Restrictions on $\tau^- \to \eta\pi^-\nu$ in two-Higgs-doublet models

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We consider several phenomenological constraints for a large class of two-Higgs-doublet models. We show that a significant ($>0.1\%$) branching ratio for the second-class decay $\tau^- \to \eta\pi^-\nu$ can be excluded for the models satisfying the "naturalness" requirements of Glashow and Weinberg. If one of these requirements is relaxed, a substantial ($>1\%$) branching ratio for this mode cannot be ruled out, but this requires a rather large coupling to the τ lepton. Implications for $p\bar{p}$ and pp experiments are briefly discussed.

I. INTRODUCTION

There is a well-known problem in accounting for all the τ-lepton decays in terms of exclusive channels.[1,2] Recently, the HRS Collaboration[3] has announced a branching ratio of $(5.1\pm1.0\pm1.2)\%$ for the decay mode $\tau^- \to \eta\pi^-\nu$, a signature which corresponds[4] to a second-class current.[5] In the standard model, this branching ratio is expected to be much smaller.[6–16] The HRS result suggests an unconventional explanation for the 7–9% discrepancy[1,2] between the sum of the exclusive modes and the inclusive measurements with a one-prong topology. The HRS result is based on the reconstruction of the $\eta \to \gamma\gamma$ decays but there is no corresponding evidence for the decay mode $\eta \to \pi^+\pi^-\pi^0$ in the three-pion invariant mass.[17] The difficulty of obtaining a clear answer comes partly from the low efficiency of the detectors in the kinematical region relevant for the observation of the latter decay mode. More recently, the TPC/Two-Gamma Collaboration[18] set an upper limit of 0.26% on the branching ratio for $\tau^- \to K^-\bar{K}^0\nu$. Using an SU(3) prediction,[14] this last result would imply that the branching ratio for $\tau^- \to \eta\pi^-\nu$ is less than 1.6%. It is thus clear that the experimental situation needs clarification. We could also wonder if the second-class resonance b_1 $[I^G(J^P)=1^+(1^+)]$ is observable for instance[4] in $\tau^- \to b_1^- \to \omega\pi^-\nu$. Experimentally[17] the branching ratio for $\tau \to \omega\pi^-\nu$ is $(1.5\pm0.3\pm0.3)\%$ but the J^P is consistent with 1^-.

In the standard model the decay mode $\tau^- \to \eta\pi^-\nu$ is forbidden by G parity and is thus suppressed by small isospin-breaking parameters such as f_{a_0} or the π^0-η mixing.[4,6–16] This implies a branching ratio smaller than 5×10^{-5} for this mode. Consequently an experimental result of 1% or 0.1% can still be considered as a significant discrepancy which would require a modification of the standard model. This justifies our interest in the question in spite of the experimental uncertainties.

The existence of a new particle has already been mentioned[1] as a possible explanation for the one-prong missing exclusive modes. One of the simplest ways to modify the standard model is to add a second Higgs doublet. This provides us with new (pseudo)scalar channels for the weak decays. In a recent publication[8] we have briefly discussed this alternative and concluded the following.

(1) For the models[19–21] which satisfy the "naturalness" requirements of Ref. 19, phenomenological constraints are sufficient to rule out a significant branching ratio for $\tau^- \to \eta\pi^-\nu$.

(2) If one of these naturalness requirements is relaxed, one can make an *ad hoc* choice of the Yukawa couplings which allows a large branching ratio for this mode.

In this paper we elaborate on these points in detail. In Sec. II we present a general class of two-Higgs-doublet models and we single out the ones for which discrete symmetries prevent flavor-changing neutral currents at the tree level.[19–21] (They will be referred to as the "natural" models I and II.) Section III is devoted to the τ decays for a general class of two-Higgs-doublet models. The result that a significant ($>0.1\%$) branching ratio for $\tau^- \to \eta\pi^-\nu$ can be ruled out in the natural models I and II is worked out in Sec. IV. This is mainly because of the strong relations among the masses and the Yukawa couplings which exist in these models and make them very predictive. In Sec. V we reconsider the phenomenological constraints for a more general class of models, unrestricted by the third naturalness requirements of Ref. 19. We discuss the K_L-K_S mass difference (because this is a small effect in the standard model) and the nonleptonic decay of the kaons [because (pseudo) scalar matrix elements are expected to be enhanced] in the case where the Yukawa couplings to the light quarks are not negligible. We show that a branching ratio larger than 1% for $\tau^- \to \eta\pi^-\nu$ cannot be ruled out phenomenologically, but implies rather large Yukawa couplings to the τ lepton. Implications for pp and $p\bar{p}$ experiments are briefly discussed.

This paper contains information which could be used independently of a possible explanation of the HRS result. In particular we discuss new bounds for the parameters of models I and II. The constraints on (pseudo) scalar couplings to the light quarks could also be used in other contexts as well.

