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# Effective-vector-boson method for high-energy collisions

P. W. Johnson, F. I. Olness, and Wu-Ki Tung
*Illinois Institute of Technology, Chicago Illinois 60616*
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The effective-vector-boson method for high-energy collisions is precisely formulated by using factorization of the helicity amplitudes. All approximations related to kinematics are eliminated. Compact formulas for exact vector-boson distribution functions are derived. The on-mass-shell continuation of the vector-boson hard-scattering cross section is isolated and clarified. Improvements over the leading-logarithmic approximation are dramatic. Application of the new formalism to heavy-fermion and Higgs-boson production processes illustrates its extended range of accuracy and usefulness.

## INTRODUCTION

For high-energy collisions in the TeV range and beyond, the vector gauge bosons $W$ and $Z$ play an increasingly important role. They will behave very much like the usual partons, but their unique couplings to other particles (especially Higgs bosons) will make them particularly valuable for tests of the standard model and also for "new physics." These considerations motivated the introduction of the "effective-vector-boson" (EVB) approximation for calculating high-energy processes initiated by virtual $W$'s and $Z$'s (Ref. 1). The original formulation of this approach uses a number of approximations of uncertain accuracy such as leading-logarithmic expansion, nonunique definition of vector-boson polarization, neglect of off-diagonal terms in summing the polarization indices, etc. Comparisons of this approach to full Feynman-diagram calculations on a case-by-case basis yield encouraging results,[1,2] but do not provide an understanding of the reliability and the limitations of this method.

Applying a recently published factorization technique for analyzing Feynman diagrams,[3] we have developed a precise formulation of the EVB method.[4] This formulation does not invoke any *kinematic* approximations such as those mentioned above. It leads to exact vector-boson distribution functions which greatly improve the accuracy and significantly extend the range of applicability of the EVB approach. The only approximation invoked is the *dynamical* one concerning the on-mass-shell continuation of the vector-boson hard-scattering cross section (which can be process dependent).[1,4,5]

In this Brief Report we summarize the general formalism, present compact expressions for the exact distribution functions, compare these with the commonly used leading-logarithmic formulas to highlight the limitations of the latter, apply the new formalism to two examples (heavy-fermion and Higgs-boson production) illustrating its accuracy and usefulness, and discuss further applications.

## EXACT RESULTS

Consider the vector-boson-exchange contribution to the generic process $f + A \rightarrow f' + X$ where $f$ and $f'$ are light fermions (leptons or quarks), $A$ is any light particle, and $X$ is an arbitrary final state with large invariant mass. The tree diagram [Fig. 1(a)] can be regarded as a two-step process:

$$f(k) \underset{(\mathrm{soft})}{\rightarrow} f'(k') + {}^{*}V(q) \ , \tag{1}$$

$$ {}^{*}V(q) + A(p) \underset{(\mathrm{hard})}{\rightarrow} X(p_X) \ , \tag{2}$$

where ${}^{*}V$ denotes the *virtual* vector boson. The amplitude for the overall process can be written in the factor-

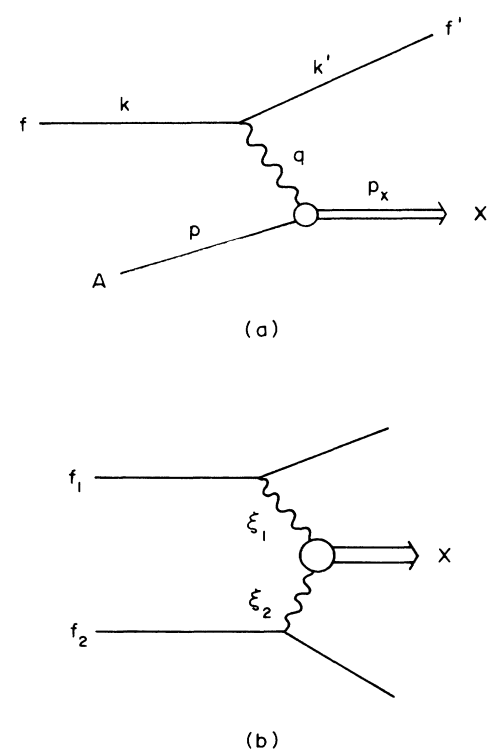


FIG. 1. General Feynman diagrams relevant for (a) single-vector-boson and (b) two-vector-boson hard-scattering processes.

