a number of cases one can find $S_i{}^j(x)$ using Eq. (2.29), $[T_i{}^n]$ being given and the value of $S_i{}^j$ being determined (from a field theory) only on a certain two-dimensional boundary lying on Σ.

(3) Of the most interest is a treatment of the problem when there is no fixed matter state in one of the regions (for example, the state of the medium inside a new phase bubble is not given). In this case, $S_i{}^j$ has to be determined completely from a field theory [generally speaking as a functional of $F(y)$]. Since a metric in one of the regions is unknown (say, in V^-), one cannot calculate $K_i{}^{j-}$ immediately and therefore it is convenient to find $K_i{}^j$ from Eq. (2.26) and then substitute into Eq. (2.30). Then one obtains

$$S_i{}^jK_j{}^{i+}-\epsilon 4\pi\kappa S_i{}^j(S_j{}^i-\tfrac{1}{2}\delta_j{}^iS_l{}^l)+[T_n{}^n]=0\ . \tag{2.31}$$

Equation (2.29) for $[T_i{}^n]$ does not change, since it contains only quantities characterizing the inherent geometry of the shell.

B. Transition to the limit $M_{\rm Pl}\to\infty$

Equations (2.29) and (2.30) are valid both in the case when one takes into account the influence of the gravitational field on the shell motion and the back reaction of the shell energy-momentum tensor $S_i{}^j$ on the spacetime geometry and in the case when one considers the shell motion in arbitrary smooth curved spacetime (in particular, in the flat spacetime) not accounting for gravity. Neglecting the influence of the shell on the metric, one obtains the following equations of motion of the shell in an arbitrary curved spacetime:

$$K_j{}^iS_i{}^j+[T_n{}^n]=0\ , \tag{2.32a}$$

$$S_i{}^j{}_{|j}+[T_i{}^n]=0\ . \tag{2.32b}$$

This is of course nothing but Eq. (2.8a) rewritten in Gaussian coordinates. In this case obviously

$$[K_j{}^i]=0\ . \tag{2.33}$$

C. Spherical shells

For the sake of simplicity we shall consider in the rest of the paper spherical shells only. Let us derive corresponding formulas for this particular case. An interval can be represented now as

$$ds^2=g_{00}dt^2+2g_{01}dt\,dq+g_{11}dq^2-r^2d\Omega^2\ , \tag{2.34}$$

where

$$d\Omega^2=d\theta^2+\sin^2\theta\,d\varphi^2\ ,$$

while g_{00}, g_{01}, g_{11} and r are functions of q and t only. One can proceed in the metric (2.34) to the new variables q and t related to the old variables through transformations

$$t=t(t,q),\quad q=q(t,q)\ , \tag{2.35}$$

which do not include angular variables. Therefore, one can use two coordinate conditions without loss of generality. If one of these conditions is the orthogonality requirement

$$g_{01}=0\ , \tag{2.36}$$

the metric then takes the form

$$ds^2=e^{\nu}dt^2-e^{\lambda}dq^2-r^2(q,t)d\Omega^2 \ . \qquad (2.37)$$

We choose the coordinate q originating from the center of a bubble on the outside.

In curved spacetime, the normal vector to the surface $r=\text{const}$ may be spacelike as well as timelike. In the first case,

$$\Delta \equiv g^{\alpha\beta}r_{,\alpha}r_{,\beta}<0 \qquad (2.38)$$

and the corresponding region is called the R region[28,17] (in flat space, the R region occupies the whole space). In the second case,

$$\Delta>0 \ . \qquad (2.39)$$

Such a region is called the T region.[28,17] In a chosen coordinate frame (2.37) we have

$$\Delta=e^{-\nu}\dot{r}^2-e^{-\lambda}r'^2 \ . \qquad (2.40)$$

As far as $\Delta>0$ in the T region, it is impossible to satisfy the condition $\dot{r}=0$ there. That is, in the T region either $\dot{r}>0$ or $\dot{r}<0$ remains true under any continuous coordinate change. The region where $\dot{r}>0$ we shall call the T region of expansion (T_+ region), while the region where $\dot{r}<0$ we shall call the T region of contraction (T_-). Similarly, the sign of $r'\equiv dr/dq$ does not depend upon the coordinate chosen in the R region (remember that q originates from the center of a bubble). We shall call R_+ the region with $r'>0$, while R_- that with $r'<0$. The metric on the shell is determined by

$$ds^2\,|_{\Sigma}=-\epsilon\, dz^2-\rho^2(z)d\Omega^2 \ , \qquad (2.41)$$

where z in the case of the timelike shell ($\epsilon=-1$) is the proper time (which we shall denote by τ) measured by an observer at rest with respect to this shell, while in the case of the spacelike shell, z is the distance from the center of the sphere of the radius $\rho(z)$ (which we shall denote by ξ). We shall choose the same angular variables in the inner and outer regions and on the shell. We shall use indices 2 and 3 for them, while the indices 0 and 1 will be used on the shell for τ and ξ, respectively.

It is sufficient to derive explicitly all necessary formulas only for the TL junction. The corresponding junction equations (in their final form) for the SL hypersurface can be obtained by the change in the indices $0\to 1$ and the substitution $\tau\to i\xi$, $S_0{}^0\to -iS_1{}^1$.

1. *Equations of motion*

In the spherically symmetric case $A_2{}^2=A_3{}^3$ for any second-rank tensor and the junction equation (2.26) take the form

$$\epsilon 4\pi\kappa S_0{}^0=[K_2{}^2] \ , \qquad (2.42a)$$

$$\epsilon 8\pi\kappa S_2{}^2=[K_0{}^0]+[K_2{}^2] \ , \qquad (2.42b)$$

and Eqs. (2.29) and (2.30) become

$$\{K_0{}^0\}S_0{}^0+2\{K_2{}^2\}S_2{}^2+[T_n{}^n]=0 \ , \qquad (2.43a)$$

$$dS_0{}^0/d\tau+2\dot{\rho}(S_0{}^0-S_2{}^2)/\rho+[T_0{}^n]=0 \ , \qquad (2.43b)$$

where $\dot{\rho}\equiv d\rho/d\tau$. With the four-dimensional energy-momentum tensor, $T_\alpha{}^\beta$, given, the components $T_n{}^n$ and $T_0{}^n$ can be evaluated using Eqs. (2.18).

Our main purpose now will be to determine the outer curvature tensor $K_i{}^j$.

2. *The shell outer curvature tensor $K_i{}^j$*

Because of the factorization of the two-sphere metric, the components of the three-tensor $A_i{}^j$ on the shell are invariants as long as we keep the angular variables θ and φ fixed. So, we can compute $K_i{}^j$ using any convenient coordinates in the whole spacetime and then reexpress it via the invariants. Let us choose now the coordinates in (2.37), so that

$$q=r \ . \qquad (2.44)$$

Then it is convenient to represent the equation of a timelike shell in the coordinates of the V^- (V^+) region in the form

$$F(y^\alpha)=r-R(t)=0 \ , \qquad (2.45)$$

where $R(t)$ is a certain unknown function. One can easily satisfy oneself that the first junction in this case is always possible.

The condition of metric continuity on the hypersurface Σ results in the relation between the time of the inner (outer) region and the proper time on the shell

$$R(t)=\rho(\tau) \ , \qquad (2.46a)$$

$$\left[e^{\nu}-e^{\lambda}\left[\frac{dR}{dt}\right]^2\right]\left[\frac{dt}{d\tau}\right]^2=1 \ ; \qquad (2.46b)$$

hence,

$$\left[\frac{dt}{d\tau}\right]^2=e^{\lambda-\nu}(\dot{\rho}^2+e^{-\lambda}) \ , \qquad (2.47a)$$

$$dR/dt=\dot{\rho}\exp\left[\frac{\nu-\lambda}{2}\right]\Big/(\dot{\rho}^2+e^{-\lambda})^{1/2} \ . \qquad (2.47b)$$

Thus, after the procedure of the first junction is carried out with given $\lambda^\pm$ and $\nu^\pm$, there remains one unknown function $\rho(\tau)$ which should obey the Einstein equations.

We shall regard for definiteness the inner region of the bubbles as the V_- region and the outer as the V_+ region, respectively; then [see Eq. (2.2)]

$$\tilde{n}=\sigma\frac{r-R(t)}{|g^{\alpha\beta}F_{,\alpha}F_{,\beta}|^{1/2}} \ , \qquad (2.48)$$

since radii may either increase in the direction of the outer normal ($\sigma=+1$, which is always the case for a flat space) or decrease ($\sigma=-1$, in an R region this corresponds to a semiclosed world). The vector of the outer normal to the shell is

$$\mathcal{N}_\alpha=\epsilon\tilde{n}_{,\alpha}\,|_{\Sigma} \ , \qquad (2.4')$$

and

$$\mathcal{N}_2=\mathcal{N}_3=0,\quad \mathcal{N}_0=-\left[\frac{dR}{dt}\right]\mathcal{N}_1\ , \tag{2.49a}$$

$$\mathcal{N}_1=\sigma\left[e^{-\lambda}-\left[\frac{dR}{dt}\right]^2 e^{-\nu}\right]^{-1/2}=\sigma e^{\lambda}(\dot\rho^{\,2}+e^{-\lambda})^{1/2}\ . \tag{2.49b}$$

We shall calculate now the outer curvature tensor of the shell (2.45) in the metric (2.37) with condition (2.44) using formula (2.25).

The $K_2{}^2$ component can be easily calculated

$$K_{22}=\left[\Gamma_{22}{}^1-\frac{dR}{dt}\Gamma_{22}{}^0\right]\mathcal{N}_1=re^{-\lambda}\mathcal{N}_1\ ,$$
$$K_2{}^2=-\frac{\sigma}{\rho}(\dot\rho^{\,2}+e^{-\lambda})^{1/2}\ . \tag{2.50}$$

Calculation of the $K_0{}^0$ component is more cumbersome; therefore we give only the final result:

$$K_0{}^0=-\sigma\left[\left[\ddot\rho+\dot\rho^{\,2}\frac{\lambda'+\nu'}{2}+\frac{\nu'}{2}e^{-\lambda}\right]\Big/(\dot\rho^{\,2}+e^{-\lambda})^{1/2}+\dot\lambda\dot\rho e^{(\lambda-\nu)/2}\right]\ , \tag{2.51}$$

where

$$\lambda'\equiv\frac{\partial\lambda}{\partial r},\quad \nu'\equiv\frac{\partial\nu}{\partial r},\quad \dot\lambda\equiv\frac{\partial\lambda}{\partial t},\quad \dot\rho\equiv d\rho/d\tau\ .$$

Now we want to rewrite the outer curvature tensor in terms of invariants. There is one invariant without derivative at our disposal: namely, the "radius" $r=r(t,q)$ $[=\rho(\tau)]$ on the shell. It is clear from expressions (2.50) and (2.51) that we need only those invariants derived from r which, being expressed in coordinates (2.44), involve derivatives up to the first order. Those invariants are Δ [see (2.38) and (2.40)] and

$$\Delta_1=(g^{\alpha\beta}r_{,\beta})_{;\alpha},\quad \Delta_2=\Delta_{,\alpha}r_{,\beta}g^{\alpha\beta}\ ,$$
$$\Delta_3=\Delta_{,\alpha}\Delta_{,\beta}g^{\alpha\beta}\ . \tag{2.52}$$

We have, in coordinates (2.44),

$$e^{-\lambda}=-\Delta,\quad \dot\lambda^2e^{\lambda-\nu}=\frac{\Delta_2{}^2}{\Delta^4}-\frac{\Delta_3}{\Delta^3}\ ,$$
$$\frac{\nu'}{2}e^{-\lambda}=-\Delta_1+\frac{2\Delta}{\rho}+\frac{1}{2}\frac{\Delta_2}{\Delta}\ , \tag{2.53}$$
$$\frac{\nu'+\lambda'}{2}=\frac{\Delta_1}{\Delta}-\frac{\Delta_2}{\Delta^2}-\frac{2}{\rho}\ .$$

So, we obtain the following formulas for $K_2{}^2$ and $K_0{}^0$, which are valid in any coordinate frame:

$$K_2{}^2=-\frac{\sigma}{\rho}(\dot\rho^{\,2}-\Delta)^{1/2}\ , \tag{2.54a}$$

$$K_0{}^0=-\sigma\left[\frac{\ddot\rho-\Delta_2/2\Delta}{(\dot\rho^{\,2}-\Delta)^{1/2}}+\left[\frac{\Delta_1}{\Delta}-\frac{\Delta_2}{\Delta^2}-\frac{2}{\rho}\right](\dot\rho^{\,2}-\Delta)^{1/2}+\frac{\dot\rho}{\Delta^2}(\Delta_2{}^2-\Delta_3\Delta)^{1/2}\right]\ . \tag{2.54b}$$

Note that the outer curvature tensor given by Eqs. (2.50) and (2.51) has been calculated in coordinates (2.44) valid in an R region only. But the final results, Eqs. (2.54), are valid everywhere. The tensor $K_i{}^j{}_{(\mathrm{SL})}$ for the SL hypersurface can be obtained form $K_i{}^j(\rho,\dot\rho,\ddot\rho)_{\mathrm{TL}}$ by the substitution

$$K_i{}^j(\rho,\rho',\rho'')_{\mathrm{SL}}=iK_i{}^j(\rho,-i\rho',-\rho'')_{\mathrm{TL}}\ .$$

Using the Einstein equations (see Appendix A) we can simplify the above expression for $K_0{}^0$. Indeed, from Eqs. (A9) and (A11) we easily derive the following formulas for Δ's:

$$\Delta_1=4\pi\kappa\rho T+\frac{\Delta-1}{\rho}\ , \tag{2.55}$$

$$\Delta_2=8\pi\kappa\rho(T\Delta-T^{\alpha\beta}\rho_{,\alpha}\rho_{,\beta})-\frac{\Delta(1+\Delta)}{\delta}\ , \tag{2.56}$$

$$\Delta_3=64\pi^2\kappa^2\rho^2(T^2\Delta-2TT^{\alpha\beta}\rho_{,\alpha}\rho_{,\beta}+T_\alpha{}^\gamma T^{\gamma\delta}\rho_{,\gamma}\rho_{,\delta})-16\pi\kappa(1+\Delta)(T\Delta-T^{\alpha\beta}\rho_{,\alpha}\rho_{,\beta})+\frac{\Delta(1+\Delta)^2}{\rho^2}\ . \tag{2.57}$$

Inserting this into Eq. (2.54b) we arrive, after some algebra, at the expression

$$K_0{}^0=-\frac{\sigma}{(\dot\rho^{\,2}-\Delta)^{1/2}}\left[\ddot\rho+\frac{1+\Delta}{2\rho}-4\pi\kappa\rho T_n{}^n\right]\ . \tag{2.58}$$

Now it easy to show that Eq. (2.43a) is in fact an algebraic combination of Eqs. (2.43a) and (2.43b). So, we are left with the following set of equations:

$$\epsilon 4\pi\kappa S_0{}^0=[K_2{}^2]\ , \tag{2.59a}$$

$$\epsilon\pi\kappa S_2{}^2=[K_0{}^0]+[K_2{}^2]\ , \tag{2.59b}$$

$$\frac{dS_0{}^0}{d\tau}+2\dot\rho(S_0{}^0-S_2{}^2)/\rho+[T_0{}^n]=0\ ; \tag{2.60}$$

the last equation can be viewed as an integrability condition for Eqs. (2.59).

D. Surface energy-momentum tensor on the shell

The derived equations are valid for any spherical symmetric shell. To proceed further one needs to set the tensor $S_j{}^i$.

Let us find the surface energy momentum tensor for the bubbles arising in cosmological phase transitions. The order parameter in such transitions is the appropriately defined average of the scalar field operator $\hat\varphi$. In equilibrium, in absence of sources, $\bar\varphi=\langle\hat\varphi\rangle$ does not depend on coordinates.

