Effective chiral Lagrangian of pseudoscalar and vector mesons

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We extend Weinberg's formulation of the Lagrangian involving pions and ρ mesons to include electromagnetism. We also extend this approach to SU(3). We obtain the results: $g_\rho=(m_\rho{}^2/g)=2g_{\rho\pi\pi}f_\pi{}^2$, $g_{\rho\pi\pi}=g=m_\rho/\sqrt{2}f_\pi$, and decay widths for $K^*\to K\pi$ and $\phi=K\bar{K}$ as well as leptonic widths of ρ, ω, and ϕ mesons and charge mean-square radius for K^+ or K^- meson in mass-mixing and current-mixing models of SU(3) symmetry breaking. These results agree reasonably well with their experimental values and require $f_K/f_\pi>1$ (1.1–1.3). We compare this formulation with the conventional formulation of the effective chiral Lagrangian involving ρ and A_1 mesons.

I. INTRODUCTION

It is generally accepted that the chiral quark model, where the fields in the effective chiral theory are those of QCD, plus the nonet of Nambu-Goldstone boson fields (pseudoscalar mesons), is a good low-energy approximation to QCD. In the linear version of the chiral Lagrangian, quarks belong to the representation (3,1) and (1,3) of $SU(3)_L\times SU(3)_R$. The nonet of pseudoscalar mesons belong to the representation $(3,3^*)+(3^*,3)$ of chiral SU(3).

Recently there has been a considerable interest in the effective low-energy chiral Lagrangian.[1–3] The usual approach[2–5] is to construct a Lagrangian from the pseudoscalar matrix field $U=\exp(i\lambda_i\pi_i/f_\pi)$ and linearly transforming gauge fields $V_{L\mu}$ and $V_{R\mu}$. The nonlinear gauge nonet fields ρ_μ can be obtained from linearly transforming gauge fields $V_{L\mu}$ and $V_{R\mu}$ (see Sec. IV).

Another interesting approach is based on noting that vector bosons are dynamical gauge bosons of a hidden local symmetry in the $U(3)_L\times U(3)_R/U(3)_V$ nonlinear[6–8] σ model. This approach is similar to that formulated by Weinberg for ρ mesons.

In this paper we follow Weinberg's formulation in writing the effective low-energy Lagrangian for a nonet of pseudoscalar and vector bosons. The plan of this paper is as follows. In Sec. II we review the Weinberg formulation for the chiral $SU(2)\times SU(2)$ group and extend it to include electromagnetism. In Sec. III we extend this formulation to SU(3). In particular, we study the effective Lagrangian for the decays $\rho\to\pi\pi$, $K^*\to K\pi$, and $\phi\to K\bar{K}$ and mean-square radius $\langle r_{K^+}{}^2\rangle$ for K^+ meson in two models of SU(3)-symmetry breaking. In Sec. IV we give a brief review of the usual effective chiral Lagrangian in order to compare the results obtained in this section to those of Sec. II. Finally, in Sec. V we give the summary and discussion of our results.

II. THE WEINBERG FORMULATION FOR THE CHIRAL SU(2)×SU(2) GROUP AND AN EXTENSION TO INCLUDE ELECTROMAGNETISM

A. SU(2)×SU(2)-invariant chiral Lagrangian

We start with the σ model and write the Lagrangian with quark fields, a triplet of pseudoscalar fields, and a singlet of a scalar field. This Lagrangian is invariant under the global chiral group $SU(2)\times SU(2)$. Under this group, a doublet of left-handed and right-handed quarks transform as

$$\begin{aligned}&\psi_L\to g_1\psi_L,\quad \psi_R\to g_1\psi_R\ ,\\ &\psi_L\to g_2\psi_L,\quad \psi_R\to g_2^\dagger\psi_R\ .\end{aligned}\tag{1}$$

Mesons transform as

$$\Sigma\to g_1\Sigma g_1^\dagger,\quad \Sigma\to g_2\Sigma g_2\ .\tag{2}$$

Here g_1 is the SU(2) (isospin) transformation and g_2 is the chiral SU(2) transformation. Σ is a 3×3 matrix:

$$\Sigma=\frac{1}{\sqrt{2}}(\sigma-i\boldsymbol{\tau}\cdot\boldsymbol{\pi}')\ .\tag{3}$$

