Calculation of diquark masses in QCD

R. T. Cahill, C. D. Roberts, and J. Praschifka
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042
(Received 17 February 1987)

Baryons in QCD are three-quark color-singlet bound states and any two of the quarks are necessarily in a $\bar{3}$ color state. These $\bar{3}$ diquark states form bound states which, if they are of sufficiently low mass, will play a significant dynamical role in the structure of baryons. Here we estimate these diquark masses by using an approximate homogeneous Bethe-Salpeter (BS) equation. A new technique for determining the mass of a bound state from the BS equation, which avoids the need to numerically solve the integral equation, is introduced and its validity demonstrated by application to various meson states. Our diquark mass values are compared with those determined from analysis of experimental data on inclusive production of protons.

I. INTRODUCTION

The standard model of hadrons is quantum chromodynamics (QCD) and in this model hadrons are understood as color-singlet bound states of quarks with the force between the quarks arising from the exchange of self-interacting gluons. In particular, baryons and mesons are primarily qqq and $\bar{q}q$ color-singlet states, respectively. The binding of these states is related to the color algebra of QCD. The baryon states of QCD are interesting because, in contrast, quantum electrodynamics does not bind three electron states. This feature of QCD may be understood by noting that in a color-singlet baryon any two quarks are necessarily in a $\bar{3}$ color state and that for $\bar{3}$ qq states, gluon exchange is attractive and believed to lead to the formation of bound states which are named diquarks. If, as the evidence suggests, diquarks have sufficiently low effective masses, then they will play a significant dynamical role in the structure of baryons. Of course, because the diquarks, like the quarks, carry color charge they are assumed to be confined; that is, they may only be "observed" as constituents of baryons. The purpose of this paper is to estimate the masses of these diquark states by using a well-defined approximation scheme for QCD and to compare our diquark mass predictions with those obtained from the analysis of experimental data.

Diquarks were introduced by Ida and Kobayashi[1] and by Lichtenberg and co-workers[2] and in recent years it has become clear that diquarks may provide a means for understanding baryonic properties and reactions. For example, Fredriksson, Jändel, and Larsson[3] have suggested that the diquarks could explain the trends in high-energy data that are usually attributed to gluon processes as described by perturbative QCD. They have also shown[4] that the bulk of data from deep-inelastic ep, eD, μp, μN, νp, and νN scattering data is fitted with a diquark model of nucleons. Fredriksson and Jändel[5] have shown that there is evidence for diquarks from the slow decrease in probability for double collisions with increasing number of hadrons produced in the first projectile-nucleon collision. An analysis of experimental data on inclusive production of protons by Laperashvili[6] confirms the dominant role of hard scattering of quarks by diquarks in regions of large transverse momenta of hadron production, and extracts a scalar (ud) diquark mass bound of >400 MeV. Dziembowski, Metzger, and Van de Walle[7] show that diquarks can provide a quantitative interpretation of the negative charge radius of the neutron.

Calculations of diquark masses have used potential models[8] and the MIT bag.[9] However, the validity of and the relationship to QCD of these model calculations is not clear. Here we will attempt to calculate diquark masses using an approximate form of the covariant homogeneous Bethe-Salpeter (BS) equation for qq bound states in QCD, and so the nature of our approximations, which is to neglect nonplanar graphs and $n>2$ gluon n-point functions, is clear. Of course, such integral equations can only be solved by difficult numerical computations. To overcome this problem an analytical procedure is introduced which reformulates the BS integral equations as a variational problem for the bound-state mass functional. While variational calculations then determine the bound-state masses and the corresponding form factors, these mass functionals also allow bounds to be easily placed on these masses. The calculation of these bounds exploits particular nonperturbative features of QCD related to the manner in which chiral symmetry is realized.

Any approximate treatment of QCD always attracts criticism of the validity of the approximations employed. Hence we also apply the same treatment to the mesons. We obtain masses for the pseudoscalar meson octet (the Nambu-Goldstone bosons of QCD) and the ρ and ω masses. For the pseudoscalar octet their masses are shown to automatically satisfy the Gell-Mann–Okubo mass formula. Thus our approximation procedure is shown to work both qualitatively and quantitatively for the mesons, and so we may attach some significance to our diquark mass results.

In Sec. II the Bethe-Salpeter equations for mesons and diquark states are presented along with the Schwinger-Dyson (SD) equation for the quark propagator, which plays a vital role in our analytical treatment of the BS equation. It is the interplay between the SD and meson BS equations which allows one to understand the dual role of the pseudoscalar mesons in QCD, namely, that of $\bar{q}q$ bound states and of Nambu-Goldstone bosons. The SD equation solutions indicate the structure of the QCD vacuum manifold and thus the mode of realization of chiral symmetry in QCD. In Sec. III explicit expressions for meson mass functionals are obtained and the results mentioned above derived. In Sec. IV the techniques are applied to the calculation of various diquark masses. The significance of the results is discussed finally in Sec. V.

II. MESONS AND DIQUARKS

We intend to estimate meson and diquark masses by truncating the SD equation for the quark propagators [Fig. 1(a)] which are then used in the truncated homogeneous BS equations for the mesons [Fig. 1(b)] and the diquarks [Fig. 1(c)] by keeping only the planar gluon exchanges. We are thus neglecting nonplanar gluon exchanges and the $n>2$ n-point gluon functions $D^{(n)}$. This of course means that we are using a non-gauge-invariant approximation to QCD. We are unaware of any practical way of making gauge-invariant approximations in QCD. However, in one attempt to address this problem, Cornwall[10] has considered the possibility of selectively summing terms in a gauge-invariant manner. At best we shall assume that the 2-point function $D(p)$ we use is an effective one arising from a scheme such as that of Cornwall.