The field $\bar{\varphi}$ obeys the equations of motion following from the effective action. In particular, the field that is not dependent upon coordinates ($\bar{\varphi}$) is determined from the condition of the effective potential extremum on this field. We consider the situation when the effective potential has several local minima. The lowest minimum corresponds to the ground (equilibrium) state of the system, the rest correspond to metastable states. In the case of a spherical bubble of one phase surrounded by another phase the value of the field $\bar{\varphi}$ at the center of the bubble coincides with the value $\bar{\varphi}_-$ of the field in one of the local minima of potential, while at $\rho \to \infty$ it coincides with the value $\bar{\varphi}_+$ in other local minima. In general, the field $\bar{\varphi}$ has m components and transforms according to some representation of gauge group G. The group G is broken down to $G_- \subset G$ by the value of the field $\bar{\varphi}_-$ and it is broken down to $G_+ \subset G$ by the value $\bar{\varphi}_+$. The subgroup G_+ (G_-) may coincide with G if $\bar{\varphi}_+ = 0$ ($\bar{\varphi}_- = 0$). The value of $\bar{\varphi}$ in the potential minimum determines the corresponding scale of violation of the group G: $M_G \sim \bar{\varphi}$. In modern grand unified theories (GUT's) several such scales are presented. Namely, one usually has at least two scales: the scale of violation of the unifying group $M_G \equiv M_X \sim 10^{14}-10^{18}$ GeV and the scale of breaking of the electroweak group $SU(2)_L \times U(1)$, $M_G \equiv M_W \sim 10^2$ GeV. Furthermore, we may consider phase transitions with "hadronization" as well, as in QCD. Then $\bar{\varphi}$ is the value of the $q\bar{q}$ or gluon condensates (in QCD, $\langle q\bar{q} \rangle \sim 1$ GeV). Such transitions occur in any theory when temperature falls down to the strong-coupling regime value.

Consider as a scaled down example the bubble arising in the minimal SU(5) model.[29] The unifying group SU(5) is violated by the vacuum expectation value of the 24-component field Υ from the adjoint representation of the group. The field Υ may be represented by traceless matrices 5×5.

Consider for definiteness a bubble of the $SU(4) \times U(1)$ phase in the $SU(3) \times SU(2) \times U(1)$-symmetric vacuum.[27] Then, inside the bubble, the field Υ has the form $\bar{\Upsilon} = \mathrm{diag}(u,u,u,u,-4u)$ while outside it is $\bar{\Upsilon} = \mathrm{diag}(u,u,u,-3u/2,-3u/2)$, where $u \sim M_X$.

In the transient layer $\bar{\Upsilon} = \mathrm{diag}(u,u,u\ -(\tfrac{3}{2}-\epsilon)u, -(\tfrac{3}{2}+\epsilon)u)$ with the boundary conditions $\epsilon = \tfrac{5}{2}$ at $\rho = 0$ and $\epsilon = 0$ at $\rho \to \infty$, respectively. The electroweak group is broken in the framework of the SU(5) model by the five-component field $\bar{H} \sim M_W$.

Let us proceed now to the derivation of the energy-momentum-tensor surface density on the shells of bubbles arising in such phase transitions.

The scalar field $\hat{\varphi}$ is described by the Lagrangian density of the form

$$\mathcal{L} = (\mathcal{D}_\mu \hat{\varphi})^* (\mathcal{D}^\mu \hat{\varphi}) - (1-\xi)\frac{R}{6}\hat{\varphi}^* \hat{\varphi} - V(\hat{\varphi}) - \tfrac{1}{4} F_{\mu\nu}{}^a F^{a\mu\nu} ,$$

$$\mathcal{D}_\mu \equiv \partial_\mu - ig A_\mu{}^a T^a , \tag{2.61}$$

$$F_{\mu\nu}{}^a \equiv \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g f^{abc} A_\mu{}^b A_\nu{}^c ,$$

where g is the gauge coupling constant of the group G, T^a are generators in the representation of the fields $\hat{\varphi}^i$, f^{abc} are structure constants of the group, R is the scalar curvature, ξ is the coupling constant of the scalar field with the background metric ($\xi = 0$ for conformal coupling and $\xi = 1$ for the minimal coupling, respectively). The potential $V(\hat{\varphi})$ is a G-invariant polynomial of degree not higher than 4 in the fields $\hat{\varphi}^i$.

Using the formula

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \left[\frac{\partial \sqrt{-g}\, \mathcal{L}}{\partial g^{\mu\nu}} \right] - \frac{2}{\sqrt{-g}} \left[\frac{\partial \sqrt{-g}\, \mathcal{L}}{\partial g^{\mu\nu}{}_{,\alpha}} \right]_{,\alpha} , \tag{2.62}$$

where $g = \det(g_{\mu\nu})$,

$$\partial \sqrt{-g} / \partial g^{\mu\nu} = -\tfrac{1}{2}\sqrt{-g}\, g_{\mu\nu} ,$$

we obtain for the energy-momentum tensor

$$\begin{aligned} T_{\mu\nu} = {} & (\mathcal{D}_\mu \hat{\varphi})^* (\mathcal{D}_\nu \hat{\varphi}) - \tfrac{1}{2} g_{\mu\nu} (\mathcal{D}^\alpha \hat{\varphi})^* (\mathcal{D}_\alpha \hat{\varphi}) \\ & + \tfrac{1}{3}(\xi - 1)[(\partial_\mu \hat{\varphi})^* (\partial_\nu \hat{\varphi}) - g_{\mu\nu} (\partial^\alpha \hat{\varphi})^* (\partial_\alpha \hat{\varphi}) + \hat{\varphi}^* \nabla_\mu \partial_\nu \hat{\varphi} - g_{\mu\nu} \hat{\varphi}^* \nabla_\alpha \nabla^\alpha \hat{\varphi} + \tfrac{1}{2} G_{\mu\nu} \hat{\varphi}^* \hat{\varphi}] \\ & + \tfrac{1}{2} V(\hat{\varphi}) g_{\mu\nu} + \mathrm{H.c.} + \tfrac{1}{4} F_{\lambda\kappa}{}^a F^{a\lambda\kappa} g_{\mu\nu} - F_{\mu\kappa}{}^a F^{a}{}_{\nu}{}^{\kappa} . \end{aligned} \tag{2.63}$$

It is clear that before the transition to the limit of vanishing thickness of the layer separating two phases with different values of $\bar{\varphi}$ we have to choose as a Gaussian normal system of coordinates, the system in which the surfaces of the constant value of coordinate n coincide with the surfaces of constant values of $\bar{\varphi}$. Let x^i be the coordinates on the surfaces $n = \text{const}$; then $\partial \bar{\varphi} / \partial x^i = 0$. Let us define the field $\hat{\phi}$ as follows: $\hat{\varphi} = \bar{\varphi} + \hat{\phi}$, i.e., $\langle \phi \rangle = 0$. Substituting $\hat{\varphi} = \bar{\varphi} + \hat{\phi}$ into Eq. (2.63) we find (we set here and in what follows $\xi = 1$)

$$T_{ij} = (\mathcal{D}_i \hat{\phi})^* (\mathcal{D}_j \hat{\phi}) - \tfrac{1}{2} F_{i\kappa}{}^a F_j^{a\kappa} + \tfrac{1}{2} g_{ij} T(\bar{\varphi}, \phi) + \mathrm{H.c.} ,$$

$$T \equiv V(\hat{\varphi}) + (\mathcal{D}_n \bar{\varphi})^* (\mathcal{D}_n \bar{\varphi}) + \tfrac{1}{4} F_{\lambda\kappa}{}^a F^{a\lambda\kappa} - (\mathcal{D}^i \hat{\phi})^* (\mathcal{D}_i \hat{\phi}) , \tag{2.64}$$

$$T_{nn} = \tfrac{1}{2} (\mathcal{D}_n \bar{\varphi})^* (\mathcal{D}_n \bar{\varphi}) - \tfrac{1}{2} V(\hat{\varphi}) + \tfrac{1}{2} (\mathcal{D}_n \hat{\phi})(\mathcal{D}_n \hat{\phi}) + \tfrac{1}{2} (\mathcal{D}_i \hat{\phi})(\mathcal{D}^i \hat{\phi}) - \tfrac{1}{2} F_{n\kappa}{}^a F_n^{a\kappa} - \tfrac{1}{8} F_{\lambda\kappa}{}^a F^{a\lambda\kappa} + \mathrm{H.c.} ,$$

where we omitted terms linear in $\hat{\phi}$ because after subsequent averaging these terms will give vanishing contribution. Averaging now the expression for T_{ij} and integrating it over n from $-\delta$ to $+\delta$, one obtains, for $S_i{}^j$ in the limit $\delta\to 0$,

$$S_{ij}=Sg_{ij}+\lim_{\delta\to 0}\int dn(\langle \mathcal{D}_i\hat{\phi}\mathcal{D}_j\hat{\phi}\rangle -\tfrac{1}{2}\langle F_{i\kappa}{}^aF_j{}^{a\kappa}\rangle+\text{H.c.}) ,$$
$$S\equiv \lim_{\delta\to 0}\int dn(\tfrac{1}{2}T+\text{H.c.}) . \tag{2.65}$$

1. The vacuum case

Neglecting field fluctuations in Eq. (2.65) (i.e., setting all correlation functions like $\langle\hat{\phi}\hat{\phi}\rangle$, etc., to be equal to zero) one obtains

$$S_i{}^j=S\delta_i{}^j . \tag{2.66}$$

We shall call such a shell a vacuum shell.

Let us find now the dependence S upon coordinates using Eq. (2.29). The vacuum energy-momentum tensor in any frame of reference is of the form $T_\mu{}^\nu=\epsilon\delta_\mu{}^\nu$, $\epsilon=$const being the vacuum energy density. Therefore, in the pure vacuum case $[T_i{}^n]=0$ and we obtain, from Eq. (2.29), $S_{,i}=0$ or

$$S=\text{const} . \tag{2.67}$$

Relation (2.67) remains valid also if the shell (2.66) is spherically symmetric and charged. Indeed, if the shell carries a charge, then in the spherically symmetric case the energy-momentum tensor has the form $T_0{}^1=T_1{}^0=0$, $T_0{}^0=T_1{}^1$, so we obtain from Eqs. (2.18b) and (2.49a) that $T_0{}^n=0$. Then from Eqs. (2.43b) and (2.66) we obtain again Eq. (2.67).

Since the quantity S does not depend on x^i (in particular, in the case of a spherical shell it does not depend upon time τ), one may find S by calculating its value at a certain point x^i. We shall not consider the calculation of this quantity in this paper; it will as a rule play the role of an input parameter of the model. Here we consider only, as a characteristic example, the neutral wall produced by one of the components of the field $\bar{\varphi}$. In this case the field $\bar{\varphi}$ configuration is described in the Gaussian frame of references by

$$\frac{1}{2}\frac{d}{d\bar{\varphi}}\left[\frac{d\bar{\varphi}}{dn}\right]^2+\Gamma_i{}^i{}_n\frac{d\bar{\varphi}}{dn}=+\frac{dV}{d\bar{\varphi}} . \tag{2.68}$$

When the transient layer is thin enough, the "singularity" of the $(d/d\bar{\varphi})(d\bar{\varphi}/dn)^2$ is stronger than that of $d\bar{\varphi}/dn$ and we may neglect the second term in the left-hand side of Eq. (2.68) (this gives by the way the criterion for validity of the thin-wall approximation). Then we obtain immediately the first integral of (2.68):

$$(d\bar{\varphi}/dn)^2=2V+C , \tag{2.69}$$

C being an integration constant. Since $d\bar{\varphi}/dn$ equals zero far from the wall we can take for C either the value $-2V(\bar{\varphi}_-)$ or $-2V(\bar{\varphi}_+)$ (remember that the thin-wall approximation is valid when $|V_--V_+|\ll V_+$).

Let us write explicitly the expressions (2.64) for the system under consideration:

$$T_{nn}=\tfrac{1}{2}(\partial_n\bar{\varphi})(\partial_n\bar{\varphi})-V(\bar{\varphi}) , \tag{2.70}$$

$$T_{ij}=g_{ij}\left[\frac{1}{2}\left[\frac{d\bar{\phi}}{dn}\right]^2+V(\bar{\varphi})\right] . \tag{2.71}$$

We see that

$$S_{nn}=\lim_{\delta\to 0}\int \tfrac{1}{2}C\,dn=0$$

as a consequence of the equation of motion (2.69) in agreement with (2.22), and

$$S_i{}^j=\delta_i{}^j\lim_{\delta\to 0}\int dn\{2V\}=\delta_i{}^j\lim_{\delta\to 0}\int dn\left\{\frac{d\bar{\varphi}}{dn}\right\}^2 . \tag{2.72}$$

Note that we can obtain $S_i{}^j$ without using the equation of motion in the form (2.68). Indeed, since we are interested only in the singular part of $T_\mu{}^\nu$, we can obtain Eqs. (2.72) just from the condition $S_{nn}=0$ using Eqs. (2.70) and (2.71). In such a way we could calculate $S_i{}^j$ but would not obtain the criterion of validity of thin-wall approximation.

The value of $S_i{}^j$ does not vanish if the field values $\bar{\varphi}_-$ and $\bar{\varphi}_+$ do not coincide, giving $d\bar{\varphi}/dn\sim\delta(n)$. Using Eq. (2.69) one can rewrite (2.72) as

$$S_i{}^j=\delta_i{}^j\left|\int_{\bar{\varphi}_-}^{\bar{\varphi}_+}d\varphi[2|V(\varphi)-V(\varphi_-)|]^{1/2}\right| , \tag{2.73}$$

Formally speaking one is integrating in expression (2.72) the uncertain quantity $\sim[\delta(n)]^2$ rather than $\delta(n)$. This uncertainty is settled by the functional relation

$$\delta^2(x)=C_1\delta(x)+C_2\delta'(x)+\cdots ,$$

the coefficient C_i being in general unknown. We can, however, find them for any given physical problem. Namely, one has to calculate first of all the integral (2.73) for a thick-wall (smoothed) field configuration and then proceed to the limit in terms of parameters of the theory leading to validity of the thin-wall approximation. Let us consider for example the potential of the form

$$V=\lambda(\varphi^2-\varphi_0{}^2)^2 . \tag{2.74}$$

We obtain in this case

$$S=\tfrac{4}{3}\sqrt{\lambda}\,\varphi_0{}^3=\frac{\sqrt{2}}{3}m_H\varphi_0{}^2 , \tag{2.75}$$

m_H being the mass of the Higgs field in the minimum of V. Since the thickness of the transient layer is $d\sim m_H{}^{-1}$, the latter becomes thin in the limit $m_H\to\infty$, so we see from (2.72) that $C_i=0$ and $C_1\sim m_H$. Similarly, the integral (2.73) can be calculated in every particular case. [When the potential does not possess the $\varphi\to-\varphi$ symmetry one may have $C_i\neq\text{const}=0$ for $i\neq 1$ but $\lim_{m_H\to\infty}(C_i/C_1)\to 0$, so one might hope that in order to obtain an approximate solution in the case $m_H\ll M_{\text{Pl}}$ one may set $C_i=0$, $i\neq 1$.] It is important, however, that by order of magnitude $S\sim M_G{}^3$, M_G being the scale of violation of the group G if only there are no special conditions imposed on coupling constants [such conditions may arise naturally in supersymmetric (SUSY) GUT's].