II. PRESENTATION OF THE TWO-HIGGS-DOUBLET MODELS

If we modify the standard model by adding a second Higgs doublet, we can always perform rotations among the various fields in such a way that[21,22] (a) only one Higgs field (ϕ) acquires a vacuum expectation value (VEV), and (b) the couplings of the fermion to the neutral component of ϕ are diagonal. In this basis, the other Higgs boson (χ) is physical and the couplings to the quarks read, in general,

$$\mathcal{L}^{\rm int}=\chi^+\left[\bar{\mathcal{U}}\left(H_u^+K\frac{1-\gamma_5}{2}-KH_d(1+\gamma_5)\right)\mathcal{D}\right]$$
$$+\chi^0[\mathcal{U}H_u^+(1-\gamma_5)\mathcal{U}+\bar{\mathcal{D}}H_d(1+\gamma_5)\mathcal{D}]+\text{H.c.}\ , \tag{1}$$

where $\mathcal{U}$ and $\mathcal{D}$ are short notations for three generations of charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks, K is the Kobayashi-Maskawa matrix, and H_d and H_u are 3×3 matrices. By construction, χ does not have a VEV and consequently H_u and H_d are not necessarily related to the fermion masses.

Flavor-changing neutral currents (FCNC's) at the tree level can be "naturally"[19] avoided by imposing discrete symmetries which prevent off-diagonal terms in H_u and H_d. There are essentially two ways of doing this, denoted by I and II. The first (model I) is[19] to couple $\mathcal{U}_R$ only to one Higgs doublet, and $\mathcal{D}_R$ only to the other. This yields

$$H_u=\frac{g}{\sqrt{2}M_W}xM_u,\quad H_d=-\frac{g}{\sqrt{2}M_W}\frac{1}{x}M_d\ , \tag{2}$$

where $M_u=\text{diag}(m_u m_c m_t)$, $M_d=\text{diag}(m_d m_s m_b)$, and x is the ratio of the VEV's (in the original basis). The second possibility[20] (model II) is to couple all the quarks to only one Higgs doublet with the result

$$H_u=\frac{g}{\sqrt{2}M_W}xM_u,\quad H_d=\frac{g}{\sqrt{2}M_W}xM_d\ . \tag{3}$$

In both of these models, the Yukawa couplings obey the mass hierarchies for each charge sector. It will be shown later that this implies negligible branching ratios for $\tau^-\to\eta\pi^-\nu$ when the available phenomenological constraints are imposed. Consequently we will consider a larger class of models where the off-diagonal terms of H_u and H_d are set to zero *by hand* and where the Yukawa couplings are, in general, unrelated to the masses. We will use the general notation

$$H_u=\begin{bmatrix}h_u&0&0\\0&h_c&0\\0&0&h_t\end{bmatrix},\quad H_d=\begin{bmatrix}h_d&0&0\\0&h_s&0\\0&0&h_b\end{bmatrix}. \tag{4}$$

This contains models I and II as particular cases but there is no natural justification for other choices of the parameters in the present context (i.e., the third naturalness requirement of Ref. 19 has been relaxed and we might have to tune some couplings at the quantum level). We have not investigated systematically the possibility of having natural couplings of type (4) but different from (2) and (3) in models with more than two Higgs doublets.

Similarily, the couplings of the charged component of χ to leptons reads, in general,

$$\mathcal{L}_{\rm int}=\chi^+\bar{\mathcal{N}}H_L\left(\frac{1+\gamma_5}{2}\right)\mathcal{E}\ , \tag{5}$$

with the same three-generation notation as before. In models I and II we impose that the leptons couple to only one Higgs doublet and obtain

$$H_L=\frac{g}{\sqrt{2}M_W}xM_L\quad\text{or}\quad H_L=-\frac{g}{\sqrt{2}M_W}\frac{1}{x}M_L\ . \tag{6}$$

Again, we will later consider a more general situation where

$$H_L=\begin{bmatrix}h_e&0&0\\0&h_\mu&0\\0&0&h_\tau\end{bmatrix}. \tag{7}$$

For simplicity we will assume that the constants appearing in Eqs. (4) and (7) are real.

III. EFFECTS OF THE HIGGS BOSONS ON τ DECAYS

In this section we mainly consider the decay rates $\Gamma(\tau^-\to\pi^-\nu)$ and $\Gamma(\tau^-\to a_0^-\nu)$ with a scalar χ and a W as intermediate boson. Other decay modes are then briefly discussed. We define

$$\begin{aligned}&\langle\pi^-(p)\,|\,\bar{d}\gamma_\mu\gamma_5u\,|\,0\rangle=if_\pi p_\mu\quad(f_\pi\simeq132\text{ MeV})\ ,\\&\langle a_0^-(p)\,|\,\bar{d}\gamma_\mu u\,|\,0\rangle=f_{a_0}p_\mu\ ,\\&\langle\pi^-(p)\,|\,\bar{d}\gamma_5u\,|\,0\rangle=S_\pi\ ,\\&\langle a_0^-(p)\,|\,\bar{d}u\,|\,0\rangle=S_{a_0}\ .\end{aligned} \tag{8}$$