The usual Feynman rules, in Euclidean metric, give for the diagrams in Fig. 1 the following equations. For the quark propagator we have

$$\Sigma(p)=M+\int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{\lambda^a\gamma^\mu}{2}\frac{1}{i\not{q}+\Sigma(q)}\frac{\gamma^\mu\lambda^a}{2} , \tag{2.1}$$

where the quark propagator is $G(q)=[i\not{q}+\Sigma(q)]^{-1}$, M is the quark mass matrix and $\{\lambda^a/2,\, a=1,\ldots,8\}$ are the generators of SU(3), the color group. Equation (2.1) is a nonlinear integral equation in spin, flavor, and color space. For massless quarks ($M=0$) the solution of Eq. (2.1) has the form[11]

$$\Sigma(q)=i[A(q)-1]\not{q}+VB(q) , \tag{2.2}$$

where the matrix V is

$$V=\exp[i\gamma_5(\eta+\pi^bT^b)] .$$

The $\{T^b;\ b=1,\ldots,N_F{}^2-1\}$ are the generators of $SU(N_F)$ where N_F is the number of quark flavors. Here η and π^a are arbitrary real constants. In the massless limit QCD has exact global $G=U_L(N_F)\otimes U_R(N_F)$ chiral symmetry and the presence of the matrix V indicates that a mixed, partial Nambu-Goldstone (NG) realization of chiral symmetry occurs. This and other aspects of the hidden chiral symmetry of QCD are discussed in Ref. 12. The functions $A(q)$ and $B(q)$ are solutions of the coupled integral equations

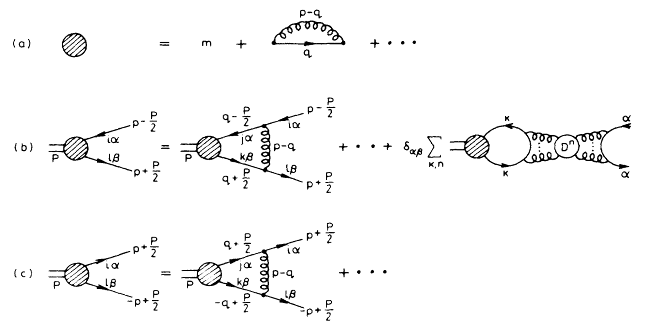

FIG. 1. Feynman diagrams for the various integral equations: (a) the nonlinear Schwinger-Dyson equation [Eq. (2.1)] for the quark mass function; (b) the Bethe-Salpeter integral equation for $\bar{q}q$ meson bound states [Eq. (2.4)]. The quark annihilation diagram, which is shown but not used in Eq. (2.4), only contributes to the isosinglet mesons. The neglect of this process means that Eq. (2.4) is only valid for isovector mesons. Misapplication of Eq. (2.4) to the isosinglet meson leads to the $U_A(1)$ "problem": namely, the underestimation of the η' pseudoscalar-meson mass. (c) The Bethe-Salpeter integral equation for qq diquark bound states. In (b) and (c) the nonperturbative quark propagator from (a) is used. The $i,j,\ldots$ denote color indices while $\alpha,\beta,\ldots$ denote flavor indices. The $+\cdots$ denote higher-order gluonic processes.

$$[A(p)-1]p^2=\frac{8}{3}\int \frac{d^4q}{(2\pi)^4}D(p-q)\frac{A(q)q\cdot p}{A^2q^2+B^2} , \tag{2.3a}$$

$$B(p)=\frac{16}{3}\int \frac{d^4q}{(2\pi)^4}D(p-q)\frac{B(q)}{A^2q^2+B^2} , \tag{2.3b}$$

where we have used $\lambda^a\lambda^a=\frac{16}{3}1$.

For the case of small quark masses the solution of Eq. (2.1) has the form $\Sigma(q)=i[A_m(q)-1]\not{q}+M+B_m(q)$, with $A_m\approx A$ and B_m given by

$$B_m(p)=\frac{16}{3}\int \frac{d^4q}{(2\pi)^4}D(p-q)\frac{M+B_m(q)}{A^2q^2+(M+B_m)^2} . \tag{2.3c}$$

Except in the critical case of the NG bosons we shall use the approximation $B_m\approx B$.

For the $\bar{q}q$ meson bound states the integral equation corresponding to Fig. 1(b) is, using matrix notation,

$$\Gamma(p,P)=-\int \frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^a\gamma^\mu}{2}\frac{1}{iA\left[\not{q}-\frac{\not{P}}{2}\right]+M+B}\Gamma(q,P)\frac{1}{iA\left[\not{q}+\frac{\not{P}}{2}\right]+M+B}\frac{\lambda^a\gamma^\mu}{2} , \tag{2.4}$$

in which we use the convention that, for example, the first A and B functions have argument $(q-P/2)$.

In Eq. (2.4) we have neglected the q-$\bar{q}$ annihilation terms, which are shown in Fig. 1(b), and which can only contribute to isosinglet meson states. Hence Eq. (2.4) is to be used only for isovector mesons. If Eq. (2.4) is incorrectly used for the isosinglet pseudoscalar η' meson, for example, then a mass is predicted which is much lower than the experimental value. This is of course the well-known $U_A(1)$ problem.

The matrix notation of Eq. (2.4) obscures one important point: namely, the difference between quarks and antiquarks. For example, the color structure of Eq. (2.4) has the form

$$\Gamma_{il}=\Gamma_{jk}\lambda_{ji}^{*a}\lambda_{kl}^{a}\ , \tag{2.5}$$

with summation convention for repeated indices. This makes it clear that antiquarks transform as the $\bar{3}$ and quarks as the 3 irreps of the SU(3)-color group. Using the algebra of the color generators Eq. (2.4) may be separated into a color-singlet state and color-octet states, since $\bar{3}\otimes 3=1\oplus 8$. For this purpose it is sufficient to use the form (2.5). Write

$$\Gamma=\Gamma_1 1+\Gamma_8^b\lambda^b\ .$$

Then using $\lambda^a\lambda^a=\frac{16}{3}1$ and $\lambda^a\lambda^b\lambda^a=-\frac{2}{3}\lambda^b$ we obtain two uncoupled integral equations:

$$\Gamma_1=\tfrac{16}{3}\Gamma_1\ , \tag{2.6}$$

$$\Gamma_8^b=-\tfrac{2}{3}\Gamma_8^b,\quad b=1,\ldots,8\ . \tag{2.7}$$

Equations (2.6) and (2.7) cryptically denote equations of the form in (2.4) but with the λ matrices now removed. The sign in Eq. (2.6) is significant as it indicates that gluon exchange is an attractive force in color-singlet $\bar{q}q$ states. This can be seen by noting that for these states Eq. (2.4) is identical to the BS equation for e^+e^- bound states in QED, provided $D(x)$ becomes the photon propagator. Conversely, the opposite sign in Eq. (2.7) indicates that gluon exchange is repulsive in $\bar{q}q$ 8 states.