2. Equation of motion of pure vacuum shell

In the previous section we found that the surface energy-momentum tensor of the vacuum shell has the form $S_i{}^j=\delta_i{}^j S$ and $S=\text{const}$. Here we would like to immediately utilize this result. We shall consider again the case of a spherical bubble. Then the solution to the Einstein equations for V^+ and V^- regions are also known. The pure vacuum metric (or electrovacuum in the case of charged shells) may be written as

$$ds^2=f\,dt^2-\frac{1}{f}dr^2-r^2d\Omega^2\ , \qquad (2.76)$$

where

$$f(r)=1-\frac{8\pi\kappa}{3}\epsilon r^2-\frac{2\kappa m}{r}+\frac{e^2\kappa}{r^2}=-\Delta\ . \qquad (2.77)$$

The parameters m and e, being the Schwarzschild mass and the charge of the shell, respectively, may not equal zero for the outer metric of the bubble only, while ϵ (which is vacuum energy density) may not equal zero for both outer and inner metrics. All these quantities are constant. Thus, in the pure vacuum case one has to find only one unknown function $\rho(\tau)$ to describe a spherical shell model. Therefore, we need only one of Eqs. (2.42a), (2.42b), and (2.43b), the rest of them being satisfied identically. It is convenient to write Eq. (2.42) in the form

$$[K_2{}^2]=\frac{4\pi}{M_{\rm Pl}{}^2}S\ . \qquad (2.78)$$

Using Eq. (2.54a) one easily finds, for the metric (2.76),

$$K_2{}^2=-\frac{\sigma}{\rho}[\dot\rho^2+f(\rho)]^{1/2}\ . \qquad (2.79)$$

Thus, the motion of the vacuum bubble is described by[5]

$$\frac{\sigma_{\rm in}}{\rho}[\dot\rho^2+f_{\rm in}(\rho)]^{1/2}-\frac{\sigma_{\rm out}}{\rho}[\dot\rho^2+f_{\rm out}(\rho)]^{1/2}=\frac{4\pi S}{M_{\rm Pl}{}^2}\ . \qquad (2.80)$$

A detailed analysis of Eq. (2.80) including gravitational effects will be carried out later. Now we shall find its form in the case when one may neglect gravity effects. Let us consider in Eq. (2.80) the transition to the limit

$$\frac{8\pi}{3}\frac{\epsilon_{\rm in(out)}\rho^2}{1+\dot\rho^2}\ll M_{\rm Pl}{}^2$$

at $m=e=0$. Taking the series expansion of the radical, we obtain

$$4\pi\rho^2S(1+\dot\rho^2)^{1/2}=\frac{4\pi}{3}\rho^3(\epsilon_{\rm out}-\epsilon_{\rm in})\ . \qquad (2.81)$$

This is the expression of the usual energy conservation law when one may neglect gravity effects. Let us find $\dot\rho$ from this equation and substitute it into Eq. (2.47) (again taking the limit $M_{\rm Pl}\to\infty$) which gives the relation of outer coordinates with the coordinates on the shell. Integrating then the derived equation we find finally

$$r^2-t^2=[3S/(\epsilon_{\rm out}-\epsilon_{\rm in})]^2\ . \qquad (2.82)$$

This is the well-known (see Refs. 1 and 2) equation of expansion of the pure vacuum bubble.

3. A nonvacuum shell

Nonvacuum shells are however most interesting. Indeed, an expansion of an even initially pure vacuum shell is followed with the particle creation. In this connection the key question arises as to whether or not a bubble remains "empty." Recently there appeared a number of papers[30,7,31] devoted to the investigation of particle production process by vacuum shells. Such a process is usually treated as a small perturbation over the vacuum solution (2.76); however, the complete investigation of the problem (in the framework of validity of thin-wall approximation) should be based on the solutions to Eqs. (2.26), (2.29), and (2.30). Moreover, besides the structure of the tensor, $S_i{}^j$ should be given.

In this paper for the nonvacuum case we shall restrict ourselves to a discussion of some peculiar properties of Eqs. (2.43) which do not depend essentially on the structure of $S_i{}^j$ (Ref. 7).

In the case of spherical symmetry one can rewrite Eq. (2.65) as

$$S_2{}^2=S_0{}^0+\tilde S_2{}^2\ ,$$
$$\tilde S_{22}=\lim_{\delta\to 0}\int dn[\,2\langle(\mathcal{D}_2\phi)^*(\mathcal{D}_2\phi)\rangle+\langle F_{0\kappa}{}^aF_0{}^{a\kappa}\rangle-(0\leftrightarrow 2)]\ . \qquad (2.83)$$

Calculating radiative corrections to $T_\mu{}^\nu$ in the case when the temperature $T=0$ and $\bar\varphi=\text{const}$ in the whole spacetime, one should necessarily obtain $T_\mu{}^\nu=\epsilon\delta_\mu{}^\nu$ due the Lorentz invariance of the ground state. However, at $T\neq 0$ we obtain $T_{\mu\nu}=(\epsilon,p,p,p)$. Similarly, we shall obtain $S_{22}\neq 0$ if $T\neq 0$, S_{22} being constant over all spacetime and the averaging in (2.83) being understood in the Gibbsian sense. Moreover, even at $T=0$ we obtain $\tilde S_{22}\neq 0$, since for a bubble $\bar\varphi=\bar\varphi(n)$ and the symmetry between the coordinates 0 and 2 which resulted in $\tilde S_{22}=0$ is now broken.

Let us consider first of all the case when the outer medium is a pure metastable vacuum $(\epsilon+p)_{\rm out}=0$ with $\epsilon_{\rm out}>0$. As far as the inner medium is concerned we shall suppose that its vacuum energy density equals zero and that its state is not a vacuum one (say, $\epsilon=3p$). In this case, Eqs. (2.43) allow the existence of a shell with $S_2{}^2=0$. However, by no means is $S_2{}^2=0$. Indeed, our shell is the source of particles; consequently the $S_2{}^2$ component of the shell stress tensor is not zero. It is seen from Eq. (2.43b) that such a shell should have $S_2{}^2>0$. If, on the contrary, just the inner medium is a pure vacuum one (being, e.g., a remnant of the old phase) and the shell collapses producing particles outside, then Eq. (2.43b) has a solution with $S_0{}^0=0$ only at $S_2{}^2<0$. We may assume further that the inner medium is in thermal equilibrium (in this case we consider the whole transient nonequilibrium layer as a thin one and attribute it to the shell). For simplicity, let the chemical potential of the inner medium be equal to zero, then $(\epsilon+p)_{\rm in}=Ts$. The inner medium possess the nonzero entropy density s while the outer medium entropy density equals zero. Consequently, the shell has a nonvanishing entropy source ω:

$$\frac{su}{(1-u^2)^{1/2}}=\omega \ , \tag{2.84}$$

u being a three-velocity of the medium relative to the shell. If $S_0{}^0=0$ [or $S_2{}^2=\tilde{S}_2{}^2$, see Eq. (2.83)] and the energy-momentum tensor of the inner medium is that of a homogeneous isotropic perfect fluid,

$$T_\mu{}^\nu=(\epsilon+p)u_\mu u^\nu-p\delta_\mu{}^\nu \ , \tag{2.85}$$

then from Eq. (2.43b) follows the relation between the entropy source ω and $\tilde{S}_2{}^2$:

$$\tilde{S}_2{}^2=\frac{\rho T}{2\dot{\rho}(1-u^2)^{1/2}}\omega \ . \tag{2.86}$$

As far as $S_0{}^0=0$ the released energy cannot be absorbed by the bubble walls as in the pure vacuum case (2.81) and should be converted in the energy of the inner medium. Therefore, a shell with $S_0{}^0=0$, $S_2{}^2\neq 0$ could apparently be considered to be a good approximation of studies of chemical burning processes. We have seen that Eqs. (2.43) allow the existence of a shell with such structure in the case of a pure vacuum equation of state of the outer medium. Therefore, we may believe that the effect of vacuum combustion[7] (with the following structure of $S_i{}^j$: $S_0{}^0=0$, $S_2{}^2=\tilde{S}_2{}^2\neq 0$) is possible in principle.

We now turn to consider several specific problems in more detail.

III. DECAY OF A METASTABLE STATE

In accordance with the classification of possible situations arising in the phase transitions (see Sec. I) we start with the consideration of a single bubble of the new phase surrounded by the old medium.

In the general case of a first-order phase transition, the spacetime is described by different metrics inside and outside the new phase bubble. The same is true also in the case of the Universe before and after a cosmological phase transition. The surface of phase separation should obey the Einstein equations; therefore, one has to match metrics on this hypersurface. The form of the TL hypersurface gives the bubble wall's motion, the investigation of which we postpone until the next section. In this section, we consider only the possibility of junction of metrics on the hypersurface separating the phases. In such a way, the decay probability of a metastable state cannot be obtained; however, we can find the range of parameters of a theory when it should vanish. One has to consider the junction on both timelike (TL) and spacelike (SL) surfaces of phase separation.

A. Timelike hypersurface of junction

Consider first a timelike hypersurface of junction. Let both the junction hypersurface and the metrics inside and outside the bubble be O(3) invariant. Then the $K_2{}^2$ and $K_0{}^0$ components of outer curvature tensor are determined by Eqs. (2.54). The case $\sigma=+1$ in these expressions corresponds to increasing radii r in the outward direction while $\sigma=-1$ indicates the opposite case. Thus, for given inner and outer metrics σ determine the global geometry (i.e., how the inner geometry is stuck together with the outer one). We find that in some cases the junction is impossible. Such a situation may arise if the sign of $\sigma_{\rm out}$ (or $\sigma_{\rm in}$) in Eqs. (2.54) following from the junction equations does not agree with the given geometry of an outer (inner) region. It follows immediately from Eqs. (2.42a) and (2.54a) that $\sigma_{\rm out}=+1$ at $S_0{}^0>0$ if

$$\Delta_{\rm out}-\Delta_{\rm in}>16\pi^2(S_0{}^0)^2\rho^2/M_{\rm Pl}{}^4 \ . \tag{3.1}$$

Correspondingly, $\sigma_{\rm out}=-1$ at the opposite sign of the inequality (3.1). One may also write easily similar inequalities for $\sigma_{\rm in}$.

Consider the case of homogeneous isotropic space described by the Friedmann-Robertson-Walker metric:

$$ds^2=dt^2-a^2(t)\left[\frac{dq^2}{1-kq^2}+q^2d\Omega^2\right] , \tag{3.2}$$

where $k=+1$, 0, and -1 distinguish the closed, spatially flat, and the open cosmological models, respectively. Using the Einstein equations we obtain (see Appendix B)

$$-\Delta=1-8\pi\epsilon\rho^2/3M_{\rm Pl}{}^2 \ , \tag{3.3}$$

ϵ being the energy density inside (outside) the bubble. For further convenience let us introduce the notation

$$\xi\equiv M_{\rm Pl}{}^2[\epsilon_{\rm out}(t_{\rm out})-\epsilon_{\rm in}(t_{\rm in})]/6\pi[S_0{}^0(\tau)]^2 \ . \tag{3.4}$$

Then, solving Eq. (2.59a) for $\dot{\rho}^2$ one has

$$\dot{\rho}^2=\left[4\pi^2\kappa^2S(\xi+1)^2+\frac{8\pi\kappa}{3}\epsilon_{\rm in}\right]\rho^2-1 \ . \tag{3.5}$$

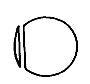
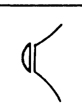
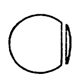
Inserting this back into Eq. (2.59a) one obtains the relation

$$\sigma_{\rm in}\,|\,\xi+1\,|-\sigma_{\rm out}\,|\,1-\xi\,|=2 \ . \tag{3.6}$$

The correspondence between ξ and σ's is now easily seen. It is shown in Table I. (This table corresponds to the case $S_0{}^0>0$. In the exotic case $S_0{}^0<0$, the signs of all the σ's would be opposite.) Let us note that in the pure vacuum case, ξ is constant and the values of $\sigma_{\rm in}$ and $\sigma_{\rm out}$ remain unchanged. In other words, the σ's are constants of motion.

We see that the case $k_{\rm out}=+1$, $\xi<-1$ can be reduced to the case $k_{\rm out}=+1$, $\xi>1$ by exchanging the outer and the inner regions; i.e., these two cases are formally equivalent. The same is true for $\xi<0$ and $\xi>0$ cases when $-1<\xi<+1$. However, these cases are physically different because we do not consider bubbles existing externally but spontaneously arising bubbles, so $\epsilon_{\rm out}$ is

TABLE I. The correspondence between the values of ξ and the signs of σ's. $\xi = M_{\rm Pl}^2(\epsilon_{\rm out}-\epsilon_{\rm in})/6\pi S^2$, and $\sigma=-1$ if radii of spheres increase in the outward direction normal to the shell, while $\sigma=-1$ if radii decrease. Schematically shown are spatial sections of resulting spacetimes at the moment of bubble creation when the shell is at rest.

	$\sigma_{\rm IN}$	$\sigma_{\rm OUT}$	SPATIAL SECTION (SCHEMATIC)		
			$K_{\rm OUT}=+1$	$K_{\rm OUT}=-1$	$K_{\rm OUT}=0$
$\xi > 1$	+1	+1			
$-1 < \xi < 1$	+1	-1		JUNCTION	
$\xi < -1$	-1	-1		IMPOSSIBLE	

also the energy of the metastable state before the phase transition. Thus, the case $\xi>0$ describes spontaneously arising bubbles with $\epsilon_{\rm in}<\epsilon_{\rm out}$, while $\xi<0$ does this with $\epsilon_{\rm in}<\epsilon_{\rm out}$ correspondingly. Transitions into the (metastable) state with the higher energy density may be possible in the closed Universe. Having in mind remarks concerning the physical difference between $\xi<0$ and $\xi>0$ cases, we may consider now the formal possibility of junction and restrict ourselves to the case $\xi \geq 0$ only.

Note that the case of domains created in a source of spontaneous breaking of a discrete symmetry (e.g., for *CP* domains[26]) one has just $\xi=0$.

As one can see from Table I the junction is not always possible. Indeed, since $\sigma = \mathrm{sgn}(\partial r/\partial q)$ in the *R* region for any coordinate q increasing along the direction from the center of a bubble [$\sigma=-\mathrm{sgn}(\partial r/\partial q)$ for q decreasing from the center in the vicinity of a shell], we may find the σ just in the coordinates (3.2). Then we obtain $r'=a(t)$ and find that in the open and spatially flat Universes $\sigma>0$ is the only possibility (in the *R* region). In a closed Universe, both $\sigma>0$ (a bubble occupies less than a half of the Universe) and $\sigma<0$ (in the opposite case) are allowed. For the bubbles nucleated in the *R* region (for example, all the bubbles which are nucleated at rest do appear in the *R* region) we arrived at the following conclusions.

(i) In a closed Universe ($k=+1$) the junction is possible at any value of ξ, so one cannot obtain any constraints in such a way. In the open Universe ($k=-1$) the junction is possible only if $\xi>1$. It is clear that if such a metric describes all the spacetime, then the decay of a metastable state occurs only if $\xi>1$. However, there may occur situations requiring an additional analysis which take place if the coordinates (3.2) with $k=-1$ do not describe all the spacetime but its part only. In these coordinates the junction is again impossible at $\xi<1$; however, this does not mean that the metastable state does not decay. It may be simply that the new phase bubble is nucleated in such a way that it goes out of the scope of the coordinate set (3.2). Let us explain this by considering as an example the junction of two metrics with pure vacuum equations of state, i.e., two de Sitter metrics with different $\epsilon_{\rm in}$'s but $\epsilon_{\rm out}>0$. In the pure de Sitter world (without bubbles) with $\epsilon>0$ one can always choose the coordinate t in such a way that the section $t=$const will be open or spatially flat. However, one needs two such coordinate sets to cover all of the manifold. At $\xi<1$ the junction in terms of these coordinates is impossible, while it may be easily shown that the junction of two de Sitter metrics with $\epsilon>0$ is possible at any ξ. It is sufficient to note that such a metric may be written in the form (3.2) with $k=+1$ and this coordinate set covers all the spacetime.