Using the general Higgs-boson couplings given in Eqs. (4) and (7) we obtain

$$\Gamma(\tau^-\to\pi^-\nu)=\frac{1}{8\pi}\cos^2\theta_C m_\tau\left[1-\frac{m_\pi{}^2}{m_\tau{}^2}\right]^2\left|\frac{G_F}{\sqrt{2}}m_\tau f_\pi+i\frac{h_\tau(h_u+h_d)}{4M_\chi{}^2}S_\pi\right|^2, \tag{9a}$$

$$\Gamma(\tau^-\to a_0^-\nu)=\frac{1}{8\pi}\cos^2\theta_C m_\tau\left[1-\frac{m_{a_0}{}^2}{m_\tau{}^2}\right]\left|\frac{G_F}{\sqrt{2}}m_\tau f_{a_0}-\frac{h_\tau(h_u-h_d)}{4M_\chi{}^2}S_{a_0}\right|^2. \tag{9b}$$

We have recently discussed[12] the value of f_{a_0} in a large class of linear σ models (including several types of "anomalous" terms) and found an invariant answer which is numerically $f_{a_0} \simeq 1.2$ MeV. The other form factors can be evaluated by using the method of Shifman, Vainshtein, and Zakharov.[23] This yields

$$S_\pi = i\frac{f_\pi}{m_u+m_d}m_\pi{}^2 \simeq i(0.43 \text{ GeV})^2 ,$$
$$S_{a_0} = \frac{f_{a_0}}{m_d-m_u}m_{a_0}{}^2 \simeq (0.55 \text{ GeV})^2 . \tag{10}$$

In this paper we use the quark masses of Ref. 24: namely, $m_u=5.1$ MeV, $m_d=8.9$ MeV, $m_s=175$ MeV, and $m_c=1350$ MeV. It is clear that the evaluation of the form factors contains some uncertainties. Nevertheless, the values given here are consistent with those obtained by other methods.[9–16]

If we impose that the branching ratio for $\tau^- \to \pi^- \nu$, which is well predicted in the standard model,[1] stays within the experimental value[25] [$(10.1\pm1)\%$], we have to require that the cross term in (9a) is less than 10% of the W-exchange rate. This gives the constraint

$$|h_\tau(h_u+h_d)| < 0.014\frac{M_\chi{}^2}{M_W{}^2} . \tag{11}$$

We have disregarded the extraordinary possibility of having (in the amplitude) a Higgs doublet contribution exactly twice as large as the W contribution but with opposite signs.

We now define B_{a_0} as the branching ratio for $\tau^- \to \chi^- \nu \to a_0^- \nu$. Taking the experimental branching ratio for $\tau^- \to \pi^- \nu$ as normalization, we obtain, from (9b) and (10),

$$B_{a_0} = 1.64\left[h_\tau(h_u-h_d)\left[\frac{M_W{}^2}{M_\chi{}^2}\right]\right]^2 . \tag{12}$$

We note that the dominant decay mode of the a_0^- is $\eta\pi^-$ (Refs. 14 and 26) and that the HRS result[3] is consistent with the hypothesis that the $\eta\pi^-$ pairs come from the a_0^- resonance. In the rest of this section we will give arguments showing that resonant channels in the Higgs-doublet mediated decays are mainly scalar or pseudoscalars. This means that B_{a_0} is a good approximation for the branching ratio of $\tau^- \to \eta\pi^- \nu$ (if significantly larger than in the standard model).

We do not expect large Higgs-boson contributions for the τ decays into vectors (ρ, a_1, b_1) and neutrino because a scalar-vector coupling requires a derivative which implies a suppression at the pole [$p^\mu \epsilon_\mu(p)=0$ for $p^2 = m_v{}^2$]. This reflects the fact that for an on-shell massive vector V_μ, we have $\partial^\mu V_\mu = 0$. This argument can be extended for higher-rank (traceless) tensors. Note that the above argument does not forbid a transition from an off-shell vector to an on-shell scalar (like $\tau^- \to W^- \nu \to \pi^- \nu$) because in that case the divergenceless condition does not hold. As a consequence, a significant second-class signature with $I^G(J^P)=1^-(0^+)$ (i.e., the a_0) and the absence of a second-class signature with $I^G(J^P)=1^+(1^+)$ (i.e., the b_1) could in principle, be interpreted in terms of a new scalar particle. [As recalled in Ref. 8 the mnemonic for a second-class resonance is $GP(-1)^J$ odd.] Experimentally, the decay mode $\tau^- \to \omega\pi^-\nu$ is observed[17] with a branching ratio of $(1.5\pm0.3\pm0.3)\%$ and a J^P consistent with 1^- instead of 1^+ for the b_1^-. (See Ref. 27 for the theoretical expectations of the standard model.)