The chiral-invariant Lagrangian is given by

$$\begin{aligned}\mathcal{L}=&-\bar{\psi}_L\gamma_\mu\partial_\mu\psi_L-\bar{\psi}_R\gamma_\mu\partial_\mu\psi_R-\sqrt{2}(\bar{\psi}_L\Sigma\psi_R+\bar{\psi}_R\Sigma^\dagger\psi_L)\\ &-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma)-V(\Sigma)\ .\end{aligned}\tag{4}$$

Now $\mathrm{Tr}(\Sigma^\dagger\Sigma)=(\sigma^2+\pi'^2)$ is chiral invariant. $\Sigma^\dagger\Sigma=\frac{1}{2}(\sigma^2+\pi'^2)$ is also chiral invariant, if $[\sigma,\boldsymbol{\pi}']=0$.

We nonlinearize the Lagrangian (4) by means of a field-dependent gauge transformation

$$\psi_L=\Omega_L(\xi)\hat{\psi}_L,\quad \psi_R=\Omega_R(\xi)\hat{\psi}_R\ ,\tag{5a}$$

$$\Sigma=\Omega_L\hat{\Sigma}\Omega_R^\dagger=\frac{1}{\sqrt{2}}f_\pi\Omega_L\Omega_R^\dagger\ .\tag{5b}$$

ξ is a pseudoscalar field and Ω_L and Ω_R are unitary matrices, and we take $\hat{\Sigma}=(1/\sqrt{2})f_\pi\hat{I}$. This will give

$$\Sigma^\dagger\Sigma=\tfrac{1}{2}f_\pi{}^2=\text{const} \ , \tag{6}$$

since we take $\Sigma^\dagger\Sigma$ to be chiral invariant. Here f_π is the pion decay constant. We shall write $U=\Omega_L\Omega_R^\dagger$.

The transformed Lagrangian is given by

$$\begin{aligned}\mathcal{L}=&-\bar{\hat{\psi}}_L\gamma_\mu\partial_\mu\hat{\psi}_L-\bar{\hat{\psi}}_R\gamma_\mu\partial_\mu\hat{\psi}_R+i\frac{g}{\sqrt{2}}\bar{\hat{\psi}}_L\gamma_\mu\mathcal{V}_{L\mu}\hat{\psi}_L\\&+i\frac{g}{\sqrt{2}}\bar{\hat{\psi}}_R\gamma_\mu\mathcal{V}_{R\mu}\hat{\psi}_R-f_\pi f(\bar{\hat{\psi}}_L\hat{\psi}_R+\bar{\hat{\psi}}_R\hat{\psi}_L)\\&-\tfrac{1}{4}f_\pi{}^2\,\mathrm{Tr}(\partial_\mu U^\dagger\partial_\mu U)-V(U) \ ,\end{aligned}\tag{7}$$

where

$$\begin{aligned}&\mathcal{V}_{L\mu}=i\frac{\sqrt{2}}{g}(\Omega_L^\dagger\partial_\mu\Omega_L), \quad \mathcal{V}_{L\mu}=\frac{1}{\sqrt{2}}\tau\cdot\boldsymbol{\mathcal{V}}_{L\mu} \ ,\\&\mathcal{V}_{R\mu}=i\frac{\sqrt{2}}{g}(\Omega_R^\dagger\partial_\mu\Omega_R), \quad \mathcal{V}_{R\mu}=\frac{1}{\sqrt{2}}\tau\cdot\boldsymbol{\mathcal{V}}_{R\mu} \ .\end{aligned}\tag{8}$$

As first noted by Weinberg, the Lagrangian (7) is gauge invariant under local transformation:

$$\begin{aligned}&\hat{\psi}_L\to h(x)\psi_L, \quad \hat{\psi}_R\to h(x)\psi_R \ ,\\&\mathcal{V}_{L\mu}\to h\mathcal{V}_{L\mu}h^\dagger+\frac{\sqrt{2}i}{g}h\,\partial_\mu h^\dagger \ ,\\&\mathcal{V}_{R\mu}\to h\mathcal{V}_{R\mu}h^\dagger+\frac{\sqrt{2}i}{g}h\,\partial_\mu h^\dagger \ ,\end{aligned}\tag{9}$$

or equivalently

$$\begin{aligned}&\mathcal{V}_\mu\to h\mathcal{V}_\mu h^\dagger+i\frac{\sqrt{2}}{g}h\,\partial_\mu h^\dagger \ ,\\&b_\mu\to hb_\mu h^\dagger \ ,\end{aligned}\tag{10}$$