Let us now set $\epsilon_{\rm out}=0$, i.e., the Universe without a bubble is a Minkowski world which is described by one coordinate set of the type (3.2) with $k=0$. We see that gravitation stabilizes such a vacuum with respect to decay into the states with negative energy density if $\xi<1$. This particular result was obtained in Ref. 3 using a completely different approach. It was found[3] that the probability of decay of a pure vacuum state equals zero at $\xi<1$ if $\epsilon_{\rm out}=0$, in agreement with our constraints following from the junction conditions.

Thus, the conclusion about the stability of a system with $\xi<1$ may always be possible if the coordinates (3.2) with $k=-1$ or 0 describe all the spacetime. Therefore, we may arrive at such a conclusion, for example, when the Universe filled with radiation only (in this case the metric $k=-1$ is inextendable[32]).

(ii) In order to obtain from Table I the constraints on the decay of a metastable state of the Universe in terms of parameters of the model of quantum field theory one has yet to find the relation between the times t and τ in (3.4). Fortunately, in practically interesting cases this procedure is unnecessary because of the large difference between ϵ and $S^2/M_{\rm Pl}^2$ at $\xi \leq 1$. Let us consider, for example, the supersymmetric SU(5) model with the low scale of supersymmetry breaking, $M \ll R_{\rm GUT}^{-1}$, $R_{\rm GUT}$ being the radius of confinement during the grand unified phase [in the case of the SU(5) model $R_{\rm SU(5)}^{-1}\sim 10^9$ GeV]. In such a theory (as generally in any theory where the barrier between phases disappears at low temperature) a strong-coupling regime occurs at $T\sim R_{\rm GUT}^{-1}$ and this is when there appears the possibility of the phase transitions occurring, energy density being equal then to $\epsilon=(\sigma^2/30)NR_{\rm GUT}^{-4}$, where N is the total number of effectively massless degree of freedom [$N\sim 200$ in the minimal SU(5) model]. The vacuum contribution dominates in the wall energy density at least at early stages of the bubble expansion. Thus, we obtain that in the nonclosed Universe, in such a case the transition is possible in principle, only if

$$S_0{}^0 \lesssim \left[\frac{\pi N}{180}\right]^{1/2} M_{\rm Pl} R_{\rm GUT}{}^{-2} \sim 10^{37}\ {\rm GeV}^3\ . \qquad (3.7)$$

The constraints on decaying vacuum parameters following from Table I may be useful in studies of any stages of the Universe evolution and not only of the very early ones. In Ref. 33 the constraint $\xi > 1$ derived in Ref. 3 for the particular case $\epsilon_{\rm out}=0$ was used to show the stability of the present $\Lambda=0$ Universe with respect to decay into the states with negative energy which arise in the framework of supergravity models (all the states arising in the model of Ref. 33 satisfy the condition $\xi < 1$). It should be noted, however, that the Universe state in fact is not a pure vacuum one. Therefore, using the results of the present paper we may conclude that the decay probability of such a state could not equal zero in a closed Universe (though it should be extremely small). In the open Universe the state considered in Ref. 33 is undoubtedly stable for it satisfies the condition $\xi < 1$.

Of particular interest is the vacuum-dominated case. Namely, let the Universe at high temperature be radiation dominated and be described by the metric (3.2) with $k=-1$. Further let at a certain temperature there begin the epoch of domination of a metastable vacuum and the parameters of a (field) theory be such that $\xi < 1$. Will there occur a phase transition in this case or not? Does the answer to this question depend on the global spacetime structure at the epoch preceding the vacuum-dominating state?[34] We would conclude with the following remark. A bubble could probably be nucleated by means of large-scale fluctuations (greater than the Hubble radius) of a scalar field in de Sitter universe. Such a bubble does not appear in the R region, but in the T region, so all our constraints are invalid in this case. The standard calculations[3] of decay probability are also invalid for these bubbles. Moreover,[35] if a vacuum bubble appears with $\xi < 1$, this might in general take place only due to the mentioned large-scale fluctuations and not due to the subbarrier tunneling.

B. Spacelike hypersurface of junction

Let us proceed now to the consideration of the junction on the SL hypersurface. The tensor $K_i^{\,j}$ of the SL hypersurface may be obtained from $K_i^{\,j}(\tau)_{\rm TL}$ given by Eq. (2.54) by the analytic continuation $K_i^{\,j}(q)_{\rm SL} = iK_i^{\,j}(iq)_{\rm TL}$. Here we consider the isotropic homogeneous space with the metric (3.2). Then Δ is given by (3.3) and Eq. (2.42a) takes the form

$$\sigma_b\left[\rho_\xi^{\,2}-1+\frac{8\pi}{3}\frac{\epsilon_b(t_b)}{M_{\rm Pl}^{\,2}}\rho^2\right]^{1/2} -\sigma_a\left[\rho_\xi^{\,2}-1+\frac{8\pi}{3}\frac{\epsilon_a(t_a)}{M_{\rm Pl}^{\,2}}\rho^2\right]^{1/2} = -\frac{4\pi}{M_{\rm Pl}^{\,2}}\rho S_1^{\,1}\ . \tag{3.8}$$

The sign of the σ's shows [see Eq. (2.48)] whether radius r is increasing ($\sigma=+1$) or decreasing ($\sigma=-1$) in the outer normal direction to the SL hypersurface. In a T region of spacetime $\sigma=+1$ means the expansion and $\sigma=-1$ correspondingly the contraction of the Universe when the normal is directed from the past to the future. If we choose the SL hypersurface to be the surface $t=$const, then such an interpretation holds in an R region also. Thus, in the case $\sigma_a\sigma_b=-1$, one has the transition from the expansion to the contraction or vice versa. Let us introduce the quantity $\bar\xi$ defined as

$$\bar\xi \equiv \frac{\epsilon_B-\epsilon_a}{6\pi(S_1^{\,1})^2}M_{\rm Pl}^{\,2}\ . \tag{3.9}$$

From Eq. (3.8) we have $\sigma_a\sigma_b=1$ if $|\bar\xi|>1$, while $\sigma_a\sigma_b=-1$ if $-1<\bar\xi<1$. We would like to note here some physical situations to which, as we may suppose, the SL junction could be applied: (i) A phase transition which proceeds through isolated bubble nucleation, collision, and subsequent thermalization, but which stops very rapidly (almost instantly); (ii) a materialization of a new phase inside one (large) bubble; (iii) a phase transition in the framework of the new inflationary scenario.[22]

In case (i) the matching is carried out across the Universe, while in cases (ii) and (iii) we are restricted by the bubble dimensions. In case (ii) we match two phases, the transition between them being a quantum tunneling process. Therefore, the applicability of the SL junction to this case is unclear. Though this junction may have proved to be rather unphysical we shall nevertheless formally investigate it too. In case (iii) we pay no attention to the quantum transition process and consider the whole region of spacetime where the slow classical rolling of the field fluctuations occurs (during which vacuum energy is essentially constant) as the region still occupied by the old phase. We attribute to the transient layer the region where the system rolls down rapidly to the minimum of the potential and then oscillates, the kinetic energy of the field being transformed into the particle energy. The duration of both latter stages is of the order $1/M_x$, that is much smaller than even dimensions of the visible part of the Universe. Therefore it is quite natural to use here the thin-wall formalism.

Consider first case (ii). The phase transition can take place only when the physical volume of new phase bubbles increases. In the expanding Universe the radius of a bubble increases (in the T region) only if the scale factor in the inner metric is also rising. So, we conclude that in this case the old phase decays only if the parameters of the field theory are chosen in such a way that $\bar\xi > 1$.

Let us consider now cases (i) and (iii). Our Universe expands, so we restrict ourselves to the case $\sigma_b=+1$ and $\sigma_a=+1$. Because of homogeneity and isotropy it is natural to choose the hypersurface of phase separation as the surface of constant t, where t is the cosmological time (3.2). In proper coordinates the equation of such a hypersurface is $\rho_q^{\,2}=1-k\rho^2/R^2$ and Eq. (3.8) now takes the form

$$\left[\frac{8\pi}{3}\frac{\epsilon_b}{M_{\rm Pl}^{\,2}}-\frac{kT^2}{N^2}\right]^{1/2}-\left[\frac{8\pi}{3}\frac{cT^4}{M_{\rm Pl}^{\,2}}-\frac{kT^2}{N^2}\right]^{1/2} = -\frac{4\pi S}{M_{\rm Pl}^{\,2}}\ , \tag{3.10}$$

where cT^4 is the radiation energy density, $N=TR$, $N^3=s3/4c$, s being the coordinate entropy density. For the inflationary scenario to work it is necessary to have $N>10^{28}$ (Ref. 23). Thus, Eq. (3.8) relates to the amount of inflation with the temperature after reheating. In general the value of k in Eq. (3.10) is not fixed. However, when the complete metric is O(4) invariant the metric inside the new phase bubble is of the open type ($k=-1$). In the new inflationary scenario the fluctuation passes the stage of vacuum expansion. One expects that in this case the visible part of the Universe is describe by (3.2) with $k=-1$. The contributions to S come from the rapid rolling down to the minimum of potential and from subsequent particle creation processes. The first contribution can be easily calculated and equals $S\sim-\sqrt{2\lambda}\,\varphi_0^3/3$, where φ_0 is the equilibrium value of the scalar field, λ being the quartic coupling constant entering the potential as $-\lambda\varphi^4$. The second contribution requires special consideration.

IV. GROWTH OF NEW PHASE BUBBLES

We shall consider here the problem of new phase bubble growth at the background of some given outer medium. This surrounding medium can be either the unperturbed matter before the phase transition (the "detonation" case) or the reheated matter with the one more nonsingular boundary (shock wave) expanding in the "initial" medium (the "deflagration" case). The investigation of these problems with the nonsingular layer of phase transition has been carried out in Refs. 11 and 12. We shall treat only the shell where the phase transition occurs, not considering at all the outgoing shock waves. In general the process of new phase bubble expansion is determined essentially by the structure of the tensor $S_i^{\,j}$.

A. New vacuum bubble expansion

First of all let us assume that the state of the inner medium is given. Let the inner and outer metrics be of the Friedmann-Robertson-Walker type (3.2). This picks out a subclass of $S_i^{\,j}$. Second junction conditions in such a problem are given by Eqs. (2.42). The first equation (2.42a), now taking the form

$$\sigma_{\rm in}\left[1+\dot\rho^{\,2}-\frac{8\pi\kappa}{3}\epsilon_{\rm in}(t)\rho^2\right]^{1/2}-\sigma_{\rm out}\left[1+\dot\rho^{\,2}-\frac{8\pi\kappa}{3}\epsilon_{\rm out}(t)\rho^2\right]^{1/2}=4\pi\kappa S_0^{\,0}(\tau)\rho\ , \tag{4.1}$$

determines $\rho(\tau)$ at any given $S_0^{\,0}(\tau)$. Then we have to consider the second equation (2.42b) as the equation which determines the $S_2^{\,2}(\tau)$. Such a shell should have of course a very special structure unless we do not consider the pure vacuum case [see (2.66)]. At arbitrary $S_i^{\,j}$ the space does not remain homogeneous and isotropic and this case will be considered later.

Twice squaring Eq. (4.1) we obtain

$$\dot\rho^{\,2}=B^2\rho^2-1\ , \tag{4.2a}$$

where

$$B^{-1}=\frac{3S_0^{\,0}(\tau)}{\{[\epsilon_{\rm out}+\epsilon_{\rm in}+6\pi\kappa(S_0^{\,0})^2]^2-4\epsilon_{\rm in}\epsilon_{\rm out}\}^{1/2}}\ . \tag{4.2b}$$

In the pure vacuum case ϵ's and S are constant and the quantity B^{-1} determines the bubble radius at the rest moment in the frame of reference connected with the shell and coincides with the bubble radius at its moment of materialization. In the vacuum case Eq. (4.2) can be easily integrated

$$\rho(\tau)=\frac{1}{B}\cosh B\tau\ . \tag{4.3}$$

We see that in a proper time the bubble expands exponentially.

Let us find now the equation of motion of the bubble in coordinates of inner and outer regions. In both cases the equation looks identical so we omit the index out (in) keeping in mind that corresponding quantities carry this index. The equation to be found is

$$\frac{dR}{dt}=\frac{\left[\left(\frac{8\pi\kappa}{3}\epsilon-\frac{k}{a^2}\right)\left(1-\frac{k\rho^2}{a^2}\right)\right]^{1/2}\pm\left[(B^2\rho^2-1)\left(B^2-\frac{8\pi\kappa}{3}\epsilon\right)\left(1-\frac{k\rho^2}{a^2}\right)\right]^{1/2}}{\rho a\left(B^2-\frac{k}{a^2}\right)}\ , \tag{4.4}$$

where

$$\rho=aR\ .$$

In the case of zero energy density inside (or outside) the bubble, Eq. (4.4) is readily integrated to give [cf. (2.82)]

$$\rho^2-(t-t_0)^2=B^{-2}\ , \tag{4.5}$$

where

$$B^{-1}=\frac{3S_0^{\,0}}{\epsilon+6\pi\kappa(S_0^{\,0})^2}\ .$$

This clarifies the fact that the exponential growth (4.3) is an effect of the proper-time parametrization. In any case, the shell velocity does not exceed that of light.

For simplicity let us consider Eq. (4.4) in the particular case of a spatially flat world, i.e., let us put in Eq. (4.4) $k=0$ (which is a good approximation at any k if the bubble size is much less than dimensions of the Universe):

$$\frac{dR}{dt}=\frac{1}{a}\left[1-\frac{8\pi\kappa}{3}\frac{\epsilon}{B^2}\right]^{1/2}\left[1-\frac{1}{B^2\rho^2}\right]^{1/2}-\frac{1}{\rho B}\left[\frac{8\pi\kappa}{3}\frac{\epsilon}{B^2}\right]^{1/2}\ . \tag{4.6}$$

A light radial geodesic would be described in the coordinates (3.2) by the equation $dR/dt=1/a$. The bubble shell (4.6) will tend to move along the light geodesic from the very beginning only in the case $1/B\equiv 0$ or, equivalently, $S_0{}^0\rightarrow 0$. [This does not mean, however, that the shell propagating along the light geodesic is a nonsingular one (see Sec. VI).] Moreover, the coordinate velocity of the shell during expansion does not even tend to the coordinate velocity of light. This is the consequence of two factors: (1) The shell velocity in the coordinate space does not tend to velocity of light due to medium thermal properties (see, e.g., Sec. IV B); (2) the asymptotical shell velocity in the coordinate space differs from velocity of light due to the Hubble expansion of space.[5] This statement is valid at any equation of state of matter, so in the particular case of a pure vacuum we have

$$\frac{dR_{\rm out}}{dt_{\rm out}}\underset{\rho\to\infty}{\longrightarrow}\frac{1}{a}\left[1-\frac{8\pi\kappa}{3}\frac{\epsilon}{B^2}\right]^{1/2}. \tag{4.7}$$

Note that in the vacuum case, contrary to the thermal one, only the coordinate velocity of the shell does not tend to the velocity of light. The proper velocity (4.2) does tend to the velocity of light ($\dot\rho\rightarrow\infty$), but never being equal to it.

In GUT's with $M_X \ll M_{\rm Pl}$, the velocity (4.7) only slightly differs from the velocity of light. This, however, may not be so if the wall energy contribution is essential, which may take place, for example, in supersymmetric GUT's. Moreover, $dR_{\rm out}/dt_{\rm out}\rightarrow 0$ if $\xi\rightarrow 1$, i.e., the coordinate volume of such a bubble does not increase.