The above analysis can be easily repeated for the Cabibbo-suppressed decays of the τ-lepton into K^- (493.7 MeV) or K_0^{*-} (~ 1350 MeV, $J^P=0^+$) and a neutrino. We use[28] $f_K = 1.22 f_\pi$ and a σ-model calculation gives $f_{K_0^*} \simeq f_K - f_\pi$ (see e.g., Ref. 12). Proceeding as in Eqs. (10) we obtain $|S_K| = (0.47\text{ GeV})^2$ and $|S_{K_0^*}| = (0.56 \text{ GeV})^2$. In the standard model this gives branching fractions of 0.7% (Ref. 1) [$0.67\pm0.17\%$ (Ref. 25)] and 0.5×10^{-5} [$<0.3\%$ (Ref. 29)] for $\tau^- \to K^-\nu$ and $\tau^- \to K_0^{*-}\nu$, respectively. (The quantities between the brackets are the experimental values.) Imposing that the branching ratio for $\tau^- \to K^-\nu$ stays within the experimental errors (the relative errors being now $\pm 25\%$) we obtain

$$|h_\tau(h_u+h_s)| < 0.036\left[\frac{M_\chi{}^2}{M_W{}^2}\right] . \tag{13}$$

As in Eq. (12) we define $B_{K_0^*}$ as the branching ratio for $\tau^- \to \chi^- \nu \to K_0^{*-}\nu$ and we get, for $\sin\theta_C = 0.22$,

$$B_{K_0^*} = 0.034\left[h_\tau(h_d-h_s)\left[\frac{M_W{}^2}{M_\chi{}^2}\right]\right]^2 . \tag{14}$$

IV. LIMITS ON $\tau^- \to \eta\pi^-\nu$ IN THE "NATURAL" TWO HIGGS-DOUBLET MODELS

The aim of this section is to show that in the "natural" two Higgs-doublet models (referred to as models I and II in Sec. II) the branching ratio for $\tau^- \to \eta\pi^-\nu$ is as negligible as in the standard model. In both models the only free parameters are x (the ratio of VEV's) and M_χ. The constraints arising from K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixings have been studied in these models by Abbott, Sikivie, and Wise[21] with the conclusion that, in both models,

$$x^2 < 2\frac{M_\chi}{m_c} . \tag{15}$$

An even better bound can be found in Ref. 30 but its precise expression depends on the values of m_t and the relevant mixing angles. In model I, the bound on $1/x^2$ given in Ref. 21 is less stringent: namely,

$$\frac{1}{x^2} < 100\frac{M_\chi}{m_s} . \tag{16}$$

As suggested in Ref. 14, more restrictions could be obtained from the semileptonic decays of B mesons into τ. Up to now, the inclusive branching ratio for $B \to \tau+$anything has not been measured. However, by considering the effect of the Higgs bosons on the B lifetime, for instance in model I with the lepton coupling proportional to $1/x$ [see Eq. (7)], we could reasonably infer that $1/x^2<10M_\chi{}^2/(m_b m_\tau)$. Nevertheless, for our purposes, such a bound will not be necessary. For model I, the constraints arising from $\tau^-\to\pi^-\nu$ [Eq. (11)] and the branching ratio $\tau^-\to a_0^-\nu$ [noted B_{a_0} in Eq. (12)] read

$$\frac{g^2}{2}\left|x^{\pm1}m_\tau\left[-xm_u+\frac{1}{x}m_d\right]\right|<0.014M_\chi{}^2\ , \qquad (17a)$$

$$B_{a_0}=1.64\left|\frac{g^2}{2}x^{\pm1}m_\tau\left[xm_u+\frac{1}{x}m_d\right]\left[\frac{1}{M_\chi{}^2}\right]\right|^2\ , \qquad (17b)$$

where the + or − refer to the two ways of coupling the leptons to the Higgs bosons as shown in Eq. (7). For the + (−) case, significant effects can only be obtained for $x^2\gg1$ ($0<x^2\ll1$). In these two cases, Eq. (17a) gives, respectively,

$$x^2<0.064\frac{M_\chi{}^2}{m_u m_\tau}\quad\text{and}\quad\frac{1}{x^2}<0.064\frac{M_\chi{}^2}{m_d m_\tau}\ , \qquad (18)$$

and in either case we have $B_{a_0}<3.3\times10^{-4}$. In the + case, we can also use the bound (15) on large values of x, with the result

$$B_{a_0}<4.57\times10^{-6}\left[\frac{m_\tau}{M_\chi}\right]^2 . \qquad (19)$$

The lower bound on a Higgs-boson mass being 17 GeV (Ref. 25), we see that this last equation implies $B_{a_0}<5\times10^{-8}$. In the − case, Eq. (13) yields

$$\frac{1}{x^2}<0.165\frac{M_\chi{}^2}{m_\tau m_s}\ , \qquad (20)$$

which implies $B_{a_0}<5.6\times10^{-6}$.