Note that in the meson equations above, and those for the diquarks below, the quark propagators from Eq. (2.1) are used. This is important because, as we shall see later, it leads to the emergence of current-algebra results for the Nambu-Goldstone bosons. It is also related to the confinement problem of quarks.

For the qq diquark bound states which, because they always carry color charge, can presumably only occur as dynamical constituents of color-singlet states, the equation for Fig. 1(c) is

$$\Gamma(p,P)=-\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^{aT}\gamma^{\mu T}}{2}\frac{1}{iA\left[\not{q}+\frac{\not{P}}{2}\right]^T+M+B}\Gamma(q,P)\frac{1}{iA\left[-\not{q}+\frac{\not{P}}{2}\right]+M+B}\frac{\lambda^a\gamma^\mu}{2}\ , \tag{2.8}$$

using matrix notation and where T denotes the transpose. Defining $\Gamma^c=C\Gamma$ where $C=\gamma^2\gamma^4$ is the charge-conjugation matrix, and using $C^{-1}\gamma^\mu C=-\gamma^{\mu T}$, Eq. (2.8) becomes

$$\Gamma^c(p,P)=-\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^{aT}\gamma^{\mu}}{2}\frac{1}{iA\left[\not{q}+\frac{\not{P}}{2}\right]-M-B}\Gamma^c(q,P)\frac{1}{iA\left[-\not{q}+\frac{\not{P}}{2}\right]+M+B}\frac{\lambda^a\gamma^\mu}{2}\ . \tag{2.9}$$

The color structure of Eq. (2.8) or (2.9) is explicitly shown by

$$\Gamma_{il}=\Gamma_{jk}\lambda_{ji}^{a}\lambda_{kl}^{a}\ , \tag{2.10}$$

which may be separated into $\bar{3}$ and 6 SU(3)-color states, since $3\otimes 3=\bar{3}\oplus 6$, which are simply the antisymmetric and symmetric parts of Γ_{il}, respectively. Using $\lambda_{ji}^a\lambda_{kl}^a=2(\delta_{ki}\delta_{lj}-\frac{1}{3}\delta_{kl}\delta_{ij})$, Eq. (2.10) becomes

$$\Gamma_{\bar{3}}^a=-\tfrac{8}{3}\Gamma_{\bar{3}}^a,\quad a=1,2,3\ , \tag{2.11}$$

$$\Gamma_6^b=\tfrac{4}{3}\Gamma_6^b,\quad b=1,\ldots,6\ , \tag{2.12}$$

which denote equations of the form (2.8) or (2.9) but with the λ matrices removed. Again the difference in sign between Eqs. (2.11) and (2.12) results in gluon exchange being attractive in $\bar{3}$ states, but repulsive in qq 6 states. The justification for these statements is given in Sec. IV.

That any two quarks in a three-quark color-singlet baryon are necessarily in a $\bar{3}$ color state is seen by noting that one can simply rewrite the color state of such a baryon:

$$\frac{1}{\sqrt{6}}(RBG-RGB+GRB-GBR+BGR-BRG)$$
$$=\frac{1}{\sqrt{3}}\left[R\frac{1}{\sqrt{2}}(BG-GB)+G\frac{1}{\sqrt{2}}(RB-BR)+B\frac{1}{\sqrt{2}}(GR-RG)\right],$$

where $\{(1/\sqrt{2})(BG-GB),\ldots\}$ are recognized as three basis states for the $\bar{3}$ representation.

The determination of meson masses from Eq. (2.4) and the diquark masses from Eq. (2.9), after finding solutions of Eq. (2.3), is not an easy computational problem. Hence in Sec. III we introduce an analytical technique for obtaining approximate solutions to Eq. (2.4). The same technique is applied to diquarks in Sec. IV.

III. MESON MASS FUNCTIONALS

Of fundamental significance in QCD are the pseudoscalar isovector meson states which emerge from Eq. (2.4). Experimentally, in the limit of $N_F=3$ light-quark masses, these are the three pions, four kaons, and one η meson forming the well-known pseudoscalar-meson octet. They have low mass because in the limit of massless quarks they are the massless Nambu-Goldstone bosons which arise from the hidden chiral symmetry of QCD, as we now show.