 

ized form[3]

$$T = J_m(q^2, M_X{}^2) \frac{e^{-im\phi} d^1(\xi)^m{}_n}{q^2 + M_V{}^2} j^n(q^2) , \quad (3)$$

where $j^n(q^2) = \langle k' | J \cdot \epsilon^{n*} | k \rangle$ is the helicity vertex function for "emission" of the $^*V$ [Eq. (1)] and $J_m(q^2, M_X{}^2)$ is the corresponding amplitude for the "hard process" [Eq. (2)]. The factor in the middle originates from the vector-boson propagator. The numerator corresponds to elements of the "spin-1" representation matrix of the little group of the vector-boson momentum $q^\mu$. For spacelike $q$, the group is SO(2,1). Its elements are characterized by a Lorentz boost along the $x$ axis by the hyperbolic angle $\xi$ and a rotation around the $z$ axis by the angle $\phi$, both defined in the brick-wall (BW) frame where $q^\mu = (0,0,0,Q)$ (Refs. 3 and 4).

We are interested in the cross section for a given hard-scattering final state, integrated over the unobserved fermion momentum $k'$. A distinct advantage of using the factorized form of the amplitude, Eq. (3), is that the group-theoretical expression suggests that *the helicity density matrix* $J_m^\dagger J_{m'} e^{i(m-m')\phi}$ *should diagonalize* upon integration over the angle $\phi$. To prove this important result, it is necessary to establish that the exponential factor $e^{i(m-m')\phi}$ contains the only $\phi$ dependence in the cross-section formula. This is not obvious as the BW frame is not "inertial": the frame varies with the integration variable $k'$. Thus, the integration measure, in general, depends on $\xi$ and $\phi$. However, it can be shown that the BW frame angle $\phi$ coincides with the c.m. azimuthal angle $\phi_{\text{c.m.}}$ provided the mass of the particle $A$ vanishes ($M_A = 0$). Hence, the relevant phase-space integration measure is simply $d\phi$, and the proof goes through.[6] Even if $M_A$ is nonzero, the off-diagonal elements of the density matrix will be negligible compared to the diagonal ones at high energies, as they will be suppressed by a kinematic factor of order $M_A{}^2/s$ resulting from the $\phi$ integration.

The cross section for a given hard-scattering final state can then be written as a one-dimensional integral over the remaining variable,[4] say $q^2$ (or $\xi$). The integrand consists of the cross section for the $^*V$-initiated hard process (2) (proportional to the trace of the density matrix, $J_m^\dagger J^m$) multiplied by a known function [proportional to the square of the remaining factors of Eq. (3)] which will be identified with the EVB distribution function. (See the next section.)

## THE EFFECTIVE-VECTOR-BOSON FORMULA AND THE EXACT VECTOR-BOSON DISTRIBUTION FUNCTIONS

A precise EVB formula is obtained by replacing the hard-scattering cross section of the virtual vector boson $^*V$ with that of an on-shell vector boson $V$, and bringing it outside the $q^2$ integral described above. This procedure is reliable provided the $^*V$ cross section is a smooth function of $q^2$ over the important range of integration.[4] Because of the propagator factor in the amplitude, Eq. (3), the significant range is $0 < q^2 < M_V{}^2$. The continuation of the cross section to its on-shell value is safe unless it contains a singular factor such as in the case of the longitudinal cross section where we have $^*\epsilon_0(q) = (\,|\mathbf{q}|, 0,0,q^0)/\sqrt{q^2}$ for the *virtual* vector boson. We can take care of known singularities by keeping all rapidly varying factors inside the integral, and applying the continuation only to a *regularized* hard-scattering cross section. In the case of longitudinal polarization, this amounts to writing $^*\epsilon_0(q) = \epsilon_0(q)(M_V/\sqrt{q^2})$—where $\epsilon_0(q) = (\,|\mathbf{q}|, 0,0,q^0)/M_V$ is the longitudinal polarization vector for an *on-shell* vector boson—and keeping the singular factor $(M_V/\sqrt{q^2})$ inside the integral. In this way we obtain the EVB formula

$$d\sigma(s, \ldots) = f_n(x)\, d\hat{\sigma}^{\,n}(\hat{s}, \ldots)\, dx , \quad (4)$$

where $s = -(k+p)^2$, $\hat{s} = -p_X{}^2 = M_X{}^2$, $x = \hat{s}/s$. $d\hat{\sigma}^{\,n}$ is the hard-scattering cross section for an on-shell $V$ of helicity $n$ ($n = 1, 0, -1$). $f_n(x)$ is the exact vector-boson distribution function (VBDF); it has the usual parton interpretation as the probability of finding a vector boson of helicity $n$ with fractional momentum $x$ in a incoming high-energy fermion $f$.