For model II, Eqs. (17a) and (17b) become

$$\frac{g^2}{2}\,|x^{\pm1}m_\tau(xm_u+xm_d)|<0.014M_\chi{}^2\ , \qquad (21a)$$

$$B_{a_0}=1.64\left|\frac{g^2}{2}x^{\pm1}m_\tau(xm_u-xm_d)\left[\frac{1}{M_\chi{}^2}\right]\right|^2 . \qquad (21b)$$

In the − case, expressions become x independent and give $B_{a_0}<10^{-10}$. In the + case, Eqs. (21a) and (21b) imply

$$B_{a_0}<2.4\times10^{-5}\ . \qquad (22)$$

Again, a better bound ($B_{a_0}<310^{-8}$) can be obtained from Eq. (15). We thus conclude that a significant branching ratio for $\tau^-\to a_0^-\nu$ and consequently for $\tau^-\to\eta\pi^-\nu$ (see discussion in Sec. III) can be excluded in models I and II.

V. RESTRICTIONS ON GENERAL TWO-HIGGS-DOUBLET MODELS AND THEIR IMPLICATIONS FOR $\tau^-\to\eta\pi^-\nu$

A significant branching ratio for $\tau^-\to\eta\pi^-\nu$ would require larger couplings of the Higgs bosons to up and down quarks on the τ lepton than the ones allowed in models I and II. We will thus consider the more general class of models mentioned in Sec. II, where the mass hierarchies are not imposed on the Yukawa couplings. In this case, h_u and h_d are not necessarily negligible and the phenomenological constraints have to be reconsidered in detail in order to determine how large they can be. We show that, in this context, a branching ratio of 1% for $\tau^-\to a_0^-\nu$ cannot be ruled out.

One of the less attractive features of the models where the third naturalness condition[19] has been relaxed is their lack of predictivity; i.e., they contain a large number of arbitrary parameters. Some of them are not directly relevant to the present discussion: namely, h_e, h_μ, h_b, and h_t. They can be as small as we want and, in order to simplify the discussion, we will assume $h_e=h_\mu=h_b=h_t=0$, but this is not an essential feature of the models suggested here.

We first discuss the constraints of the K_L-K_S mass difference and the nonleptonic decays of the kaons. The study of these decays is motivated by the fact that (pseudo)scalar matrix elements involving light quarks are expected to be enhanced.[23] These two constraints will be sufficient to impose reasonably small (but not negligible) couplings to the quarks, and thus other nonleptonic processes or radiatively induced FCNC's will not be discussed. A branching ratio of 1% then requires a rather large coupling to the τ lepton. In a model-independent way we can say that such a branching ratio implies a non-negligible effect for at least one of the modes $B^+\to\tau^+\nu$ or $B^+\to\pi^0\tau^+\nu$, but it is difficult at present to obtain an experimental measurement of these Kobayashi-Maskawa-suppressed modes.

With these remarks in mind we can compute the effective Hamiltonian relevant for the K_L-K_S mass difference. For this purpose we calculate the box diagrams with two Higgs bosons and with one Higgs boson and one W (see Ref. 21 for technical details). Up to first order in $m_c{}^2/M_{W,\chi}{}^2$ and neglecting $m_u{}^2/M_{W,\chi}{}^2$ terms, we obtain, for two generations of quarks,

$$\mathcal{H}^{\Delta S=2}=\cos^2\theta_C\sin^2\theta_C\frac{1}{2^9\pi^2M_\chi{}^2}\times\left\{\bar d\gamma^\mu(1-\gamma_5)s\,\bar d\gamma_\mu(1-\gamma_5)s\left[(h_u{}^2-h_c{}^2)^2+h_c{}^4\frac{m_c{}^2}{M_\chi{}^2}\left[3+2\ln\frac{m_c{}^2}{M_\chi{}^2}\right]+h_c{}^2g^2\frac{m_c{}^2}{M_W{}^2}\left[-4-4\ln\frac{m_c{}^2}{M_\chi{}^2}-\frac{3M_\chi{}^2}{M_W{}^2-M_\chi{}^2}\ln\frac{m_W{}^2}{M_\chi{}^2}\right]\right]\right.$$

$$+\bar{d}\gamma^{\mu}(1-\gamma_5)s\bar{d}\gamma_{\mu}(1+\gamma_5)s\left[-2h_d h_s h_c^2\frac{m_c^2}{M_\chi^2}+2h_d h_s g^2\frac{m_c^2}{M_W^2}\left[1+2\ln\frac{m_c^2}{M_\chi^2}+\frac{2M_\chi^2}{M_W^2-M_\chi^2}\ln\frac{M_W^2}{M_\chi^2}\right]\right]$$
$$+\bar{d}\gamma^{\mu}(1+\gamma_5)s\bar{d}\gamma_{\mu}(1+\gamma_5)s\left[h_d^2h_s^2\frac{m_c^2}{M_\chi^2}\left[3+2\ln\frac{m_c^2}{M_\chi^2}\right]\right]$$
$$+\bar{d}(1+\gamma_5)s\bar{d}(1+\gamma_5)s\left[h_c^2h_s^2\frac{m_c^2}{M_\chi^2}\left[8+4\ln\frac{m_c^2}{M_\chi^2}\right]\right]$$
$$+\bar{d}(1-\gamma_5)s\bar{d}(1-\gamma_5)s\left[h_d^2h_c^2\frac{m_c^2}{M_\chi^2}\left[8+4\ln\frac{m_c^2}{M_\chi^2}\right]\right]\Bigg\}\ . \tag{23}$$