where

$$\begin{aligned}&\mathcal{V}_\mu=\tfrac{1}{2}(\mathcal{V}_{L\mu}+\mathcal{V}_{R\mu})=\frac{i}{\sqrt{2}g}(\Omega_L^\dagger\partial_\mu\Omega_L+\Omega_R^\dagger\partial_\mu\Omega_R) \ ,\\&b_\mu=\tfrac{1}{2}(\mathcal{V}_{L\mu}-\mathcal{V}_{R\mu})=\frac{i}{\sqrt{2}g}(\Omega_L^\dagger\partial_\mu\Omega_L-\Omega_R^\dagger\partial_\mu\Omega_R) \ .\end{aligned}\tag{11}$$

The above transformation is possible provided Ω_L and Ω_R transform as

$$\begin{aligned}&\Omega_L\to g_2\Omega_L h^\dagger, \quad \Omega_R\to g_2^\dagger\Omega_R h^\dagger \ ,\\&U\to g_2Ug_2 \ .\end{aligned}\tag{12}$$

The hidden gauge symmetry is defined by transformation equations (9). Thus, it is possible to introduce a vector field V_μ in a gauge-invariant way,[1] if V_μ transforms in the same way as $\mathcal{V}_\mu$, by replacing $\partial_\mu\hat{\psi}$ by

$$[\partial_\mu-\tfrac{1}{2}ig\tau\cdot(\mathbf{V}_\mu-\boldsymbol{\mathcal{V}}_\mu)]\hat{\psi}$$

and adding the mass term

$$-\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}[(V_\mu-\mathcal{V}_\mu)^2]$$

together with the kinetic energy term $-\frac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})$ in the Lagrangian. Hence, the gauge-invariant Lagrangian (under h) (removing the caret) is given by

$$\begin{aligned}\mathcal{L}=&-\bar{\psi}\gamma_\mu\partial_\mu\psi+i\frac{g}{\sqrt{2}}\bar{\psi}\gamma_\mu V_\mu\psi\\&+i\frac{g}{\sqrt{2}}\bar{\psi}\gamma_\mu b_\mu\psi-f_\pi f\,\bar{\psi}\psi\\&-\tfrac{1}{4}f_\pi{}^2\,\mathrm{Tr}[\partial_\mu U^\dagger\partial_\mu U]\\&-\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}[(V_\mu-\mathcal{V}_\mu)^2]-\tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu}) \ ,\end{aligned}\tag{13}$$

where

$$V_{\mu\nu}=\partial_\nu V_\mu-\partial_\mu V_\nu+\frac{i}{\sqrt{2}}g[V_\mu,V_\nu] \ . \tag{14}$$

We select the gauge such that

$$\begin{aligned}&\Omega_L=e^{i\xi}=\Omega_R^\dagger\equiv\Omega \ ,\\&\xi=\frac{\pi}{\sqrt{2}f_\pi}=\frac{i\tau\cdot\pi}{2f_\pi}, \quad f_\pi=93\text{ MeV} \ ,\end{aligned}\tag{15}$$

and identify V_μ with the ρ_μ field.

For low-energy phenomena, we use the approximations

$$\begin{aligned}&e^{i\xi}\approx 1+i\xi+\frac{1}{2!}(i\xi)^2 \ ,\\&\mathcal{V}_\mu\approx\frac{i}{\sqrt{2}g}[\xi,\partial_\mu\xi] \ ,\\&b_\mu\approx-\frac{\sqrt{2}}{g}\partial_\mu\xi \ .\end{aligned}\tag{16}$$

Hence, we have from Eq. (13) the low-energy effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}=&-\bar{\psi}\gamma_\mu\left[\partial_\mu-\frac{ig}{\sqrt{2}}\left[\rho_\mu-\frac{1}{f_\pi g}\gamma_5\partial_\mu\pi\right]\right]\psi\\&-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu\pi\,\partial_\mu\pi)\\&-\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}\left[\rho_\mu{}^2-i\frac{1}{\sqrt{2}gf_\pi{}^2}\rho_\mu[\pi,\partial_\mu\pi]\right]\\&-\tfrac{1}{2}\mathrm{Tr}(\rho_{\mu\nu}\rho_{\mu\nu}) \ .\end{aligned}\tag{17}$$

This gives the result

$$m_\rho{}^2=m_0{}^2, \quad g_{\rho\pi\pi}=m_\rho{}^2/2f_\pi{}^2g \ . \tag{18}$$

Equation (18) gives the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[9] if $g_{\rho\pi\pi}=g$. We will now confine ourselves to the meson part of the Lagrangian (17) in the rest of this section.