Because Eq. (2.4) is only valid for isovector mesons we must project out the required states. For the color-singlet pseudoscalar states we thus write $\Gamma=\Gamma^f T^f i\gamma_5$, and Eq. (2.6) becomes

$$\Gamma^f(p,P)=\frac{8}{3}\int\frac{d^4q}{(2\pi)^4}D(p-q)\mathrm{tr}_{\mathrm{SF}}\left[\frac{1}{iA\left[\not{q}-\frac{\not{P}}{2}\right]+M+B}T^g\frac{1}{iA\left[-\not{q}-\frac{\not{P}}{2}\right]+M+B}T^f\right]\Gamma^g(q,P)\ , \tag{3.1}$$

where we have used $\mathrm{tr}(T^fT^g)=\frac{1}{2}\delta_{fg}$, $\{\gamma^\mu,\gamma_5\}=0$, and $\gamma^\mu\gamma^\mu=4$. Now consider the case $M=0$. Then for $P=0$ Eq. (3.1) becomes, after calculating the spin-flavor trace,

$$\Gamma^f(p)=\frac{16}{3}\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{\Gamma^f(q)}{A^2q^2+B^2}\ ,$$

which, on comparing with Eq. (2.3b), is seen to have solutions $\Gamma^f(q)=B(q)$. Thus the crucial result emerges that in the limit of massless quarks the pseudoscalar-meson equation (3.1) has solutions for $P^2=0$; that is, these mesons are massless, and furthermore their form factors Γ^f are the quark mass function $B(q)$. This result was in fact already apparent from the $\{\pi^a\}$ occurring in the solution (2.2) of the SD equation in the massless limit. This important result was first obtained in a four-fermion model by Nambu and Jona-Lasinio[13] and further studied by Delbourgo and Scadron.[14] It arises naturally in the approximate bosonization of QCD in Ref. 11 and in the bosonization discussed in Refs. 12 and 15. It is also intimately related to the fact, discussed in Ref. 12, that the vacuum manifold of QCD has the structure of the coset space $G/H=\mathrm{U}_A(N_F)$ where $H=\mathrm{U}_V(N_F)$ is a subgroup of the chiral group G. This is most easily seen in Ref. 11, though there the language of coset spaces was not used.

In the case of low-mass quarks, $M\approx 0$, these pseudoscalar mesons acquire mass, and we now present a technique for obtaining these masses. Consider first the simpler case of equal-mass quarks, $M=m\mathbf{1}$, so that the flavor trace in Eq. (3.1) is easily evaluated. Now expand the A and B functions in Eq. (3.1) as power series in P_μ. We obtain

$$\Gamma^f(p)=-\frac{2}{9}\int\frac{d^4q}{(2\pi)^4}D(p-q)[P^2I_0(q^2)+4(P\cdot q)^2I_1(q^2)]\Gamma^f(q)$$
$$+\frac{16}{3}\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{1}{A(q)^2q^2+[m+B_m(q)]^2}\Gamma^f(q)+\cdots\ , \tag{3.2}$$

where

$$I_0(s)=\frac{6}{(A^2s+B^2)^2}\left[3A^2+2B\frac{dB}{ds}\right]\ ,$$
$$I_1(s)=\frac{6}{(A^2s+B^2)^2}\left[3\left[\frac{dB}{ds}\right]^2-\frac{\left[A^2+2B\frac{dB}{ds}\right]^2}{A^2s+B^2}+B\frac{d^2B}{ds^2}\right]\ , \tag{3.3}$$

where $s=q^2$ and terms involving dA/ds are not shown.

Changing the left-hand side (LHS) of Eq. (3.1) [or Eq. (3.2)] from Γ to $\lambda\Gamma$, these equations become eigenvalue problems with eigenvalue $\lambda(P^2)$. Then the requirement that the eigenvalue has value 1, in order to recover the original form of these equations, determines $P^2=-M_f{}^2$, and gives the meson mass spectrum $\{M_f\}$. The usefulness of Eq. (3.2) is that it may be converted into a variational problem. By taking Fourier transforms, multiplying throughout by $\Gamma^f(x)/D(x)$, and integrating we obtain, using the rest frame [in which $P=(iM,0)$], the pseudoscalar mass functional $M_\pi[\Gamma]$:

$$M_\pi[\Gamma]^2=-\frac{24}{f_\pi{}^2}\int\frac{d^4q}{(2\pi)^4}\frac{\Gamma(q)^2}{A(q)^2q^2+[m+B_m(q)]^2}+\frac{9}{2f_\pi{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)}\ , \tag{3.4}$$

where

$$f_\pi[\Gamma]^2=\int\frac{d^4q}{(2\pi)^4}[I_0(q^2)+4q_4{}^2I_1(q^2)]\Gamma(q)^2\ , \tag{3.5}$$

where $\Gamma(q)$ is the form factor in the rest frame ($\mathbf{P}=0$). It is easily checked that the variational equation

$\delta M[\Gamma]^2/\delta\Gamma=0$ reproduces Eq. (3.2). The advantage of the variational formulation is that we can use approximate forms for Γ to estimate masses rather than numerically solve the eigenvalue problem of Eq. (3.2).

We will assume that $f_\pi{}^2 M_\pi[\Gamma]^2\geq 0$, and thus $\int d^4x\,\Gamma^2/D>0$. This relates to the stability analysis of the "vacuum" which arises in the bosonization approach[11,12,15] to these mass functionals and is more appropriately considered in that context. Later, analogous assumptions are made for other mass functionals.

In the limit of zero-quark masses we have already determined that the pseudoscalars are massless and that their form factor is $\Gamma=B$. Noting that Eq. (2.3b), after Fourier transforming, has the form

$$B(x)=\tfrac{16}{3}D(x)\mathrm{FT}\left[\frac{B(q)}{A(q)^2q^2+B(q)^2}\right], \tag{3.6a}$$

and using $\Gamma=B$ we find that Eq. (3.4) also gives $M_\pi=0$ as required.

For low mass quarks Eq. (3.6a) becomes, from Eq. (2.3c),

$$B_m(x)=\tfrac{16}{3}D(x)\mathrm{FT}\left[\frac{m+B_m(q)}{A(q)^2q^2+[m+B_m(q)]^2}\right], \tag{3.6b}$$

and using the approximate form factor $\Gamma\approx B_m$, Eq. (3.4) gives

$$m_\pi{}^2<\frac{m\langle\bar{q}q\rangle}{f_\pi{}^2}, \tag{3.7}$$

where

$$\langle\bar{q}q\rangle\approx 24\int\frac{d^4q}{(2\pi)^4}\frac{B(q)}{A(q)^2q^2+B(q)^2},$$

where we have finally used $B_m\approx B$. Eq. (3.7) reproduces the standard current-algebra result for NG pseudoscalar-meson masses.