It is important to know the value of the asymptotic velocity of the shell in estimates of the number density of produced magnetic monopoles,[36] in studies of the percolation problem in phase transitions,[20,37] etc. All the possible Penrose diagrams for the pure vacuum bubbles are shown in Fig. 1.

B. Vacuum burning

Let us turn now to the more general case of the arbitrary $S_i{}^j$. Now the metric inside the bubble is unknown. Since we are primarily interested in the matter content inside the bubble (but not in the inner metric) it is appropriate to rewrite the second junction equations in the form (2.31). Now we shall sacrifice the generality and consider the case when both inside and outside the bubble the energy-momentum tensor of a medium $T_\mu{}^\nu$ is that of a perfect fluid (2.85). Thus, all possible entropy production (and all particle production in general) we relate with the transient layer and attribute it to the shell.

Consider first the case when the metric outside the bubble is the pure vacuum one. Then $(K_i{}^j)^+$ is determined by expressions (B7) and (B11) with $\epsilon+p=0$, and Eqs. (2.29) and (2.31) take the form

$$\sigma_{\rm out}\left[S_0{}^0\frac{\ddot\rho-8\pi\kappa\epsilon_{\rm out}\rho/3}{(\dot\rho^2+1-8\pi\kappa\epsilon_{\rm out}\rho^2/3)^{1/2}}+2S_2{}^2\frac{(\dot\rho^2+1-8\pi\kappa\epsilon_{\rm out}\rho^2/3)^{1/2}}{\rho}+2\pi\kappa S_0{}^0(4S_2{}^2-S_0{}^0)\right]=[T_n{}^n]=-p_{\rm out}+\frac{\epsilon u^2+p}{1-u^2}, \tag{4.8}$$

$$\dot S_0{}^0+\frac{2\dot\rho}{\rho}(S_0{}^0-S_2{}^2)=-[T_0{}^n]=-(\epsilon+p)\frac{u}{1-u^2}, \tag{4.9}$$

ϵ and p being the proper energy and pressure, respectively, and u is the value of three-velocity of the inner medium relative to the shell.

At $S_0{}^0=0$ these equations are considerably simplified. This is just the case of "vacuum burning."[7] The equation of motion of the shell at $S_0{}^0=0$ may be rewritten as

$$\sigma_{\rm out}(f+\dot\rho^2)^{1/2}=\frac{(\epsilon_{\rm out}-\epsilon_{\rm in})(\epsilon_{\rm out}+p_{\rm in})\rho^2+4(S_2{}^2)^2f}{2S_2{}^2[(\epsilon_{\rm out}-\epsilon_{\rm in})+(\epsilon_{\rm out}+p_{\rm in})]\rho}, \tag{4.10}$$

$$u^2=\frac{\rho^2(p_{\rm in}+\epsilon_{\rm out})-4(S_2{}^2)^2f}{\rho^2(\epsilon_{\rm out}-\epsilon_{\rm in})-4(S_2{}^2)^2f}. \tag{4.11}$$

Consider the case when one may neglect the effects of gravitation. Then $f=1$ and $\sigma_{\rm out}=+1$. Take in these equations the limit $\rho\rightarrow\infty$. In this limiting case, the shell velocity tends to the velocity of light ($\dot\rho\rightarrow\infty$), while inner medium parameters lie on the detonation adiabat $\epsilon_{\rm in}u-p_{\rm in}=\epsilon_{\rm out}(1+u)$ if $(S_2{}^2/\rho)\rightarrow 0$. If $(S_2{}^2/\rho)\rightarrow{\rm const}\neq 0$, then the motion of the shell with the constant velocity is possible. Finally, at $(S_2{}^2/\rho)\rightarrow\infty$ the shell velocity tends to the velocity of light again, but $u\rightarrow 1$; i.e., in this latter case, at $\rho\rightarrow\infty$ there is a "constant" which is not the velocity of the inner medium with respect to the shell but the medium velocity with respect to the bubble center. In particular, a solution is possible with the homogeneous inner medium resting with respect to the bubble center. It follows also from Eq. (4.10) that in the limit $\rho\rightarrow\infty$ one has $\epsilon_{\rm in}>2\epsilon_{\rm out}+p_{\rm in}$ if $(S_2{}^2/\rho)\rightarrow 0$, while $\epsilon_{\rm in}<2\epsilon_{\rm out}+p_{\rm in}$ if $(S_2{}^2/\rho)\rightarrow\infty$, but $\epsilon_{\rm in}\geq\epsilon_{\rm out}$ always.

Independent of the value of $\tilde S_2{}^2$, the total metastable vacuum energy is processed into the inner medium energy. It is obviously the direct consequence of $S_0{}^0=0$, so that the energy release in the phase transition cannot be attributed to the wall kinetic energy as in the pure vacuum case. On the magnitude of $\tilde S_2{}^2$ there depends the velocity of the shell expansion only, i.e., the rate of processing of the vacuum energy into the inner medium

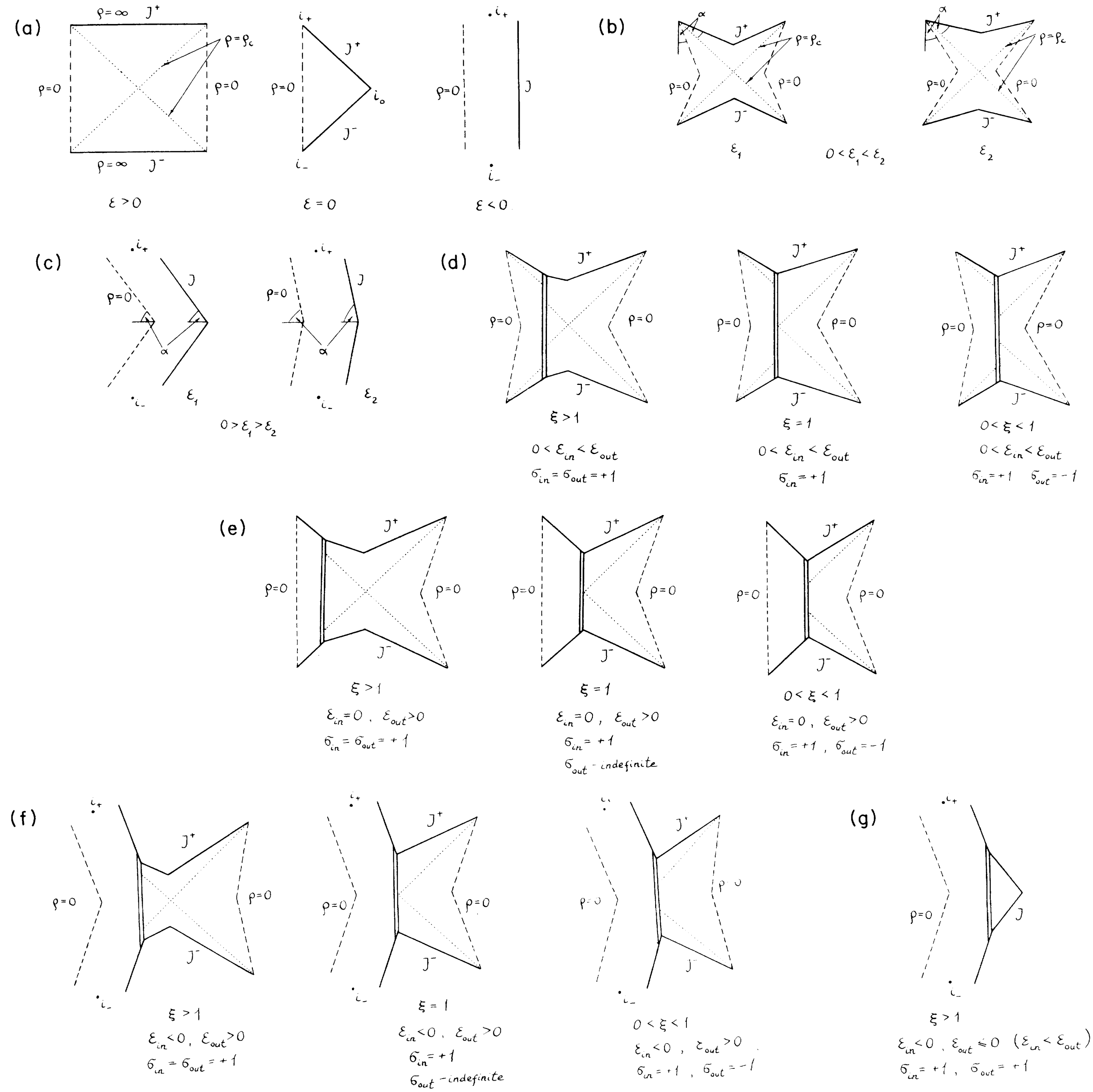


FIG. 1. Construction of Penrose diagrams for true vacuum bubbles in false vacuum surroundings. (a) From left to right: Penrose diagrams for de Sitter, Minkowski, and anti–de Sitter spacetimes. Null lines are at $\pm 45°$. (b) Distorted Penrose diagrams for de Sitter spacetimes with different values of ϵ. The larger angle α corresponds to the larger value of ϵ, α being equal to zero for Minkowski spacetime and $\alpha \to 45°$ for $\epsilon \to \infty$. Null lines are at $\pm 45°$. (c) Distorted Penrose diagrams for anti–de Sitter spacetimes with different values of $\epsilon < 0$. The larger angle α corresponds to the larger absolute value of ϵ, α approaching 45° when $\epsilon \to 0$ and $\alpha \to 90°$ when $|\epsilon| \to \infty$. The lines J and $\rho = 0$ are parallel. Null lines are at $\pm 45°$. Note that there is no continuous transition to the limiting case of the Penrose diagram for $\epsilon = 0$. (d) Matching of two de Sitter spacetimes with different values of positive vacuum energy density ϵ for three values of ξ: $\xi > 1$, $\xi = 1$, and $0 < \xi < 1$, respectively. The world line of a shell with surface energy density S is shown by the double line, while dotted lines denote apparent horizons of the inner and outer de Sitter spacetimes. $\xi = (\epsilon_{\text{out}} - \epsilon_{\text{in}})/6\pi\kappa S^2$. (e) Matching of outer de Sitter spacetime ($\epsilon_{\text{out}} > 0$) with inner Minkowski spacetime ($\epsilon_{\text{in}} = 0$) for three values of ξ: $\xi > 1$, $\xi = 1$, and $0 < \xi < 1$, respectively. The world line of a shell is shown by a double line, while the dotted lines represent apparent horizons of the outer de Sitter spacetime. (f) Matching of the outer de Sitter spacetime ($\epsilon_{\text{out}} > 0$) with the inner anti–de Sitter spacetime ($\epsilon_{\text{in}} < 0$) for three values of ξ: $\xi > 1$, $\xi = 1$, and $0 < \xi < 1$. (g) Matching of the outer Minkowski ($\epsilon_{\text{out}} = 0$) or anti–de Sitter ($\epsilon_{\text{out}} < 0$) spacetime with the inner anti–de Sitter spacetime ($\epsilon_{\text{in}} < \epsilon_{\text{out}}$). In this case only $\xi > 1$ is possible.

one.

The quantity $S_0{}^0$ gives the amount of energy contained in the transient layer between two phases under consideration. Therefore, though formally speaking $S_0{}^0$ may be equal to zero if, in the limiting case of an infinitely thin transient layer, $T_0{}^0$ has only finite discontinuity and does not possess any δ-functional singularities, nevertheless in the case of a phase transition the value of $S_0{}^0$ may prove not to vanish.

Let us now proceed to the consideration of bubbles with $S_0{}^0\neq 0$. Let us find the conditions at which one may say that there takes place the vacuum-burning phenomenon.

Particles created during the bubble expansion can form bound states with the wall[38] so that $S_0{}^0=S_0{}^0(\rho)$ and $S_0{}^0$ is growing together with the rise of ρ. We shall neglect this effect and shall take that $S_0{}^0=\text{const}$. The particle production results in $S_2{}^2=S_0{}^0+\tilde{S}_2{}^2$, where the quantity $\tilde{S}_2{}^2$ is connected with the entropy source (2.84). Generally speaking, $\tilde{S}_2{}^2$ is a complicated function of the state of a medium and of the shell motion. For example, $\tilde{S}_2{}^2$ could contain terms proportional to ρ, $\dot\rho$, $\ddot\rho$, etc. Since we are interested here in the question of whether or not a new phase bubble is empty we shall investigate in what follows the bubble motion taking $\tilde{S}_2{}^2=\text{const}$ as a model example.

One can easily integrate Eq. (4.9) assuming that all quantities except for $\rho(\tau)$ are constant:

$$(1+\dot\rho^2)^{1/2}/\rho=\frac{\epsilon_{\text{out}}-\epsilon_{\text{in}}+(\epsilon+p)_{\text{in}}/(1-u^2)}{3S_0{}^0+2\tilde{S}_2{}^2}+C'\rho^{-(3S_0{}^0+2\tilde{S}_2{}^2)/S_0{}^0}, \qquad (4.12)$$

C' being the integration constant. $C'\neq 0$ corresponds to the nonzero Schwarzschild mass in the vacuum case [see (5.3) below], so initial conditions for a spontaneously nucleated bubble require $C'=0$. The critical radius of a bubble nucleated in a thermostat (where a bubble may be nucleated with a nonvanishing mass) is determined by the conditions of equilibrium of a bubble in a medium: $\dot\rho=0$, $\ddot\rho=0$. In this case one obtains from Eq. (4.9) that $\rho_0=2S_2{}^2/(p_{\text{in}}-p_{\text{out}})$, which is the well-known formula of thermodynamics. In any case, at large enough ρ the specific value of C' becomes nonessential. From Eqs. (4.12) and (4.8) at $\rho\to\infty$ we then obtain

$$2\tilde{S}_2{}^2[\epsilon_{\text{out}}+p_{\text{in}}-u^2(\epsilon_{\text{out}}-\epsilon_{\text{in}})]=(3S_0{}^0+2\tilde{S}_2{}^2)u(\epsilon+p)_{\text{in}}\ . \qquad (4.13)$$

One can easily see from Eq. (4.13) that $\epsilon_{\text{in}}\sim\epsilon_{\text{out}}$ at any arbitrarily small value of $\tilde{S}_2{}^2$ provided $u\sim\tilde{S}_2{}^2/S_0{}^0$. We would note that the magnitude of ϵ_{in} may not serve as an adequate criterion in the question of whether or not the vacuum is burning. In order to find such a criterion let us proceed as follows. Denote by E_{part} the part of vacuum energy release absorbed by the inner medium while E_{kin} is the kinetic energy of the bubble walls,

$$E_{\text{kin}}=4\pi\rho^2S_0{}^0(1+\dot\rho^2)^{1/2}\ .$$

Clearly,

$$E_{\text{part}}/E_{\text{kin}}=4\rho^3\epsilon_{\text{out}}/3E_{\text{kin}}-1\ . \qquad (4.14)$$

We shall say that a vacuum is burning if $E_{\text{part}}/E_{\text{kin}}\gtrsim 1$. From Eqs. (4.14) and (4.12) we find

$$E_{\text{part}}/E_{\text{kin}}=[2\epsilon_{\text{out}}\tilde{S}_2{}^2(u-1)+3uS_0{}^0\epsilon_{\text{in}}]/3S_0{}^0u(\epsilon_{\text{out}}-\epsilon_{\text{in}})\ . \qquad (4.15)$$

Using the relation between ϵ_{in} and u following from Eq. (4.13) with the condition that the inner medium has the equation of state $p_{\text{in}}=V_S{}^2\epsilon_{\text{in}}$, V_S being sound velocity, we finally obtain

$$E_{\text{part}}/E_{\text{kin}}=2\tilde{S}_2{}^2(1-u)(u-V_S{}^2)/3S_0{}^0u(1+V_S{}^2)\ . \qquad (4.16)$$

We see that $E_{\text{part}}/E_{\text{kin}}$ vanishes at $u=1$ or at $u=V_S{}^2$ and takes its maximum at $u=V_S$. Thus, at $S_0{}^0=\text{const}$ and $\tilde{S}_2{}^2=\text{const}$ we obtain

$$E_{\text{part}}/E_{\text{kin}}\sim\tilde{S}_2{}^2/S_0{}^0\ .$$

What could be the value of this ratio?