B. Electromagnetism and vector-meson dominance

We introduce electromagnetism in the usual way, viz., replacing $\partial_\mu\pi$ by

$$\partial_\mu\pi+\frac{ie}{\sqrt{2}}[\pi,Q]\mathcal{A}_\mu$$

and $\partial_\mu \rho_\nu$ by

$$\partial_\mu \rho_\nu + \frac{ie}{\sqrt{2}}[\rho_\nu, Q]\mathcal{A}_\mu$$

in the kinetic-energy part of Lagrangian (17). The Lagrangian that we obtain by this substitution does not give the vector-meson dominance in the usual sense.[10] In order to get the vector-meson dominance, we add an additional gauge-invariant term

$$-\tfrac{1}{4}\,\mathrm{Tr}\left[2\frac{e}{g}Q\tilde{\rho}_{\mu\nu}F_{\mu\nu}\right]$$

to the Lagrangian. Here $Q=\tau_3/\sqrt{2}$, $\mathcal{A}_\mu$ is the electromagnetic field, and

$$\tilde{\rho}_{\mu\nu}=\rho_{\mu\nu}+\frac{ie}{\sqrt{2}}([\rho_\mu,Q]\mathcal{A}_\nu-[\rho_\nu,Q]\mathcal{A}_\mu)\ ,$$
$$F_{\mu\nu}=\partial_\nu\mathcal{A}_\mu-\partial_\mu\mathcal{A}_\nu\ . \tag{19}$$

With the above prescription, the Lagrangian is given by

$$\mathcal{L}=\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}\,\mathrm{Tr}(\tilde{\rho}_{\mu\nu}\tilde{\rho}_{\mu\nu})-\frac{1}{4}\frac{2e}{g}\mathrm{Tr}(Q\tilde{\rho}_{\mu\nu}F_{\mu\nu})$$
$$-\tfrac{1}{2}\,\mathrm{Tr}\left[\left(\partial_\mu\pi+\frac{ie}{\sqrt{2}}[\pi,Q]\mathcal{A}_\mu\right)\times\left(\partial_\mu\pi+\frac{ie}{\sqrt{2}}[\pi,Q]\mathcal{A}_\mu\right)\right]$$
$$-\tfrac{1}{2}m_\rho{}^2\,\mathrm{Tr}[(\rho_\mu-\mathcal{V}_\mu)^2]\ . \tag{20}$$

In order to explicitly see the vector-meson dominance, we make the change of variable[10]

$$\rho_\mu=\rho'_\mu-\frac{e}{g}Q\mathcal{A}_\mu\ ,$$

then

$$\tilde{\rho}_{\mu\nu}=\rho'_{\mu\nu}-\frac{e}{g}QF_{\mu\nu}$$

and we obtain the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F'_{\mu\nu}F'_{\mu\nu}-\tfrac{1}{4}\,\mathrm{Tr}(\rho'_{\mu\nu}\rho'_{\mu\nu})$$
$$-\tfrac{1}{2}m_\rho{}^2\,\mathrm{Tr}\left[\rho'^2_\mu-2\frac{e'}{g}\rho'_\mu Q\mathcal{A}_\mu+\frac{e'^2}{g^2}Q^2\mathcal{A}'^2_\mu-i\frac{\sqrt{2}}{g}\frac{1}{2f_\pi{}^2}[\pi,\partial_\mu\pi]\rho'_\mu+\frac{i\sqrt{2}}{g}\frac{1}{2f_\pi{}^2}[\pi,\partial_\mu\pi]Q\mathcal{A}'_\mu\right]$$
$$-\tfrac{1}{2}\,\mathrm{Tr}\left[\partial_\mu\pi\,\partial_\mu\pi+\frac{2ie'}{\sqrt{2}}\partial_\mu\pi[\pi,Q]\mathcal{A}'_\mu-\frac{e'^2}{2}[\pi,Q][\pi,Q]\mathcal{A}'^2_\mu\right]\ , \tag{21}$$

where

$$\mathcal{A}'_\mu=\left[1-\left(\frac{e}{g}\right)^2\right]^{1/2}\mathcal{A}_\mu,\quad F'_{\mu\nu}=\left[1-\left(\frac{e}{g}\right)^2\right]^{1/2}F_{\mu\nu},\quad e'=\frac{e}{[1-(e/g)^2]^{1/2}}\ . \tag{22}$$