It is important to notice that we have used the identity

$$\int d^4x\frac{\Gamma(x)^2}{D(x)}=\frac{16}{3}\int\frac{d^4q}{(2\pi)^4}\frac{B_m(q)[m+B_m(q)]}{A(q)^2q^2+[m+B_m(q)]^2}, \tag{3.8}$$

which is valid when $\Gamma(x)=B_m(x)$, and follows from Eq. (3.6b). This is why the gluon propagator $D(x)$, which specifies the nature of the force between the quarks, does not appear explicitly in Eq. (3.7). It is significant that the same result, Eq. (3.7), emerges from the bosonization of QCD in Refs. 11 and 15, where Eq. (3.8) is also used.

The above techniques can be extended to the case of unequal-quark masses and we again obtain Eq. (3.7) but with $2\,\mathrm{tr}(MT^fT^f)$ (no f summation) in place of m. For $N_F=3$ and a diagonal quark mass matrix M with elements m_u, m_d, m_s, with $m_u=m_d$, this result gives states which we identify as the three pions, four kaons, and one η, with their masses given by

$$m_\pi{}^2=\mu m_u,\quad m_K{}^2=\mu\frac{m_u+m_s}{2},$$

$$m_\eta{}^2=\mu\frac{m_u+2m_s}{3},\quad \mu=\frac{\langle\bar{q}q\rangle}{f_\pi{}^2}.$$

It is to be noted that these masses satisfy the Gell-Mann–Okubo mass formula $4m_K{}^2=3m_\eta{}^2+m_\pi{}^2$.

By the same technique mass functionals may also be obtained for non-NG bosons such as the $\rho,\omega,a_1,\ldots$ mesons. We briefly present the results for the ρ meson. For this color-singlet, spin-1, isovector state, we use Eq. (2.4) with $\Gamma_\mu=(\delta_{\mu\nu}-P_\mu P_\nu/P^2)\gamma_\nu\Gamma^fT^f$. Projecting out this state in Eq. (2.4) and again expanding the P dependence of the A and B functions in the quark propagators we obtain, by the above procedures, the ρ-meson mass functional

$$M_\rho[\Gamma]^2=-\frac{24}{f_\rho{}^2}\int\frac{d^4q}{(2\pi)^4}\frac{\frac{1}{2}sA^2+(m+B)^2}{[sA^2+(m+B)^2]^2}\Gamma(q)^2+\frac{9}{f_\rho{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)}, \tag{3.9}$$

where

$$f_\rho[\Gamma]^2=6\int\frac{d^4q}{(2\pi)^4}\left[\frac{A^2+s\left[\frac{dB}{ds}\right]^2-2B\frac{dB}{ds}-sB\frac{d^2B}{ds^2}}{(sA^2+B^2)^2}+\frac{2\left[A^2+2B\frac{dB}{ds}\right](\frac{1}{2}sA^2+B^2)}{(sA^2+B^2)^3}\right.$$
$$\left.+\frac{4q_4{}^2(\frac{2}{3}sA^2+B^2)\left\{2(sA^2+B^2)\left[\left[\frac{dB}{ds}\right]^2+B\frac{d^2B}{ds^2}\right]-\left[A^2+2B\frac{dB}{ds}\right]^2\right\}}{(sA^2+B^2)^4}\right]\Gamma(q)^2+\cdots,$$

where terms involving dA/ds are not shown. Repeating the above derivation for the isosinglet ω meson we obtain the same mass functional. Hence in the above approximation to QCD the ρ and ω mesons are degenerate. We show elsewhere that this degeneracy is split by $\rho\to\pi\pi\to\rho$ fluctuations which lower the ρ mass.

We also present the mass functional for the a_1 meson (previously known as the A_1 meson) as it will be used, in the next section, to provide a useful lower bound on the spin-1 diquark mass. For the a_1 meson we project the state $\Gamma_\mu=(\delta_{\mu\nu}-P_\mu P_\nu/P^2)\gamma_\nu i\gamma_5\Gamma^fT^f$ from Eq. (2.4) and use again the above techniques to obtain the following mass functional:

$$M_a[\Gamma]^2 = -\frac{24}{f_a{}^2}\int \frac{d^4q}{(2\pi)^4}\Gamma(q)^2\frac{\frac{1}{2}sA^2-(m+B_m)^2}{[sA^2+(m+B_m)^2]^2}+\frac{9}{f_a{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)}\ , \tag{3.10}$$

where

$$f_a[\Gamma]^2 = 6\int\frac{d^4q}{(2\pi)^4}\left\{\frac{A^2-s\left[\frac{dB}{ds}\right]^2+2B\frac{dB}{ds}+sB\frac{d^2B}{ds^2}}{(sA^2+B^2)^2}+\frac{2\left[A^2+2B\frac{dB}{ds}\right](\frac{1}{2}sA^2-B^2)}{(sA^2+B^2)^3}\right.$$
$$\left.+\frac{4q_4{}^2(\frac{2}{3}sA^2-B^2)\left\{2(sA^2+B^2)\left[\left[\frac{dB}{ds}\right]^2+B\frac{d^2B}{ds^2}\right]-\left[A^2+2B\frac{dB}{ds}\right]^2\right\}}{(sA^2+B^2)^4}\right\}\Gamma(q)^2+\cdots\ . \tag{3.11}$$

The above ρ, ω, and a_1-meson mass functionals, and as well expressions for various coupling constants, have been previously derived from the bosonization in Ref. 15. Hence the connection between the BS approach to mesons and the more powerful bosonization methods of Refs. 11, 12, and 15 becomes apparent.