For comparison the effective Hamiltonian corresponding to two-W exchange is[31]

$$\cos^2\theta\sin^2\theta\frac{g^4}{2^9\pi^2}\frac{m_c^2}{M_W^4}[\bar{d}\gamma^{\mu}(1-\gamma_5)s\bar{d}\gamma_{\mu}(1-\gamma_5)s]\ . \tag{24}$$

We have not written the contribution of the top quark which, even with h_t equal to zero, is present in the terms proportional to $h_d h_s$. A limit on these effects can be obtained by taking the upper bounds of the mixing angles[25] and $m_t \sim M_W$. Note that terms with the structure $(P\pm S)\times(P\pm S)$ appear in Eq. (23). According to Ref. 23 their matrix elements could be enhanced by a factor $(m_K/m_s)^2$. However, for the term $(V-A)\times(V+A)$ the procedure used by these authors is not consistent with the vacuum saturation, and that is why we have not attempted to estimate directly the K_L-K_S mass difference corresponding to Eq. (23). Overestimating the abovementioned effects, replacing the slowly varying function by their maximal values for 17 GeV $<M_\chi<1$ TeV, and taking into account the possibility that all the terms could add coherently, we can obtain a "safe" region of the parameter space in which the Higgs-boson contribution is smaller than the two-W contribution (24), namely,

$$|h_u^2-h_c^2|<0.005\frac{M_\chi}{M_W},\quad |h_c|<0.1\frac{M_\chi}{M_W}\ ,$$
$$|h_d|<0.02\frac{M_\chi}{M_W},\quad |h_s|<0.02\frac{M_\chi}{M_W}\ . \tag{25}$$

This is not the most general allowed region. The uncertainties can be reduced considerably when h_c and h_s are set to zero. In this case we have

$$h_c=h_s=0,\quad h_u^2<0.007\frac{M_\chi}{M_W}\ , \tag{26}$$

and no constraints on h_d. Following the methods described in Refs. 21 and 28, the only additional information which we can obtain from D^0-$\bar{D}^0$ or B^0-$\bar{B}^0$ mixing is, if h_t and h_b are neglected,

$$|h_d^2-h_s^2|<0.05\frac{M_\chi}{M_W}\ . \tag{27}$$

A new scalar interaction can also have implications for the nonleptonic decays of kaons. The $\Delta S=1$ effective Hamiltonian corresponding to Eq. (4) reads

$$\mathcal{H}^{\Delta S=1}=-\frac{\cos\theta_C\sin\theta_C}{4M_\chi^2}[(h_s h_d)\bar{s}(1-\gamma_5)u\bar{u}(1+\gamma_5)d-h_s h_u\bar{s}(1-\gamma_5)u\bar{u}(1-\gamma_5)d$$
$$-h_u h_d\bar{s}(1+\gamma_5)u\bar{u}(1+\gamma_5)d+h_u^2\bar{s}(1+\gamma_5)u\bar{u}(1-\gamma_5)d]\ . \tag{28}$$

Using the methods of Ref. 23, we obtain the following amplitudes for $K^+\to\pi^+\pi^0$ and $K_s^0\to\pi^+\pi^-$:

$$\mathcal{M}(K^+\to\pi^+\pi^0)\simeq -i\frac{\cos\theta_C\sin\theta_C}{4(2M_\chi^2)^{1/2}}f_\pi m_K^2\frac{m_\pi^2}{(m_u+m_d)m_s}\{h_s h_d-h_u^2+h_u(h_s-h_d)-\tfrac{1}{6}[h_u(h_s-h_d)]\}\ , \tag{29a}$$

$$\mathcal{M}(K_s^0\to\pi^+\pi^-)\simeq -i\frac{\cos\theta_C\sin\theta_C}{2(2M_\chi^2)^{1/2}}f_\pi m_K^2\frac{m_\pi^2}{(m_u+m_d)m_s}\left[h_s h_d-h_u^2+h_u(h_s-h_d)-\frac{1}{6}\frac{f_K}{f_\pi}\left[1+\frac{m_K^2}{m_\sigma^2}\right][h_u(h_s-h_d)]\right]\ . \tag{29b}$$