It seems to us rather plausible that it could be of order 1 in realistic models of field theory. First of all it seems that $S_0{}^0$ is determined by the value of a scalar coupling constant λ while $S_2{}^2$ is determined by the value of a maximal coupling constant of a model (gauge coupling constant g in GUT's). The most interesting from a cosmological viewpoint are models with strong supercooling of a metastable phase which take place just in GUT's with $\lambda\sim g^4$ (see, e.g., Ref. 27). In addition, in GUT phase transitions with supercooling, the transitions occur at temperatures of order of inverse confinement radius in a metastable phase where all the couplings increase.

Now let the outer medium not be a pure vacuum as well, $(\epsilon+p)_{\text{out}}=(TS)_{\text{out}}$. If one can ignore gravity then the corresponding equations of motion are easily obtained by setting $\kappa=0$ in (4.8) and (4.9) and taking for $T_i{}^n$ the expressions

$$[T_0{}^n]=\left.\frac{(\epsilon+p)u}{1-u^2}\right|_{\text{in}}-(\text{in}\to\text{out})\ ,$$

$$[T_n{}^n]=(\epsilon u^2+p)/(1-u^2)\,|_{\text{in}}-(\text{in}\to\text{out})\ .$$

In this case as well as in the vacuum case, equations of motion allow the shell motion with $\dot\rho\to\infty$ at $\rho\to\infty$ if

$$v\to[(\epsilon+p)_{\text{in}}-(\epsilon+p)_{\text{out}}]/[(\epsilon+p)_{\text{in}}+(\epsilon+p)_{\text{out}}]\ , \qquad (4.17)$$

where $v\equiv v^1/v^0$ is the three-velocity of the outer medium. We are discussing here the cases $S_0{}^0\to\text{const}$, $S_2{}^2\to\text{const}$. However, besides solution (4.17) there is at $(\epsilon+p)_{\text{out}}\neq 0$ the possibility of the shell motion with constant velocity ($\dot\rho\to\text{const}$) at $\rho\to\infty$. At large enough ρ, such a shell takes the regime of a detonation wave whose behavior is well known (see, e.g., Refs. 19 and 12). We see that the singular shell may indeed expand as a detonation wave and have learned at what conditions this takes place. At small values of $(\epsilon+p)_{\rm out}$ the shell velocity tends to the constant value close to the velocity of light (but not equal to it) and the inner medium characteristics u, $\epsilon_{\rm in}$, $p_{\rm in}$ obey Eq. (4.11). This regime is rather remarkable because in this case there takes place the total processing of vacuum energy into the medium energy independently of the magnitudes of $S_0{}^0$ and $\tilde{S}{}_2{}^2$ (because of constancy $\dot\rho$, the energy released cannot lead to the increase of a shell kinetic energy at large ρ).

Thus, using the energy-momentum-conservation law we have shown that the metastable vacuum-burning phenomenon is possible in principle. In order to calculate the magnitude of the effect in a realistic field theory model one has to investigate the process of particle production by the classical field with varying in time field gradient (i.e., by expanding the bubble wall). The number of particles produced and consequently the magnitude of $\tilde{S}{}_2{}^2$ should depend, in particular, on how the wall moves. The $\tilde{S}{}_2{}^2$ in turn comes into the equation of motion of the shell (2.26).

So, completing Eq. (2.26) with the calculation of $S_i{}^j$ in the framework of a field theory one may obtain a closed system of equations. We did not carry out this program in full. Suppose nevertheless that the vacuum bubble expansion in realistic grand unified models is indeed similar to the combustion process. We may expect at least that if the initial temperature inside a bubble is high enough, then this temperature will be maintained during the expansion. The vacuum burning could lead to some interesting cosmological consequences discussed in part in Ref. 7.

V. OLD PHASE REMNANTS AND DOMAINS

Up to now we considered an evolution of a new phase bubble immersed in the old phase region. Such a process takes place at the very beginning of the phase transition. The formal description of an intermediate stage of the phase transition would be extremely complicated, so we omit this stage and consider the final stage of the transition. Suppose that at a certain moment the new phase (or one of the new phases if a domain structure takes place) starts percolating. Moreover, suppose that the old phase does not percolate starting from some moment; i.e., there exists an old phase remnant of finite maximal size. Such a moment has definitively existed during most cosmological phase transitions (see the discussion in Sec. I).

A shape of a remnant is extremely angular just after formation. It is reminiscent of an amoeba. However, after the time has passed at least a part of these remnants will take a spherical form (for example, due to surface tension effects or dissipation) and we may think of such remnants of old phase as spherical bubbles.

Such a bubble, however, differs essentially from a spontaneously created bubble of the new phase. Namely, an old phase remnant can have a nonzero total mass. This mass, of course, shows itself not at once but after settling the appropriate boundary conditions at "infinity" (during this process a part of energy is carried away by gravitational and electromagnetic radiation). Thus, the case of old phase remnant is reduced to the case of a spherically symmetric bubble with the Schwarzschild metric as an exterior.

A. General equations

For generality, let us consider the case when an external vacuum has also nonzero energy density and a remnant itself carries electric or magnetic charge. Then a metric both inside and outside the bubble has the form (2.76), where we have inside the bubble $f_{\rm in}=1-8\pi\kappa\epsilon_{\rm in}\rho^2/3$, while outside the bubble

$$f_{\rm out}=1-8\pi\kappa\epsilon_{\rm out}\rho^2/3-2\kappa m/\rho+g^2\kappa/\rho \ .$$

The motion of bubble walls under consideration is described just by Eq. (2.80). We find, solving Eq. (2.80) with respect to m,

$$m=\frac{g^2}{2\rho}+\frac{4\pi}{3}(\epsilon_{\rm in}-\epsilon_{\rm out})\rho^3 +4\pi\rho^2 S\sigma_{\rm in}(\dot\rho^2+1-8\pi\kappa\epsilon_{\rm in}\rho^2/3)^{1/2} -8\pi^2\kappa S^2\rho^3 \ . \tag{5.1}$$

Relation (5.1) is also the equation of motion of the shell. It can be solved with respect to $\dot\rho$, but the resulting relation is too cumbersome in the case of a charged shell. So we write down the result for the case $g=0$ only:

$$\dot\rho^2=B^2\rho^2-1+\frac{m}{\rho}\left[\frac{1}{M_{\rm Pl}{}^2}+\frac{\epsilon_{\rm out}-\epsilon_{\rm in}}{6\pi S^2}\right]+\frac{m^2}{16\pi^2S^2\rho^4} \ , \tag{5.2}$$

B being defined by Eq. (4.2b). Both forms of the equation of motion (5.1) and (5.2) prove to be useful.

It is easy to see from Eq. (5.2) that the junction supersurface is O(4) invariant in imaginary time at $m=0$, the invariance being lost at $m\neq0$. So, calculating the probability of spontaneous creation of a ring of new vacuum with an old vacuum remnant at the center of the ring, the remnant having a nonzero mass, one has to take into account that the Euclidean solution to the coupled field equations may be O(3) invariant at most. Namely, the Euclidean phase separation surface is now a torus. This differs from the case of new vacuum bubble formation for which we have thus proved the O(4) invariance. In Ref. 3, O(4) invariance for the new vacuum bubble case has been conjectured and essentially used in calculations of tunneling probability.

Let us consider now the equations of motion in the form (5.1). It is easy to understand the meaning of all the terms in the right-hand side of Eq. (5.1). The first term describes a potential energy of a charged shell; the second one is a difference between old and new vacuum energy densities; the third is a kinetic energy of the shell; the fourth is an energy of gravitational self-interaction of the shell.

B. Static shells

Let us show that a charged shell has a point of stable equilibrium.

Theorem. The shell has an equilibrium point if the equation

$$\left.\frac{\partial m(\rho,\dot\rho)}{\partial\rho}\right|_{\dot\rho=0}=0 \tag{5.3}$$

has a solution at some value of ρ, say ρ_0. For $\partial m/\partial\dot\rho^2|_{\dot\rho=0}>0$ the equilibrium state is stable if the function $m(\rho,\dot\rho=0)$ takes a minimum value at the point ρ_0, and for $\partial m/\partial\dot\rho^2|_{\dot\rho=0}<0$ the equilibrium state is stable if this function takes a maximum value at the point ρ_0.

Proof. A point is an equilibrium point for a shell if the conditions $\dot\rho=0$ and $\ddot\rho=0$ are satisfied simultaneously. Furthermore,

$$\frac{dm}{d\tau}=\dot\rho\left[\frac{\partial m}{\partial\rho}+2\frac{\partial m}{\partial\dot\rho^2}\ddot\rho\right]. \tag{5.4}$$

The mass m is the integral of motion, so we have, for any $\dot\rho\neq0$,

$$\frac{\partial m}{\partial\rho}+2\frac{\partial m}{\partial\dot\rho^2}\ddot\rho=0 \tag{5.5}$$

and, hence, at the point $\ddot\rho=0$ we have $\partial m/\partial\rho=0$. The point $\dot\rho=0$ possesses the same property due to the continuity of the equations of motion. Thus, the shell equilibrium point is the solution to Eq. (5.3). The equilibrium state at the point is stable if at $\rho>\rho_0$ one has $\ddot\rho<0$, while at $\rho<\rho_0$ one has $\ddot\rho>0$. It is easy to see from Eq. (5.5) that at $\partial m/\partial\dot\rho^2|_{\dot\rho=0}<0$ the equilibrium state at the point ρ_0 is stable if at this point the function $m(\rho,\dot\rho=0)$ takes a minimum while at $\partial m/\partial\dot\rho^2|_{\dot\rho=0}<0$ the equilibrium is stable if this function takes a maximum, respectively. In the particular case of Eq. (5.1) one has $\partial m/\partial\dot\rho^2|_{\dot\rho=0}>0$ if $\sigma_{\rm in}>0$ and $\partial m/\partial\dot\rho^2|_{\dot\rho=0}<0$ if $\sigma_{\rm in}<0$.

Let us consider now as a simple example a charged shell in the particular case when one may neglect S. It is easy to find the radius of a stable configuration,

$$\rho_0{}^4=g^4/8\pi(\epsilon_{\rm in}-\epsilon_{\rm out}), \tag{5.6}$$

and its mass

$$m=\frac{2g^2}{3\rho_0}. \tag{5.7}$$

At $g^2\gg1$ the radius of the stable configuration is much larger than its Compton length $1/m$, so our treatment of the configuration as a classical one is quite appropriate. One example of such a configuration is well known; it is a magnetic monopole.

Consider the field φ^i, $i=1,2,3$ transforming as a triplet with respect to the group SU(2). Let the φ^i field direction in isotropic space coincide with the direction of the radius in the configurational space and the quantity $|\varphi^i|$ tend to its vacuum-averaged value at $\rho\to\infty$. The field φ^i in the center of such a configuration equals zero. Therefore, $\epsilon(\rho\to\infty)=0$ and $\epsilon(0)\sim M_x{}^4$. The electromagnetic field at large distances from the center reproduces the field of a magnetic monopole with the charge $g=2\pi/e$, where e is unit electric charge. We may now treat the monopole problem in the thin-wall approximation; i.e., we may regard that all of the field φ^i variation is concentrated near some value of the radius ρ_0. Then Eqs. (5.6) and (5.7) describe the magnetic monopole in the thin-wall approximation and one obtains, for the monopole mass, $m_M\sim M_x/\alpha$, where α is the fine-structure constant.

In principle, one can construct other charged configurations also; for example, islands of the SU(4)×U(1) phase in the SU(3)×SU(2)×U(1)-symmetrical vacuum.[27] Such a remnant may carry a charge with respect to the group $SU(2)_L$, since generators corresponding to weak intermediate bosons W's are broken in the SU(4)×U(1) vacuum while they are not broken in the SU(3)×SU(2)×U(1) vacuum.

If the shell radius at its rest moment does not coincide with ρ_0, which is the solution of Eq. (5.3), but differs from it slightly then the shell will oscillate around that state. At other initial conditions the shell may proceed either to a regime of unrestricted expansion or to a regime of collapse. During a phase transition there may arise, of course, rather different conditions for remnants. Let us proceed now to the study of the relation between initial conditions and the geometry of spacetime.

C. Global geometry of spacetime

Let us introduce the variable ξ, similarly to Eq. (3.4), and the new variable η as

$$\eta\equiv\left[m-\frac{g^2}{2\rho}\right]\frac{M_{\rm Pl}{}^2}{8\pi^2S^2\rho^3}. \tag{5.8}$$

In these variables the signs of $\sigma_{\rm in}$ and $\sigma_{\rm out}$ (and therefore the global spacetime geometry) are completely determined. The relation between σ and the variables η,ξ is shown in Table II. [We can derive the equation analogous to Eq. (3.6), which in this case takes the form $\sigma_{\rm in}|\xi+\eta+1|-\sigma_{\rm out}|1-\xi-\eta|=2$.] Generally speaking, the signs of σ may change at the shell motion; however, this may occur in T regions of spacetime only. Therefore, the shell classification in accordance with signs of σ is unequivocal only at the moment when the shell intersects an R region; however, it is sufficient for the construction of the global geometry.

In case A (see Table II) one has $\sigma_{\rm out}>0$ and the shell crosses the R_+ region. In such a case the shell forms a black hole if initial conditions allow the shell to collapse. The collapse with the formation of a charged black hole is possible in the case $\epsilon_{\rm out}=0$ only if $m\geq gM_{\rm Pl}$. A neutral shell in the case $\epsilon_{\rm out}=0$ (i.e., in the case of pure Schwarzschild outer metric) is collapsing always at the final stage of its evolution.

Suppose now that in the course of phase transition a magnetically charged remnant of the old vacuum was formed and that it has a mass large enough to collapse and to form a black hole. Such a black hole will then evaporate. However, the evaporation of charged black holes differs from that of neutral black holes; namely, the temperature of charged black holes differs from that of neutral blackholes: the temperature of a charged black hole increases at first, achieves its maximum value, and then begins to decrease. The evaporation stops when the mass of the hole decreases to the value $m = gM_{\rm Pl}$. Thus, if the monopole has not evaporated at the maximal heating of the hole (for this it is sufficient that the temperature of the maximal heating is less than the monopole mass) then such an old vacuum remnant at the final stage of evaporation will represent not the usual magnetic monopole but the magnetically charged black hole with the mass

$$m = 2\pi M_{\rm Pl}/e \ . \tag{5.9}$$

The problem of the stability of such a black hole requires special consideration. It might be thought that it will prove to be unstable, decaying on the usual black hole and magnetic monopole.

In cases B and C the shell crosses the R_- region. Any shell crossing the R_- region forms a wormhole.

Let us consider now neutral remnants of the old vacuum in more detail. If such a remnant has a mass obeying the condition

$$m > M_c \equiv M_{\rm Pl}{}^3/3\sqrt{8\pi\epsilon_{\rm out}} \ , \tag{5.10}$$

then the outer metric has no horizons.[39] A shell with such a mass will expand infinitely. At $m < M_c$ the outer metric has two horizons, an event horizon at $\rho = \rho_H$ and a cosmological horizon at $\rho = \rho_c > \rho_H$ (Ref. 39). Possible trajectories of the shells in the Schwarzschild–de Sitter metric in the case $m < M_c$ are shown in Fig. 2. Shells 1 and 2 cross the R_+ region, while shells 3 and 4 cross the R_- region.