We now use the above mass functionals to estimate some bounds on meson masses. In principle this first requires $D(x)$ to be specified and $A(q)$ and $B(q)$ to be found by solving Eqs. (2.3). For simplicity we here use analytic forms for A and B which follow from the analytic arguments of Ref. 11 and by numerical studies which have used forms for $D(x)$ which model its asymptotic freedom and infrared slavery characteristics. The forms used are

$$A(s)=\begin{cases}2 \quad \text{for } s<\frac{\chi^2}{4}\ ,\\ \frac{1}{2}\left[1+\left[1+2\frac{\chi^2}{s}\right]^{1/2}\right] \quad \text{for } s>\frac{\chi^2}{4}\ ,\end{cases}$$
$$B(s)=\begin{cases}(\chi^2-4s)^{1/2} \quad \text{for } s<s_m\ ,\\ \frac{4m_D{}^3}{s}\left[\ln\frac{s}{\Lambda^2}\right]^{-\gamma} \quad \text{for } s>s_m\ .\end{cases} \tag{3.12}$$

Here Λ is the usual mass scale parameter of QCD and χ is another mass parameter determined indirectly by Λ. The value of the parameter $\gamma\approx 1.8$ follows from the asymptotic behavior of $B(s)$ found in the numerical studies. We note that this value for γ is different from that obtained by the operator-product expansion[16] whereby

$$\gamma = 1-\frac{3C_2(R)}{11-\frac{2}{3}N_F}\quad\left[C_2(R)=\frac{N_C{}^2-1}{2N_C}\right]\ ,$$

which for $N_F=0$ and $N_C=3$ gives $\gamma=\frac{7}{11}<1$. ($N_F=0$ means only that quark loops are neglected in the evaluation of the QCD β function.) The OPE result can be obtained exactly from Eq. (2.3b) if the method of Ref. 17 is used. This amounts to supposing the validity of the approximation

$$\int d\Omega_q D((p-q)^2)=\theta(p^2-q^2)D(p^2)+\theta(q^2-p^2)D(q^2)\ , \tag{3.13}$$

for the angle integration in Eq. (2.3b), and that $D(q^2)=\alpha(q^2)/q^2$ with the running coupling constant $\alpha(q^2)$ having the usual asymptotic dependence on q^2. Numerical solutions of Eq. (2.3), without this approximation, give $\gamma>1$, showing that this approximation gives the incorrect asymptotic behavior for $B(q^2)$. We note also that the OPE asymptotic form for B would cause $\langle\bar{q}q\rangle$ and thus m_π to diverge, whereas the form in Eq. (3.12) gives finite values. That one must employ a bad approximation to obtain the OPE result suggests a shortcoming of the OPE approach.

Once Λ, χ, and γ have been set m_D and s_m are determined by requiring the functions B and dB/ds to be continuous at $s=s_m$. For $\Lambda=0.19$ GeV and $\chi=1.14$ GeV we obtain $m_D=0.483$ GeV and $s_m=0.258$ GeV^2. For different values of Λ the other parameter values follow by appropriate scaling. An important feature of A and B is that the quark propagator has no poles, and in this manner, quark confinement occurs.

For the pion we obtain the values $f_\pi=84$ MeV and $\langle\bar{q}q\rangle=(333\text{ MeV})^3$. Equation (3.7) then gives the bound $m_\pi<162$ MeV for a quark bare mass of $m=5$ MeV, which compares well with the experimental value of $m_\pi=140$ MeV.

We shall estimate the ρ mass by evaluating Eq. (3.9) with the approximation $\Gamma_\rho\approx\Gamma_\pi=B$, which might be reasonable since the ρ and the π mesons differ only by a spin flip by one quark. This allows us to easily evaluate the mass functional by using the identity of Eq. (3.8) and, by noting that for this approximation the integrand may be written in an O(4)-symmetric form and thus after analytic angle integrations, we are left with a simple one-dimensional quadrature. In general, the integrand has O(3,1) symmetry. We obtain $m_\rho<878$ MeV for $m=5$ MeV (<847 MeV for $m=0$) and the same values for the ω meson, which are to be compared with the experimental value $m_\omega=783$ MeV. Our calculations indicate that the ρ mass is decreased by some 20 MeV when $\pi\pi$ intermediate states are included.

Finally, for the a_1 meson the approximation $\Gamma_1\approx B_m$

which, unlike that for the ρ, has no physical justification, gives the excessively large upper bound of $m_{a_1} < 6.4$ GeV. This only serves to indicate that the structure of the a_1 is very different from that of the ρ or π mesons. To determine the proper form factor for the a_1 clearly requires a numerical variational calculation using $M_a[\Gamma]$.

IV. DIQUARK MASS FUNCTIONALS

We now come to the main results of this paper: namely, the estimation of diquark masses using the techniques developed in the previous section. The $\bar{3}$ state diquark masses are determined by Eq. (2.8) [or Eq. (2.9)] where the results of the color algebra are given in Eq. (2.11). For reasons that will become apparent shortly, the $J^P = 0^+ \bar{3}$ qq states can be of low mass. To extract these states from Eq. (2.11) we write $\Gamma = \Gamma_0 C i \gamma_5$ to explicitly display the flavor and spin matrix structure of Γ. That this is the correct Dirac matrix form for a $J^P = 0^+$ isoscalar diquark state is seen by showing that $u(1)^T C\gamma_5 u(2)$, where $u(1)$ and $u(2)$ are two-quark spinors, is a scalar under Lorentz transformations. Projecting the above state from Eq. (2.11) we obtain

$$\Gamma_0(p,P) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} D(p-q)\mathrm{tr}_{\mathrm{SF}}\left[\frac{1}{iA\left[\not{q}+\frac{\not{P}}{2}\right]+M+B}T^0\frac{1}{iA\left[-\not{q}+\frac{\not{P}}{2}\right]+M+B}T^0\right]\Gamma_0(q,P) , \quad (4.1)$$

where $T_0 = [1/\sqrt{(2N_F)}]\mathbf{1}$, i.e., $\mathrm{tr}(T^0T^0) = \frac{1}{2}$, and in which, in order to make a comparison, we have made the change of variables: $q,p \to -q, -p$, respectively. Noting that Eq. (4.1) differs from Eq. (3.1), the pion equation, for equal-mass quarks, by only a factor of 2, we see why the $J^P = 0^+$ diquark state is expected to be a low-mass state: if this factor of 2 were included in Eq. (4.1), and in the limit of massless quarks, then this bound state would be massless. This factor of 2 is of course due to the color algebra, and means that in $\bar{3}$ qq states the gluon force is attractive but somewhat weaker than for color-singlet $\bar{q}q$ states. Conversely the opposite sign for the **6** qq states in Eq. (2.12) shows that gluon exchange is repulsive for these states.