We could, in principle, try to fit these new contributions with the "missing" parts of the amplitudes which appear in Ref. 23 if the QCD effects are kept at their calculated values. This amounts to requiring the amplitudes (29a) and (29b) to be -0.15 and 1.6, respectively, in $G_F m_K{}^2 f_\pi \cos\theta_C \sin\theta_C$ units. However, one can see that, up to an overall factor 2, the two amplitudes of Eqs. (29a) and (29b) tend to be the same value in the limit $f_K \to f_\pi$ and $m_K/m_\sigma \to 0$. The fit proposed above requires a fine-tuning involving rather large Yukawa couplings, namely, $h_u(h_s-h_d) \simeq 4.8 G_F M_\chi{}^2$ and $h_d h_s - h_u{}^2 \simeq -3.9 G_F M_\chi{}^2$, a possibility we will not consider seriously here. We will rather impose that, for both amplitudes, the contribution of the scalar is smaller than the strong-interaction uncertainties, which implies

$$|h_u(h_s-h_d)| < 0.004 \frac{M_\chi{}^2}{M_W{}^2} ,$$
$$|h_d(h_s-h_u{}^2)| < 0.004 \frac{M_\chi{}^2}{M_W{}^2} . \quad (30)$$

Finally, we have to take into account the constraints coming from the τ decays (see Sec. III). In particular, Eqs. (11), (12), and (13) yield

$$\left| \frac{h_u+h_d}{h_u-h_d} \right| < 0.18 \left[\frac{0.01}{B_{a_0}} \right]^{1/2} ,$$
$$\left| \frac{h_u+h_s}{h_u-h_d} \right| < 0.47 \left[\frac{0.01}{B_{a_0}} \right]^{1/2} . \quad (31)$$

For B_{a_0} larger than 0.01, this imposes approximately $h_u \approx -h_d$ and $h_s = -\alpha h_u$ with $0 \lesssim \alpha \lesssim 2$.

We will now consider two special cases where the constraints can be expressed very simply. We first take the situation where

$$\pm h_c = h_s = h_d = -h_u \equiv h . \quad (32)$$

The main constraint which remains comes from the $m_c{}^2/M_{W,\chi}{}^2$ terms of the box diagram in Eq. (23), and has the form

$$|h| < \delta \frac{M_\chi}{M_W}, \quad 0.1 < \delta < 0.02 . \quad (33)$$

The parameter δ encodes the uncertainties of the top-quark contribution and of the (pseudo)scalar matrix elements. Substituting this last inequality in the expression of the branching ratio B_{a_0} of Eq. (12), we get

$$|h_\tau| > \frac{0.039}{\delta} \left[\frac{B_{a_0}}{0.01} \right]^{1/2} \left[\frac{M_\chi}{M_W} \right] . \quad (34)$$

From Eq. (14), we have the relation $B_{K_0^*} = 0.02 B_{a_0}$. The other constraints are automatically satisfied by the choice (32) and we see that $B_{a_0} = 0.01$ cannot be excluded except if a stringent upper limit on h_τ can be imposed.

Another simple situation can be obtained by setting

$$h_c = h_s = 0, \quad h_d = -h_u \equiv h' . \quad (35)$$

In this case, Eqs. (26) and (30) imply

$$|h'| < 0.084 \left[\frac{M_\chi}{M_W} \right]^{1/2} , \quad |h'| < 0.063 \left[\frac{M_\chi}{M_W} \right] , \quad (36)$$

and

$$|h_\tau| > 0.47 \left[\frac{B_{a_0}}{0.01} \right]^{1/2} \left[\frac{M_\chi}{M_W} \right]^{3/2} ,$$
$$|h_\tau| > 0.62 \left[\frac{B_{a_0}}{0.01} \right]^{1/2} \left[\frac{M_\chi}{M_W} \right] . \quad (37)$$

Equation (13) gives upper limits on B_{a_0} and $B_{K_0^*}$: namely,

$$B_{a_0} < 0.9\%, \quad B_{K_0^*} < 4.5 \times 10^{-5} . \quad (38)$$

This situation is more restrictive than the previous one, but $B_{a_0} = 0.9\%$ is still more than 2 orders of magnitude larger than what we expect in the standard model. Again h_τ has to be rather large if we want to get a substantial B_{a_0}. From Eqs. (36) and (37) with $B_{a_0} = 0.9\%$, we can, for instance, derive the inequality $|h_\tau/h'| > 10$.

It is clear from the previous discussion that an upper limit on h_τ would be welcome. Assuming that B_{a_0} is significantly larger than in the standard model, such a limit would exclude arbitrarily small values of $h_{u,d}/M_\chi{}^2$ [see Eq. (12)]. Note that in order to produce the Higgs boson considered here in $p\bar{p}$ collisions, it is important to have $h_{u,d}$ not too small and M_χ not too large. A restriction on large values of h_τ could be obtained by calculating the two-loop corrections to the ρ parameter. However, the results of Ref. 32 indicate that these corrections might be suppressed by numerical factors (typically $2^{-9}\pi^{-4}$). Within the context of perturbation theory we could impose the customary limit $h_\tau{}^2 < 4\pi^2$. For the choice of parameters given in Eq. (35) and $B_{a_0} = 0.9\%$, this yields $M_\chi < 480$ GeV and $|h_u/M_\chi{}^2| > 8.6 \times 10^{-7}$. A more complete treatment of this question would require investigations beyond perturbation theory, but this is beyond the scope of this paper.