Since $f_n(x)$ is defined as an integral of known factors, it can be precisely determined. If we write the elementary coupling between the fermions and the vector boson as $\bar{\Psi}\Gamma_\mu \Psi V^\mu$ with

$$\Gamma_\mu = g_R \gamma_\mu (1+\gamma_5)/2 + g_L \gamma_\mu (1-\gamma_5)/2 \quad (5)$$

and carry out the calculations described in the last two sections, we obtain the following exact distribution functions:

$$f_0 = (g_L{}^2 + g_R{}^2) \left[ \frac{x}{16\pi^2} \right] \left[ \frac{2(1-x)\xi}{\omega^2 x} - \frac{2\Delta(2-\omega)}{\omega^3} \ln \left[ \frac{x}{\Delta'} \right] \right] , \quad (6)$$

$$f_T = f_{+1} + f_{-1} = (g_L{}^2 + g_R{}^2)(h_1 + h_2) , \quad (7)$$

$$f_\Delta = f_{+1} - f_{-1} = (g_L{}^2 - g_R{}^2)(h_1 - h_2) , \quad (8)$$

where

$$h_1 = \left[ \frac{x}{16\pi^2} \right] \left[ \frac{-(1-x)(2-\omega)}{\omega^2} + \frac{(1-\omega)(\xi-\omega^2)}{\omega^3} \ln \left[ \frac{1}{\Delta'} \right] - \frac{\xi - 2x\omega}{\omega^3} \ln \left[ \frac{1}{x} \right] \right] , \quad (9)$$

$$h_2 = \left[ \frac{x}{16\pi^2} \right] \left[ \frac{-(1-x)(2-\omega)}{\omega^2(1-\omega)} + \frac{\xi}{\omega^3} \ln \left[ \frac{x}{\Delta'} \right] \right] , \quad (10)$$

$\omega = x - \Delta$, $\xi = x + \Delta$, $\Delta = M_V{}^2/s$, $\Delta' = \Delta/(1-\omega)$, and the $\pm 1, 0$ subscript on $f$ refers to the vector-boson helicity. The parity-odd amplitude $f_\Delta$ is a measure of the polarization of the vector boson induced by its chiral coupling.[7]

The above distribution functions reduce to those of the "leading-logarithmic" (LL) approximation in the limit $\Delta \to 0$. Unfortunately, the LL approximation can be quite misleading except in certain restricted kinematic ranges, as we will discuss in the next section.

The above formalism can clearly be applied to the two-vector-boson process $f_1+f_2 \to f_{1'}+f_{2'}+X$, where both $f_1$ and $f_2$ serve as the source of a vector boson [Fig. 1(b)]. The EVB formula takes the general parton-model form

$$\sigma(s,\ldots)=\int d\xi_1 \int d\xi_2 f_n(\xi_1) f_m(\xi_2)\hat{\sigma}^{nm}(\hat{s},\ldots) \ . \qquad (11)$$

Because of factorization, the VBDF's $f_n(\xi)$ are exactly the same as obtained above.[8]

## LIMITATIONS OF THE LEADING-LOGARITHMIC FORMULAS

Numerical calculations show that the commonly used LL formula for the longitudinal distribution approximates $f_0$ given in Eq. (6) fairly well for $x>0.05$ at, say, $(\hat{s})^{1/2}=1$ TeV—a typical "high-energy" value for parton-parton subprocesses in next-generation colliders.[4] *The discrepancy grows rapidly as x decreases,* reaching a factor of about 4 at $x=0.01$ (see Fig. 2). The situation for the transverse distributions is much worse: for the same $(\hat{s})^{1/2}$, *the LL expression is more than a factor of 2 above the exact result for all x, and the discrepancy grows to more than an order of magnitude when* $x\leq 0.01$. The case-by-case tests of the EVB approximation in the literature usually yield satisfactory results because almost all cases studied so far are dominated by the longitudinal cross section, *and* all involve $x$ ranges above 0.05. In addition to providing a general understanding of this fact, our results clearly mark the limits beyond which the usual LL approximation must break down. The gross overestimate of the transverse distributions in the LL approximation shown here was not noted before. A close examination of the relevant formulas, Eqs. (7)–(10), reveals that this unexpected large discrepancy results from the near cancellation of the LL term with the neglected "constant term" in the small $\Delta$ $(=M_V{}^2/s)$ expansion. (Terms neglected in the corresponding expansion for the longitudinal distribution function are of order $\Delta$, and hence are indeed small.)

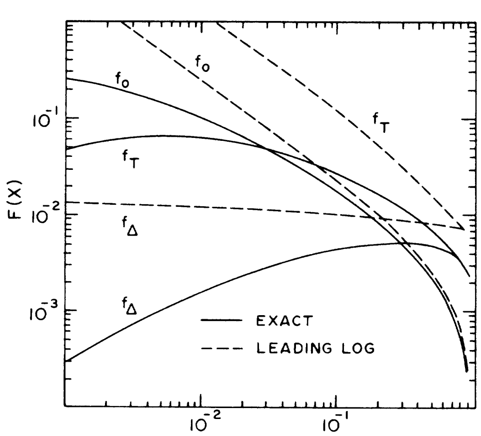


FIG. 2. Comparison of exact vector-boson distribution functions, Eqs. (6)–(10), with their leading-logarithmic approximations for $(\hat{s})^{1/2}=1$ TeV.