We may now apply the method of Sec. III to convert Eq. (4.1) into a variational problem and we obtain, for equal quark masses, the diquark mass functional.

$$M_0[\Gamma]^2 = -\frac{24}{f_0{}^2}\int\frac{d^4q}{(2\pi)^4}\frac{\Gamma(q)^2}{A(q)^2q^2+(m+B_m)^2} + \frac{9}{f_0{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)} , \quad (4.2)$$

where $f_0[\Gamma] = f_\pi[\Gamma]$ which is defined in Eq. (3.5). We shall use Eq. (4.2) to estimate an upper limit for the diquark mass by using the approximation $\Gamma_0 \approx \Gamma_\pi \approx B_m$, rather than performing a proper variational calculation. We may then use the identity of Eq. (3.8) to considerably simplify the calculation, giving

$$m_0{}^2 < \frac{24}{f_\pi{}^2}\int\frac{d^4q}{(2\pi)^4}\frac{B_m(2m+B_m)}{sA^2+(m+B_m)^2} , \quad (4.3)$$

as an upper bound on m_0, the scalar diquark mass. We may also use Eq. (4.2) to obtain a lower bound. Using Eq. (3.4), the pion mass functional, Eq. (4.2) becomes

$$M_0[\Gamma]^2 = M_\pi[\Gamma]^2 + \frac{9}{2f_0{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)} > M_\pi[\Gamma]^2 > m_\pi{}^2 , \quad (4.4)$$

and we obtain, from Eqs. (4.2) and (4.4), $m_\pi < m_0 < M_0[B_m]$ which, using $B_m \approx B$ and Eq. (3.12) for A and B, gives 140 MeV $< m_0 < 742$ MeV for $m = 5$ MeV. In the case of zero-mass quarks these bounds become $0 < m_0 < 708$ MeV, giving some indication of the dependence of the scalar-diquark mass on the quark bare masses. These results are to be compared with the value $m_0 > 400$ MeV, extracted by Laperashvili[6] from the analysis of experimental data on inclusive production of protons, which used the dominance of the hard scattering of quarks by diquarks. While our upper bound could be lowered by a detailed numerical minimization of $M_0[\Gamma]$, the above bound certainly indicates the dynamical importance of the scalar diquark in baryon structure, as argued by Fredriksson and by Lichtenberg and their co-workers.

It is also important to consider the $J^P = 1^+ \bar{3}$ diquark state since it is, in principle, also a possible constituent of color-singlet spin-$\frac{1}{2}$ baryons if its mass is sufficiently low. For these states we project the state $\Gamma_\mu = (\delta_{\mu\nu} - P_\mu P_\nu / P^2)\gamma_\nu C i \gamma_5 \Gamma_1^g T^g$ from Eq. (2.11), and using again the method of Sec. III, we obtain the mass functional

$$M_1[\Gamma]^2 = -\frac{24}{f_1{}^2}\int\frac{d^4q}{(2\pi)^4}\Gamma(q)^2\frac{\frac{1}{2}sA^2-(m+B_m)^2}{[sA^2+(m+B_m)^2]^2} + \frac{18}{f_1{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)} , \quad (4.5)$$

where we find $f_1[\Gamma] = f_a[\Gamma]$.

Again one upper bound on the mass is obtained by using the form factor $\Gamma_1 \approx \Gamma_\pi \approx B_m$ which allows the identity of Eq. (3.8) to be used. Similarly to the spin-0 diquark, Eq. (4.5) may be used to obtain a lower bound on the spin-1 diquark mass. Using Eq. (3.11), the a_1-meson mass functional, Eq. (4.5) gives

$$M_1[\Gamma]^2 = M_a[\Gamma]^2 + \frac{9}{f_1{}^2}\int d^4x\frac{\Gamma(x)^2}{D(x)} > M_a[\Gamma]^2 > m_{a_1}{}^2 , \quad (4.6)$$

and we obtain $m_{a_1} < m_1 < M[B_m]$, which gives 1.27 GeV $< m_1 < 9$ GeV ($m = 5$ MeV). Our upper bound is, like that for the a_1 meson, excessive, indicating that the spin-1 diquark form factor is very different from that of the pion, and thus should be determined from a proper minimization of $M_1[\Gamma]$ or, if its mass is indeed very large, by solving the BS integral equation. Nevertheless, the lower bound is very useful, indicating that this diquark state is unlikely to play a significant role in nucleon structure, and thus supporting the same conclusion by Fredriksson, Jändel, and Larsson.[18] Our lower bound is also to be compared with $m_1 > 500$ MeV obtained from experimental data by Laperashvili[6] which, while consistent with our bound, does not by itself argue for or against a role for spin-1 diquarks in nucleon structure.

V. CONCLUSION

The growing experimental evidence that the $\bar{3}$ $J^P = 0^+$ diquark state may play a significant dynamical role in the structure of nucleons has now been supported by the detailed theoretical analysis reported here.

Our study of the diquark masses has been based on a well-defined approximation scheme for QCD. This may be extended, in a systematic manner, to include higher-order gluonic processes, if desired. The analysis used the covariant Bethe-Salpeter equation for bound states. To overcome the difficult computational problems in solving these integral equations we have introduced the technique of mass functionals, which allow simpler variational computations. In these equations it is important to use the nonperturbative quark propagators which are determined by solving the Schwinger-Dyson equation. This is because it is through this equation that the mode of realization of the hidden chiral symmetry and the quark confinement are manifested. These features are consequences of the infrared slavery and asymptotic-freedom characteristics of the gluon propagator.