In general, the mass spectrum starts from zero for both black holes and wormholes. It may be shown, however, that the mass of any shell crossing the R_+ region is bounded from above if $\xi < 1$.

Indeed, let the shell cross the R_+ region; then we have $\eta(\rho) > 1-\xi$ (see Table II). In the R region $\rho > \rho_H$, so for any ρ in the R region we have

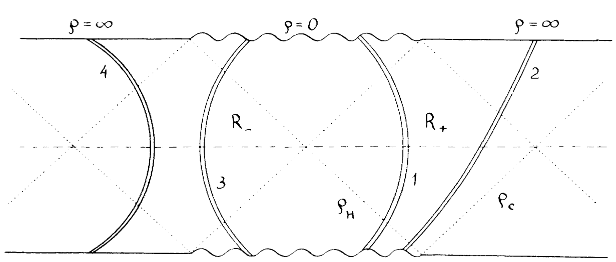


FIG. 2. Penrose diagram for a Schwarzschild–de Sitter geometry with $m < M_{\rm Pl}{}^3/3\sqrt{8\pi\epsilon_{\rm out}}$. The straight lines at ±45° are either a black-hole horizon (ρ_H) or a cosmological horizon (ρ_c). The curvy lines $\rho=0$ denote singularities. The double curves represent all possible paths of shells, the Schwarzschild–de Sitter geometry being the part of the spacetime external to the shell. Shell 1 forms a black hole, while shell 2 expands infinitely. Shells 3 and 4 are crossing the R_- region and form the wormholes (viewing from the R_+ region).

$\eta(\rho_H) > \eta(\rho) > 1-\xi$. Furthermore, we have in the case of the Schwarzschild–de Sitter metric $\rho_H \geq 2\kappa m$; i.e., a shell may cross the R_+ region only if $\eta(2\kappa m) > 1-\xi$ or equivalently

$$m < m_c/\sqrt{1-\xi} \ , \quad m_c \equiv M_{\rm Pl}{}^4/8\pi S \ . \tag{5.11}$$

Thus, a black-hole mass is bounded by the value $m_c \sim (M_{\rm Pl}/M_G)^3 M_{\rm Pl}$ if a wall energy dominates ($|\xi| \leq 1$) and by the value $(M_{\rm Pl}/M_G)^2 M_{\rm Pl}$ if vacuum energy dominates ($|\xi| \gg 1$), M_G being the unification mass scale. For GUT black holes with $M_G \sim 10^{15}$ GeV we obtain $m < 10^{27}$ GeV in the case of vacuum-energy domination and $m < 10^{35}$ GeV in the case of wall-energy domination. Black holes with masses in this range have already evaporated long ago (until now only black holes with masses larger or equal to 10^{39} GeV could survive).

For phase transitions with breaking of the Weinberg-Salam electroweak gauge group, the shell energy does not dominate. Consequently, the maximum mass of the black holes created during this phase transition is of the order 10^{53} GeV $\sim 10^{-4} M_\odot$. These are very unusual black holes. Indeed, recall that black holes originating from collapsing stars are in any case heavier than $1{,}2 M_\odot$.

It may be shown, similarly to (5.11), that a wormhole mass of type B is also bounded at $\xi < -1$ by the value

$$m < m_c/\sqrt{-1-\xi} \ . \tag{5.12}$$

Note also that for the shell having a turning point ($\dot\rho = 0$) we may give the following criterion: if the shell radius at the turning point obeys the inequality

$$\rho < \rho_c \ , \quad \rho_c \equiv \frac{M_{\rm Pl}}{4\pi}\left[\frac{S^2}{M_{\rm Pl}{}^2} + \frac{\epsilon_{\rm in}}{6\pi}\right]^{-1/2} , \tag{5.13}$$

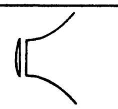
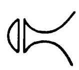
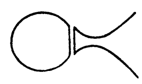
then an object is formed which collapses towards a black hole; otherwise ($\rho > \rho_c$) we are dealing with a wormhole.

D. New phase bubble with the remnant inside

In this section we will consider a new phase bubble with a remnant (BWR) of the old phase inside it; i.e., we consider a configuration which consists of two concentric shells. The metric inside the inner shell is the de Sitter metric; in between the first and the second shells it is of the Schwarzschild–de Sitter type with $m \neq 0$ (but may be with $\epsilon = 0$) and the outer metric coincides with the inner metric, so we denote its parameters by the subscript in also. The values of σ for the first (inner) shell (say $\sigma^{(1)}$) can be directly determined using the previously constructed classification (see Table II, for example). To determine σ's for the second (outer) shell let us write the junction equation for it:

$$\sigma_{\rm in}^{(2)}\left[\dot\rho^2 + 1 - \frac{8\pi\kappa}{3}\epsilon_{\rm out}\rho^2 - \frac{2\kappa m}{\rho}\right]^{1/2} - \sigma_{\rm out}^{(2)}\left[\dot\rho^2 + 1 - \frac{\rho^2}{a_{\rm in}{}^2}\right]^{1/2} = 4\pi\kappa\rho S \ . \tag{5.14}$$

TABLE II. The correspondence between the values of $(\xi+\eta)$ and the signs of σ's; $\xi=M_{\rm Pl}^2(\epsilon_{\rm out}-\epsilon_{\rm in})/6\pi S^2$; $\eta=mM_{\rm Pl}^2/8\pi^2S^2\rho^3$. Schematically shown are spatial sections of resulting spacetimes at the moment when the shell is at rest.

		$\sigma_{\rm IN}$	$\sigma_{\rm OUT}$	SPATIAL SECTION (SCHEMATIC)
A	$\eta+\xi>1$	+1	+1	
B	$-1<\eta+\xi<1$	+1	-1	
C	$\eta+\xi<-1$	-1	-1	

We see that the substitutions

$$\sigma_{\rm in}^{(2)}=-\bar\sigma_{\rm out}^{(2)}\ ,\quad \sigma_{\rm out}^{(2)}=-\bar\sigma_{\rm in}^{(2)} \tag{5.15}$$

bring this equation to the previously constructed classification. Such a configuration describes the spontaneously created BWR if the shells have the rest points.

In view of the spontaneous creation of a BWR, the shells possessing the rest points are of particular interest, so we investigate them in more detail. Taking into account Eq. (5.15), it is clear that it is sufficient to investigate the inner shell only.

To begin, it is convenient to represent the dependence of the mass m of the shell on ρ at the moment when $\dot\rho=0$. In terms of the variables

$$y\equiv-\frac{m}{8\pi^2(1+\xi)\kappa S^2a_{\rm in}^{\ 3}}\ ,\quad x\equiv\rho/a_{\rm in}\ ,$$
$$A\equiv(1+\xi)2\pi Sa_{\rm in}/M_{\rm Pl}^{\ 2} \tag{5.16}$$

[ξ is determined by (3.4)] Eq. (5.1) takes the form

$$y=x^3-\frac{\sigma_{\rm in}}{A}x^2(1-x^2)^{1/2}\ . \tag{5.17}$$

We will distinguish now two cases $A>0$ and $A<0$.

(i) Let $A<0$ (i.e., $1+\xi<0$). Then $\sigma_{\rm in}$ is changed along the curve $y(x)$ (see Fig. 3). Equation (5.17) now takes the form

$$y=x^3+\frac{\sigma_{\rm in}}{|A|}x^2(1-x^2)^{1/2}\ . \tag{5.17$'$}$$

The sign of $\sigma_{\rm in}$ is changed at the point $x=1$ and the sign of $\sigma_{\rm out}$ is changed on the upper branch of the curve $y(x)$ at the point x_σ:

$$x_\sigma^{\ 2}=\frac{1}{1+4[A/(1+\xi)]^2}\ . \tag{5.18}$$

The function $y=y(x)$ is shown in Fig. 3 (if $A<0$, we are dealing with the part of this figure corresponding to $y>0$ only).

(ii) Let $A>0$ (i.e., $1+\xi>0$). The $\sigma_{\rm in}$ is not changed now and Eq. (5.17) takes the form

$$y=x^3-\frac{1}{|A|}x^2(1-x^2)^{1/2}\ . \tag{5.19}$$

The function $y=y(x)$ given by (5.19) is shown in the region $y<0$ of Fig. 3. The sign of $\sigma_{\rm out}$ is changed at the point x_σ as above.

It is convenient to consider A and x_σ as the input parameters when constructing the overall classification. However, the situations with $1+\xi=0$ or $a_{\rm in}\to\infty$ should be described separately. We have for $m(\rho)$ in these degenerate cases, (1) $1+\xi=0$, then $\sigma_{\rm out}$ changes the sign at $x=x_\sigma$ and

$$\frac{m}{4\pi Sa_{\rm in}^{\ 2}}=x^2(1-x^2)^{1/2}\ , \tag{5.20}$$

(2) $a_{\rm in}=\infty$, then

$$\frac{m}{4\pi S}=-\frac{1+\xi}{2\rho_\sigma}\rho^3+\rho^2\ , \tag{5.21}$$

and $\sigma_{\rm out}$ changes the sign at $\rho=\rho_\sigma$, where $\rho_\sigma=M_{\rm Pl}^{\ 2}/4\pi S$. In both cases, $\sigma_{\rm in}=+1$.

The remarkable property of the function $m(\rho,\dot\rho=0)$ is as follows: there exist two shells (if any) with different values of ρ at the rest point for the given value of m [the only exclusions are the extremum points of $m(\rho)$ and the case $1+\xi<0$, $a_{\rm in}=\infty$]. Note that two shells with $m=0$ possess $x=0$ and

$$x^2(m=0)=\frac{1}{1+A^2}\ . \tag{5.22}$$

It is easy to see that ρ determined by this equation coin-

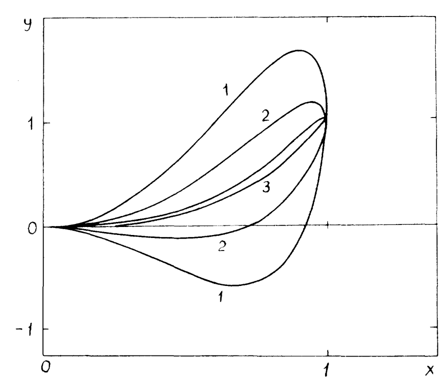


FIG. 3. Dependence of the mass of the shell on the shell radius ρ at the moment of rest. The variables are $y=-mM_{\rm Pl}^2/8\pi^2(1+\xi)S^2a_{\rm in}^3$ and $x=\rho/a_{\rm in}$, where $\xi=(\epsilon_{\rm out}-\epsilon_{\rm in})M_{\rm Pl}^2/6\pi S^2$, $a_{\rm in}^{-2}=8\pi\epsilon_{\rm in}/3M_{\rm Pl}^2$. The curves are parametrized by the values of $|A|$, where $A=(1+\xi)2\pi Sa_{\rm in}/M_{\rm Pl}^2$. The upper part of each curve $(y>0)$ corresponds to values of $A<0$, while the lower part to $A>0$. Curves corresponding to larger values of $|A|$ are inside those corresponding to smaller values of $|A|$. At the point $x_\sigma=\{1+4[A/(1+\xi)]^2\}^{-1/2}$ the quantity $\sigma_{\rm out}$ changes sign, while $\sigma_{\rm in}$ changes sign at the point $x=1$. For a given value of $m(y)$ there exist two shells with different radii.

cides with the radius B^{-1} given by Eq. (4.2b) for the empty new phase bubble.

Now we are able to construct the configuration consisting of two concentric shells. The procedure is as follows. First we put the first (inner) shell at any point on the curve $y(x)$ [or on the curves given by (5.20) and (5.21)] and obtain the values of $\sigma^{(1)}$ directly from Fig. 3. Then we have to set the second shell at any of two points with the same values of y (i.e., the same values of m). The same procedure has to be used for obtaining $\sigma^{(2)}$: we first get the $\bar{\sigma}^{(2)}$'s using $y(x)$ (see Fig. 3) and then obtain $\sigma^{(2)}$ by the rules (5.19).

The shells lying to the left from the extremum point of $y(x)$ contract, while the ones to the right expand. When both shells are at the same point on the curve $y(x)$ it does not mean that they coincide in the configurational space. In fact, the shells turn up to lie on different sides of the horizon. The whole configuration contains from the very beginning either the cosmological or the black-hole horizon inbetween two shells (and describes mostly the spontaneously created wormholes), the only exception is the case $\xi<-1$ when we put the first shell in the region where $\sigma_{\rm in}^{(1)}=\sigma_{\rm out}^{(1)}=+1$, and the second shell lies at the point with $\bar{\sigma}_{\rm in}^{(2)}=\bar{\sigma}_{\rm out}^{(1)}=-1$ on the curve $y(x)$. Then the values $\sigma_{\rm in}^{(2)}=\sigma_{\rm out}^{(2)}=+1$ will correspond to the second shell and the whole configuration does not contain initially any horizons. In the course of evolution the inner shell will collapse to form the black hole. We find from Fig. 3 that the radius of the outer shell in the case of the bubble with the remnant but without any horizon (the black-hole case) is larger than B^{-1} and the mass of the resulting black hole is constrained by

$$m<\frac{M_{\rm Pl}{}^4 x_0{}^3(1-\xi)}{\pi S(4x_0{}^2+1)^{3/2}} . \qquad (5.20')$$

The mass of any configuration is obviously bounded from above by the value corresponding to the maximum of $y(x)$. The consideration and the calculation of decay probabilities onto such configurations leading to the creation of black holes and wormholes will be carried out in our forthcoming papers.

VI. NULL SHELLS

Until now we have considered only timelike and spacelike shells. As it was shown, timelike shells describe the motion of walls of bubbles nucleated in the course of first-order phase transitions when there is a barrier between different phases in the effective potential. The spontaneous creation of a bubble is described by a spacelike shell. Let us consider now the case when the effective potential has one minimum only, but in a certain region of space the magnitude of free energy of a system is for some reason larger than on average. The state of the system in this region is obviously a nonequilibrium one. This is just the case, for example, in the chaotic inflationary-universe scenario.[24] Suppose that gradients of the scalar field nearby to boundaries of this nonequilibrium region are large enough so that one may treat the scalar field distribution approximately as a step function. (There would be correspondingly δ-functional singularities in the energy-momentum tensor.) Such scalar field discontinuities propagate along characteristic surfaces. In other words, the boundaries of the region move with the velocity of light. It is interesting, therefore, to study null shells and junction conditions for metric tensor and its derivatives on lightlike hypersurfaces.

A. Junction conditions

Let a null hypersurface be determined by Eq. (2.1), $F(y^\mu)=0$. The normal vector to this surface is given as before by the condition

$$dF=F_{,\alpha}dy^\alpha=0 , \quad N_\alpha=F_{,\alpha} . \qquad (6.1)$$

However, now Eq. (2.1) should describe the null surface, so one cannot choose the normal vector as a unit vector one, because

$$F_{,\alpha}F^{,\alpha}=0 . \qquad (6.2)$$

Thus, the formalism described in Sec. II is useless here; in particular, one cannot construct the Gaussian coordinates. Instead, let us choose one of the coordinates as follows:

$$u\equiv F(y^\mu) , \qquad (6.3)$$

so in these coordinates the surface equation is simply $u=0$ and the normal vector has the following covariant components:

$$N_u=1 , \quad N_i=0 , \quad i=1,2,3 . \qquad (6.4)$$

From condition (6.2) $g^{\alpha\rho}N_\alpha N_\beta=0$ it follows that $g^{uu}=0$. Insofar as

$$N^\beta=g^{\beta u} , \qquad (6.5)$$

we have $N^u=0$. This means that the normal vector to the surface $u=$const lies just on this surface. We may choose one of three coordinate curves on the surface (denoted by v) in such a way that the tangential vector $L^\beta\equiv(0,dv,0,0)$ is proportional to the normal vector N^β [Eq. (6.5)]. In this frame of reference we obtain $g^{2u}=0$, $g^{3u}=0$. Furthermore, $0=N^\alpha N^\beta g_{\alpha\beta}=(g^{vu})^2 g_{vv}$, so, in addition, $g_{vv}=0$. We have, therefore,

$$g^{\alpha\beta}=\begin{bmatrix} 0 & g^{uv} & 0 & 0 \\ g^{vu} & g^{vv} & g^{v2} & g^{v3} \\ 0 & g^{2v} & & \\ 0 & g^{3v} & & \alpha^{ij} \end{bmatrix} , \quad i,j=1,2 . \qquad (6.6)$$

The tensor $g_{\alpha\beta}$ being inverse to $g^{\alpha\beta}$ has the following structure:

$$g^{\alpha\beta}=\begin{bmatrix} g_{uu} & g_{uv} & g_{u2} & g_{u3} \\ g_{vu} & 0 & 0 & 0 \\ g^{2u} & 0 & & \\ g^{3u} & 0 & & \alpha_{ij} \end{bmatrix} , \qquad (6.7)$$

with $g^{uv}g_{uv}=1$.