As a general feature, the phenomenologically acceptable choices of Yukawa couplings compatible with a B_{a_0}

TABLE I. The ratios of cross sections for $p\overset{(-)}{p} \to \tau^+\nu$ with intermediate Higgs boson and $W(R_\tau)$ are given for $p\bar{p}$ at 2 TeV and pp at 40 TeV. The values of M_χ, $|h_u|$, and $|h_\tau|$ correspond to a branching ratio of 1% for $\tau^- \to a_0^- \nu$. The values of δ refer to the bound imposed on h_u.

M_χ (GeV)	δ	$\|h_u\|$	$\|h_\tau\|$	R_τ $p\bar{p}$ (2 TeV)	R_τ pp (40 TeV)
50	0.1	0.06	0.24	0.390	0.263
50	0.05	0.03	0.48	0.098	0.066
50	0.02	0.01	1.45	0.011	0.007
100	0.1	0.1	0.58	0.155	0.162
100	0.05	0.06	0.97	0.056	0.058
100	0.02	0.02	2.90	0.006	0.006
200	0.1	0.2	1.16	0.062	0.108
200	0.05	0.1	2.32	0.015	0.027
200	0.02	0.05	4.64	0.004	0.007

of the order 1% require h_τ much larger than the other couplings. This means that when this Higgs boson (χ) is produced, it will predominantly decay into the third generation of lepton. As an example, we can estimate the ratio of the cross sections $\overset{(-)}{p\,p} \to \tau^+\nu$ with an intermediate W^+ and χ^+ (at the pole). For $h_u = h_d$, we obtain[33]

$$R_\tau \equiv \frac{\sigma(p\overset{(-)}{p} \to \chi^+ \to \tau^+\nu)}{\sigma(p\overset{(-)}{p} \to W^+ \to \tau^+\nu)} = \frac{12h_u{}^2}{g^2} \frac{\left.\frac{\tau}{\hat{s}}\frac{\partial \mathcal{L}_{u\bar{d}}}{\partial \tau}\right|_{\hat{s}=M_\chi{}^2}}{\left.\frac{\tau}{\hat{s}}\frac{\partial \mathcal{L}_{u\bar{d}}}{\partial \tau}\right|_{\hat{s}=M_W{}^2}} . \quad (39)$$

We have taken the branching ratios for $\chi^+ \to \tau^+\nu$ and $W^+ \to \tau^+\nu$ to be 1 and $\frac{1}{12}$, respectively. Details concerning Eq. (39) can be found in Sec. IV of Ref. 33. The values of R_τ corresponding to $B_{a_0} = 1\%$ and various values of $|h_u|$ and M_χ are given in Table I for $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV and pp collisions at $\sqrt{s} = 40$ TeV. We have selected three different masses of the Higgs bosons (50, 100, and 200 GeV) and for each of them we have assumed Eq. (32), and saturated the bound (33) on $|h_u|$ with $\delta = 0.1$, 0.05, and 0.02. The value of $|h_\tau|$ is completely fixed by $B_{a_0} = 1\%$. We have estimated the parton-parton luminosities from Ref. 33. We see that for $\delta = 0.05$, values of R_τ of 2–10% are not excluded. However, smaller values of $|h_u|$ or larger values of M_χ can be chosen if we do not have an upper limit on $|h_\tau|$. For $\delta = 0.1$ and $M_\chi \lesssim 100$ GeV rather large values of R_τ can be obtained. A careful analysis of the UA1 data might rule out some of these possibilities.

VI. CONCLUSIONS

We have shown that a branching ratio larger than 0.1% for $\tau^- \to \eta\pi^-\nu$ can be excluded in the two-Higgs-doublet models where discrete symmetries prevent tree-level FCNC's ("natural" models I and II). There remains the possibility of relaxing the third naturalness requirement of Ref. 19 and adjusting the free parameters in such a way that the new interactions have no large observable effects except for $\tau^- \to \eta\pi^-\nu$. This possibility cannot be completely ruled out but has several unsatisfactory features (lack of predictivity and fine-tunings) and requires a rather large Yukawa coupling to the τ. The situation would be different if one could naturally obtain one of the patterns suggested in Sec. V in another theory beyond the standard model (for instance, Yukawa couplings with a hierarchy opposite to the masses for the quarks, but the usual hierarchy for the leptons). On the other hand, the experimental situation needs clarification. A firmly established branching ratio of more than 0.1% for $\tau^- \to \eta\pi^-\nu$ can be considered as a significant departure from the standard model in which case a peculiar two-Higgs-doublet explanation cannot be excluded. At the present time it seems, however, premature to claim that there is evidence for such a scenario.

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