To illustrate the usefulness of our mass-functional formulation of the bound-state problem, we have also applied the technique to the mesons. One significant result for mesons is that the interplay of the Schwinger-Dyson equation and the Bethe-Salpeter equation, in its mass-functional form, allows one to understand the dual role of the pseudoscalar mesons in QCD: namely, that of $\bar{q}q$ bound states and also of Nambu-Goldstone bosons. This dual role appears to have led to some confusion in the literature.[19] The mass functionals for some non-Nambu-Goldstone bosons were also derived, and in the case of the a_1 meson, were useful in allowing a lower bound on the mass of the $\bar{3}$ $J^P = 1^+$ diquark state to be determined, which implied that this state, because of its large mass, is unlikely to play a role in nucleon structure. This is an agreement with conclusions drawn from the analysis of experimental data.

In contrast the $\bar{3}$ $J^P = 0^+$ diquark state is certainly of sufficiently low mass to be important to nucleon structure. In this paper we have not performed numerical studies using the variational formulation but instead used the mass functionals to provide analytically determined bounds on meson and diquark masses. These analytical bounds exploited a particular nonperturbative feature of QCD related to the manner in which chiral symmetry is realized: namely, that the quark mass function, in the chiral limit, is the same as the Nambu-Goldstone $\bar{q}q$ pseudoscalar-meson bound-state form factor, a result that does not appear to be well-known despite its extreme importance in QCD. For example, it is an essential step in deriving an expression, known from current algebra, for the Nambu-Goldstone pion mass. This nonzero mass arises when the chiral symmetry is explicitly broken by giving the quarks small current masses. In the three-flavor case this dual role of the quark mass function is essential to our derivation of the Gell-Mann–Okubo pseudoscalar-octet-meson mass formula. Of course the analogous current-algebra result for the isosinglet pseudoscalar meson, the η', underestimates the mass of this meson—the $U_A(1)$ "problem." But, as we have emphasized in Sec. II, the current-algebra result follows from Eq. (2.4), and this is only valid for isovector mesons, because for the isosinglet mesons there exists the additional term corresponding to the q-$\bar{q}$ annihilation process shown in Fig. 1(b).

These results, along with all of our other results, involve convergent integral expressions for the masses (and in our other works, coupling constants) where the convergence comes about because of the presence of the bound-state form factors. While this is a seemingly obvious and trivial property it is, amazingly, a feature that is often ignored by other workers, leading to arbitrary cutoff procedures and the loss of testable numerical results.

Finally we wish to emphasize that many of the results reported here for the mesons have been previously obtained by us from our bosonization technique, which generates effective actions for the meson sector. This bosonization is a considerably more powerful technique for analyzing the meson sector of QCD, leading not only to the mass functionals we have derived here, but also to various meson couplings and to a clear understanding of chiral anomalies. It has also allowed us to construct a refutation[20] of the very topical Skyrmion model for baryons. The essential point being that this model does not adequately incorporate the color algebra, which is so clearly a feature of QCD, and which we have seen here is so essential to understanding the diquark role in baryon structure.

We hope that our rederivation here of some of the meson mass functionals arising from our bosonization of QCD, by a different but perhaps more familiar Bethe-Salpeter equation starting point, may facilitate an understanding of our bosonization of QCD.

[1]M. Ida and R. Kobayashi, Prog. Theor. Phys. **36**, 846 (1966).
[2]D. B. Lichtenberg, L. J. Tassie, and P. J. Keleman, Phys. Rev. **167**, 1535 (1968); J. Carroll, D. B. Lichtenberg, and J. Franklin, *ibid.* **174**, 1681 (1968); D. B. Lichtenberg, *ibid.*

178, 2197 (1969).
[3]S. Fredriksson, M. Jändel, and T. I. Larsson, Phys. Rev. Lett. **51**, 2179 (1983).
[4]S. Fredriksson, M. Jändel, and T. I. Larsson, Z. Phys. C **14**, 35 (1982).
[5]S. Fredriksson and M. Jändel, Z. Phys. C **14**, 41 (1982).
[6]L. V. Laperashvili, Yad. Fiz. **35**, 431 (1982) [Sov. J. Nucl. Phys. **35**, 431 (1982)].
[7]Z. Dziembowski, W. J. Metzger, and R. T. Van de Walle, Z. Phys. C **10**, 231 (1981).
[8]D. B. Lichtenberg, W. Namgung, E. Predazzi, and J. G. Wills, Phys. Rev. Lett. **48**, 1653 (1982).
[9]S. Eklin, S. Fredriksson, M. Jändel, and T. I. Larsson, Phys. Rev. D **28**, 257 (1983).
[10]J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
[11]R. T. Cahill and C. D. Roberts, Phys. Rev. D **32**, 2419 (1985).
[12]C. D. Roberts and R. T. Cahill, Aust. J. Phys. **40**, 499 (1987); C. D. Roberts, R. T. Cahill, and J. Praschifka, Report No. FPPG-R-3-87 (unpublished).
[13]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[14]R. Delbourgo and M. D. Scadron, Nucl. Phys. **5**, 1621 (1979).
[15]J. Praschifka, C. D. Roberts, and R. T. Cahill, Phys. Rev. D **36**, 209 (1987).
[16]H. D. Politzer, Nucl. Phys. **B117**, 397 (1976).
[17]K. Higashijima, Phys. Rev. D **29**, 1228 (1983).
[18]S. Fredriksson, M. Jändel, and T. Larsson, Z. Phys. C **19**, 53 (1983).
[19]P. Simic, Phys. Rev. D **34**, 1903 (1986).
[20]R. T. Cahill, C. D. Roberts, and J. Praschifka, Report No. FPPG-R-1-86 (unpublished).