By definition, the metric tensor may be

nondifferentiable only in the u direction. That is, the Ricci tensor may contain a δ function only in the terms $\partial_u \Gamma_{u\mu}{}^{\mu}$ and $\partial_u \Gamma_{\alpha\beta}{}^{u}$. The first one is

$$\partial_u \Gamma_{u\mu}{}^{\mu} = \frac{\partial^2 \ln\sqrt{-g}}{\partial u^2} \tag{6.8}$$

and the second, in view of Eqs. (6.6) and (6.7), may contain δ functions only due to the $\Gamma_{uu}{}^{u}$ [see (2.11)]:

$$\Gamma_{uu}{}^{u} = g^{uv} g_{vu,u} \ . \tag{6.9}$$

In other words, only the G_{uu} component contains a δ function; so extracting the $\partial^2/\partial u^2$ term only, we have, using Eqs. (6.6) and (6.7),

$$G_{uu} = R_{uu} = -(\ln\sqrt{\det\alpha_{ij}})_{,uu} \ . \tag{6.10}$$

Integrating the uu component of the Einstein equation, we obtain the desired junction equation

$$[(\ln\sqrt{\det\alpha_{ij}})_{,u}] = -8\pi\kappa S_{uu} \ . \tag{6.11}$$

All other components of the tensor

$$S_{\mu\nu} = \lim_{\delta\to 0} \int_{-\delta}^{\delta} T_{\mu\nu} du$$

are equal to zero. We see that the lightlike shells have the specific structure of the surface energy-momentum tensor $S_\alpha{}^\beta$ with the $S_u{}^v = g^{vu} S_{uu}$ being the only nonzero component.

Let us find now the connection of the $S_\alpha{}^\beta$ to the (possible) discontinuities of $T_\mu{}^\nu$ [i.e., the analogue of Eqs. (2.29) and (2.30) for a lightlike shell]. Integrating the $T_\mu{}^\nu$ continuity equation (2.7), we obtain, finally,

$$[T_v{}^u] = [T_i{}^u] = 0 \ , \tag{6.12a}$$

$$\frac{(g_{uv}\sqrt{\det\alpha_{ij}} S_u{}^v)_{,v}}{g_{uv}\sqrt{\det\alpha_{ij}}} + [T_u{}^u] = 0 \ . \tag{6.12b}$$

B. Spherical shells

The spherical symmetry means that the spacetime ${}^4V$ is the product ${}^4V = {}^2V \otimes {}^2S$, where ${}^2S$ is a two-dimensional sphere, so the interval is

$$ds^2 = 2H(u,v) du\, dv - r^2(u,v) d\Omega^2 \ , \tag{6.13}$$

and the junction equation (6.11) takes the form

$$[r,u] = -4\pi\kappa r S_{uu} \ . \tag{6.14}$$

Multiplying Eq. (6.14) by $g^{uv} r_{,v}$ we obtain the invariant form of the junction equation:

$$[\Delta] = -4\pi\kappa r S_u{}^v r_{,v} = -4\pi\kappa S \ . \tag{6.15}$$

Note that the shape of the phase separation surface is now completely determined. Given the metrics inside and outside the bubble, Eq. (6.15) gives the junction.

Let us consider in more detail the junction of two de Sitter metrics with different values of vacuum energy densities. It follows from Eq. (6.15) that

$$\frac{8\pi\kappa}{3} r^3 (\epsilon_{\text{out}} - \epsilon_{\text{in}}) = -4\pi\kappa r^2 S \ . \tag{6.16}$$

Let $\epsilon_{\text{in}} > \epsilon_{\text{out}}$, then $S > 0$. Supposing that the shell under consideration is due to the scalar field, one obtains

$$S_{uu} = \int_{-\delta}^{\delta} \varphi_{,u}{}^2 du > 0 \ . \tag{6.17}$$

Therefore, $r_{,v}/H > 0$. Recall that we have chosen the coordinates v and u in such a way that the tangential vector to the v-coordinate line is directed along the shell and is future directed while the tangential vector to the u-coordinate line is directed from the in region to the out region.

Let the shell be moving in such a way that the coordinate space diminishes. The physical size of the shell (its radius) will then decrease if the motion takes place in the R_+ region or in the T_- region, while it will increase if the motion takes place in the T_+ region or in the R_- region. Then the u-coordinate line is future directed and $H > 0$. Hence, $r_{,v} > 0$, so that the radius of the shell (the shell's physical size) increases along with time; moreover, such a motion is only possible in the T_+ region or in the R_- region.

If on the contrary the coordinate space of the internal region is increasing, then the u-coordinate line is past directed (i.e., this case could be obtained from the previous one by the substitution $u \to v$, $v \to -u$) and $H < 0$. Hence, $r_{,v} < 0$, so that the shell radius is decreasing along with time. Such a motion is only possible in the T_- region or in the R_- region.

Despite the fact that the considered example of the junction of two de Sitter metrics is unrealistic it allows some important conclusions to be drawn. First, the coordinate volume of the fluctuation giving rise to a nonequilibrium configuration of the scalar field in some region of space (as in the chaotic inflation scenario) should decrease with time, while its physical size should increase. Therefore, the boundary of the fluctuation should lie either in the T_+ region or in the R_- region. The latter case is only possible if this fluctuation forms a wormhole or if it occupies, from the very beginning, more than a half of a closed universe.[10] Since the visible part of our Universe is obviously in an R_+ region, the creation of such a fluctuation at the moment, say, at laboratory conditions, is impossible. Second, if in the very early Universe, when, as it is assumed, there existed opportunities for the creation of such a fluctuation (i.e., the T_+ region had occupied larger coordinate volume) and the energy dominance condition was not violated (the Λ term was equal to zero), then the very existence of the T_+ region requires the occurrence of an initial singularity. The detailed investigation of a more realistic model of evolution of the nonequilibrium scalar field fluctuation will be carried out in a forthcoming paper .

Note added. After this work was completed we became aware of new important results on bubble dynamics.[40]

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APPENDIX A: EINSTEIN EQUATIONS FOR A SPHERICALLY SYMMETRIC METRIC

Our purpose here is to derive the Einstein equations for a spherically symmetric metric in the form convenient for further use.

Let the metric of spacetime be

$$ds^2=e^{\nu}dt^2-e^{\lambda}dq^2-r^2(q,t)d\Omega^2 \; . \tag{A1}$$

As was already mentioned, one can always write the metric in this form in the case of spherical symmetry. [Note, by the way, that the coordinate system still is not fixed completely; namely, one may establish one more coordinate condition. We shall not however do that, since it is good enough for our purposes to use metric in the form (A1).] Let us write the Einstein equations as

$$-e^{-\lambda}\left[2\frac{r''}{r}+\frac{r'^2}{r^2}-\frac{r'\lambda'}{r}\right]+e^{-\nu}\left[\frac{\dot r\dot\lambda}{r}+\frac{\dot r^2}{r^2}\right]+\frac{1}{r^2}=8\pi\kappa T_0{}^0 \; , \tag{A2}$$

$$-e^{-\lambda}\left[\frac{\nu' r'}{r}+\frac{r'^2}{r^2}\right]+e^{-\nu}\left[2\frac{\ddot r}{r}+\frac{\dot r^2}{r^2}-\frac{\dot r\dot\nu}{r}\right]+\frac{1}{r^2}=8\pi\kappa T_1{}^1 \; , \tag{A3}$$

$$-2\frac{\dot r'}{r}+\frac{r'\dot\lambda}{r}+\frac{\dot r\nu'}{r}=8\pi\kappa T_{01} \; , \tag{A4}$$

$$-e^{-\lambda}\left[\frac{\nu''}{2}+\frac{\nu'^2}{4}+\frac{r''}{r}-\frac{r'\lambda'}{2}+\frac{r'\nu'}{2}-\frac{\nu'\lambda'}{4}\right]+e^{-\nu}\left[\frac{\ddot\lambda}{2}+\frac{\dot\lambda^2}{4}+\frac{\ddot r}{r}-\frac{\dot r\dot\nu}{2}+\frac{\dot r\dot\lambda}{2}-\frac{\dot\nu\dot\lambda}{4}\right]=8\pi\kappa T_2{}^2=8\pi\kappa T_3{}^3 \; . \tag{A5}$$

Multiplying now Eq. (A2) by $r^2 r'$ and Eq. (A3) by $-e^{-\nu}r^2\dot r$ and then summing, one obtains

$$e^{-\nu}(2\dot r\,'\dot r-r\dot r^2\nu'+\dot r^2 r')-e^{-\lambda}(2rr''r'+r'^3-rr'^2\lambda')+r'=8\pi\kappa r^2(T_0{}^0 r'-T_1{}^0\dot r) \; . \tag{A6}$$

This equation may be rewritten in the form

$$[r(1+\Delta)]'=8\pi\kappa r^2[(T_0{}^0+T_1{}^1)r'-T_1{}^0\dot r-T_1{}^1 r'] \; . \tag{A7}$$

Similarly, multiplying Eq. (A3) by $r^2\dot r$ and Eq. (A4) by $e^{-\lambda}r^2 r'$ and then summing, one obtains

$$[r(1+\Delta)]^{\cdot}=8\pi\kappa r^2[(T_0{}^0+T_1{}^1)\dot r-T_0{}^0\dot r-T_0{}^1 r'] \; . \tag{A8}$$

Equations (A7) and (A8) may be now combined in one vectorial equation:

$$[r(1+\Delta)]_{,\mu}=8\pi\kappa r^2(Tr_{,\mu}-T_\mu{}^\nu r_{,\nu}) \; , \quad \mu,\nu=0,1 \; . \tag{A9}$$

Note that at spherical symmetry the components $T_2{}^2=T_3{}^3$ are invariant under the transformations $t=t(t,q)$, $q=q(t,q)$ and therefore the sum $T=T_0{}^0+T_1{}^1$ is also invariant.

This vectorial equation (which is equivalent to two scalar equations) has to be complemented by the conservation equations

$$T_\mu{}^\nu{}_{;\nu}=0 \; . \tag{A10}$$

Here a semicolon means the covariant derivative with respect to the total four-dimensional metric (A1). In the case of spherical symmetry we have here, in fact, only two equations in (A10). These also may be reduced to a two-dimensional vectorial equation. One obtains as a result

$$T_\mu{}^\nu{}_{|\nu}+\frac{2}{r}(-r_{,\mu}T_2{}^2+r_{,\nu}T_\mu{}^\nu)=0 \; , \tag{A11}$$

where the vertical bar denotes covariant differentiation with respect to the metric of two-dimensional space (t,q).

The set of Eq. (A9) and (A11) is equivalent to the Einstein equations in the case of spherical symmetry.

APPENDIX B: OUTER CURVATURE TENSOR OF SPHERICAL SHELL FOR THE FRIEDMANN-ROBERTSON-WALKER METRIC

Let us rewrite the expressions for $K_2{}^2$ and $K_0{}^0$ [Eqs. (2.54a) and (2.58), respectively]:

$$K_2{}^2=-\frac{\sigma}{\rho}(\dot\rho^2-\Delta)^{1/2} \; , \tag{B1}$$

$$K_0{}^0=-\frac{\sigma}{(\dot\rho^2-\Delta)^{1/2}}\left[\ddot\rho+\frac{1+\Delta}{2\rho}-4\pi\kappa\rho T_n{}^n\right] \; , \tag{B2}$$

where

$$\Delta=g^{\alpha\beta}\rho_{,\alpha}\rho_{,\beta} \; ,$$

and $T_\alpha{}^\beta$ is in our case the energy-momentum tensor for a perfect fluid:

$$T_\alpha{}^\beta=(\epsilon+p)u_\alpha u^\beta-p\delta_\alpha{}^\beta , \tag{B3}$$

ϵ, p, and u^α being the energy density, pressure, and four-velocity, respectively. Thus, we need only to calculate Δ and $T_n{}^n$ for the Friedmann-Robertson-Walker (FRW) metric

$$ds^2=dt^2-a^2(t)\left[\frac{dq^2}{1-kq^2}+q^2d\Omega^2\right] ,$$
$$\rho=aq \; , \quad u^0=u_0=1 \; , \quad u^1=u_1=0 \; . \tag{B4}$$

From the Einstein equation for the FRW metric,

$$\frac{\dot a^2+k}{a^2}=\frac{8\pi\kappa}{3}\epsilon , \tag{B5}$$

we find easily that

$$\Delta=-1+\frac{8\pi\kappa}{3}\epsilon\rho^2 , \tag{B6}$$

so

$$K_2{}^2=-\frac{\sigma}{\rho}\left[\dot\rho^2+1-\frac{8\pi\kappa}{3}\epsilon\rho^2\right]^{1/2} \tag{B7}$$

($\dot\rho\equiv d\rho/d\tau$, τ being the proper time on the shell).

It is convenient to put Eq. (B2) for $K_0{}^0$ in the form

$$K_0{}^0=-\frac{\sigma}{(\dot\rho^2-\Delta)^{1/2}}\left[\ddot\rho-\frac{1+\Delta}{\rho}+4\pi\kappa\rho(\epsilon+p)(u^2+1)\right] , \tag{B8}$$

where $u=u^n=-u_n$. To calculate u we need to transform the FRW metric (B4) into the normal Gaussian form, and after some algebra we obtain the relation

$$\dot\rho=\dot a q(1+u^2)^{1/2}-u(1-kq^2)^{1/2} . \tag{B9}$$

Solving the above equation for $x=(1+u^2)^{1/2}$, we obtain

$$x=\frac{1}{\Delta}\{H\rho\dot\rho\pm[H^2\rho^2\dot\rho^2+\Delta^2-\Delta(\dot\rho^2+H^2)]^{1/2}\} , \tag{B10}$$

where $H=\dot a/a$ is the Hubble radius. We must choose the lower sign in this expression to be sure x is positive both in R ($\Delta<0$) and T regions ($\Delta>0$) and to have a nonsingular transition through the surface $\Delta=0$. Therefore, we obtain the following expression for $K_0{}^0$:

$$K_0{}^0=-\frac{\sigma}{(\dot\rho^2-\Delta)^{1/2}}\left[\ddot\rho-\frac{1+\Delta}{\rho}+4\pi\kappa\rho(\epsilon+p)\left[1+2\frac{H^2\rho^2\dot\rho^2}{\Delta^2}-\frac{1}{\Delta}\{\dot\rho^2+H^2+2H\rho\dot\rho[H^2\rho^2\dot\rho^2+\Delta^2-\Delta(\dot\rho^2+H^2)]^{1/2}\}\right]\right] . \tag{B11}$$

In the pure vacuum case ($\epsilon+p=0$) only the first two terms survive.

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