## ACCURACY OF THE IMPROVED EVB FORMULA

Adopting the exact VBDF's in the effective vector-boson formulas, Eqs. (4) and (11), avoids all approximations which are kinematic in origin. The only source of error comes from the on-shell continuation of the regularized hard-scattering amplitudes. We have examined the accuracy of this "improved" EVB method by comparing its numerical predictions with full Feynman-diagram calculations for two cases: (i) heavy-fermion production by single-vector-boson scattering[4] and (ii) Higgs-boson production by $W^+W^-$ fusion.

In case (i) the use of the exact VBDF's, Eqs. (6)–(8), in the improved EVB formula is found to lead to an accuracy better than a few percent for longitudinal polarization (vs a factor of 2 for the LL approximation), and better than 25% for transverse polarization (vs a factor of 10 for the LL approximation).[4]

Case (ii), Higgs-boson production by $W^+W^-$ fusion [Fig. 1(b)], is of much interest for collider physics and has been calculated in the LL approximation of the EVB method[1,2] as well as in full Feynman-diagram evaluation.[9] We have repeated the relevant calculations pertaining to the EVB method and compared the results with those obtained with our improved VBDF's. Some typical results on the total cross-section ratios, $\sigma$(LL)/$\sigma$(full) and $\sigma$(improved EVB)/$\sigma$(full), are presented in Fig. 3. As expected, the bulk of the error of the LL-EVB approximation is eliminated by using the exact VBDF's, Eqs. (6)–(8). We observe the following. (i) The error of our EVB formula is expected to be of order $M_W{}^2/M_H{}^2$, due to the on-shell continuation. The numerical results confirm this expectation. (ii) Most LL calculations in the literature ignore the contribution from the transverse polarization.[10] This is

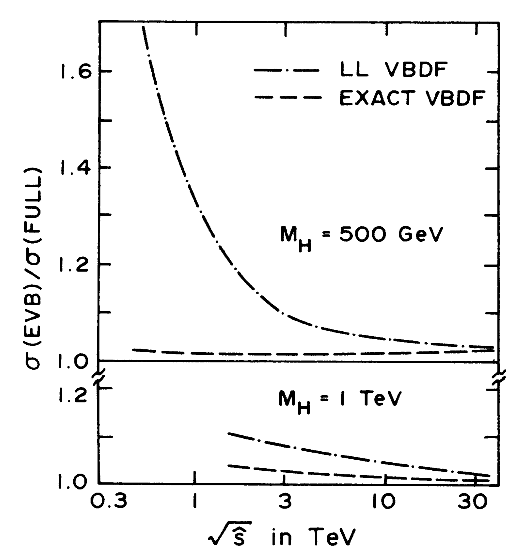


FIG. 3. Ratios of EVB cross sections to full Feynman-diagram calculations for Higgs-boson production as a function of $(\hat{s})^{1/2}$.

safe for heavy Higgs bosons (say $M_H > 1$ TeV). However, the transverse contribution rises rapidly with decreasing $M_H$, reaching about 30% of the total cross section for $M_H$ of the order 300 GeV (not shown in the figure).

## CONCLUDING REMARKS

The precise formulation of the EVB method preserves the calculational simplicity and the attractive physical interpretation of the parton-model approach. The use of the exact distribution functions reduces the errors of this approximation to truly negligible proportions. This significantly increases the range of applicability of the EVB method—to cover almost all regions of interest.

Our formulation of the problem clearly identifies the on-shell continuation of the hard-scattering amplitude as the only source of error in the EVB approach. Thus, the method is guaranteed to work if potential singularities in the variable of continuation are handled appropriately. We have explicitly treated the kinematic singularity associated with the longitudinal-polarization vector. It is not hard to see that certain dynamical singularities can be treated in a similar fashion. There are questions raised by the fact that Feynman diagrams of the general kind represented in Fig. 1 are not strictly gauge invariant by themselves. We have treated the problem in the unitary gauge which is normally the natural one to use for tree diagrams. For certain processes, care might be required to make sure that the relevant diagrams are indeed the dominant ones, and that the $q^2$ continuation is smooth in the particular gauge used for the calculation. These considerations are, of course, process dependent. One of the advantages of our approach is the natural separation of the precise handling of the kinematic (process-independent) factor from the approximate treatment of the dynamic (process-dependent) factor. An important application of the EVB method in which most of the above considerations come into play is $WW$ scattering.[11,12] The presence of a dynamical singularity due to the photon exchange in the $t$ channel,[11,12] and the complications arising from gauge dependence[5] have both received attention recently.

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