Removing the prime, we obtain the main result of this section:

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}_{\mu\nu}-\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}-\tfrac{1}{2}m_\rho{}^2\boldsymbol{\rho}_\mu{}^2+e(m_\rho{}^2/g)\rho^0_\mu\mathcal{A}_\mu-\tfrac{1}{2}m_\rho{}^2\frac{e^2}{g^2}\mathcal{A}_\mu{}^2$$
$$-\frac{m_\rho{}^2}{2gf_\pi{}^2}\boldsymbol{\rho}_\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})+e\left[\frac{m_\rho{}^2}{2g^2f_\pi{}^2}-1\right](\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})_3\mathcal{A}_\mu+e^2(\boldsymbol{\pi}\cdot\boldsymbol{\pi}-\pi^{02})\mathcal{A}_\mu{}^2\ . \tag{23}$$

Independent of any other assumption, we obtain the results

$$g_{\rho\pi\pi}=m_\rho{}^2/2gf_\pi{}^2,\quad g_\rho\equiv m_\rho{}^2/g=2g_{\rho\pi\pi}f_\pi{}^2\ . \tag{24}$$

If we assume that the pion form factor is completely dominated by ρ meson, i.e., direct interaction of pion with photon as given in Eq. (23) is zero, we obtain the result

$$(m_\rho{}^2/2g^2f_\pi{}^2)=1\ . \tag{25}$$

This would give the result

$$g_{\rho\pi\pi}=g=m_\rho/\sqrt{2}f_\pi,\quad g_\rho=\sqrt{2}m_\rho f_\pi\ , \tag{26}$$

i.e., the KSRF relation and the universality. Equations (24)–(26) are the main results of this section.

Finally, we want to remark that we obtain the same Lagrangian as given by Eq. (23), first replacing[11] ρ_μ by $\rho_\mu+(eQ/g)\mathcal{A}_\mu$ in the kinetic-energy term of ρ_μ in the Lagrangian (17), but with the usual prescription for pions, and then making the change of variable $\rho_\mu=\rho'_\mu-(e/g)Q\mathcal{A}_\mu$ and dropping the prime.

III. THE WEINBERG FORMULATION EXTENDED TO SU(3)

A. Effective SU(3) chiral Lagrangian

The Lagrangian (4) is a global chiral-SU(3)-invariant Lagrangian, provided that we regard quarks as SU(3) triplet and

$$\Sigma=\frac{1}{\sqrt{2}}(\lambda_i\sigma_i-i\lambda_i\pi'_i)\ ,$$

where λ_i $(i=0,\ldots,8)$ are Gell-Mann matrices, σ_i and π'_i are nonets of scalar and pseudoscalar mesons. As before, the nonlinear version of the Lagrangian (4) is given in Eq. (7), with

$$\Omega_L=\Omega_R^\dagger\equiv\Omega=e^{i\xi},\quad \xi=\frac{1}{\sqrt{2}}\lambda_i\xi_i\ ,$$
$$\mathcal{V}_{L\mu}=\frac{1}{\sqrt{2}}\lambda_i\mathcal{V}_{Li\mu},\quad \mathcal{V}_{R\mu}=\frac{1}{\sqrt{2}}\lambda_i\mathcal{V}_{Ri\mu}\ . \tag{27}$$

Finally, we note that the Lagrangian (13) also holds for chiral SU(3). Note that in Eq. (13), $V_\mu=\lambda_i V_{i\mu}$ are nonet of vector bosons. It may be noted that the meson part of the Lagrangian is chiral U(3) invariant.

We now take into account SU(3)-symmetry breaking. There are two kinds of symmetry breaking, one is in the kinetic-energy term and the other is in the mass term of Eq. (13). For the symmetry breaking in the kinetic-energy term, we consider two cases.

(i) We replace Σ as given in Eq. (5b) by

$$\Sigma=\frac{1}{\sqrt{2}}f_\pi\Omega_L\hat{C}\Omega_R^\dagger=\frac{1}{\sqrt{2}}f_\pi U'\ , \tag{28}$$

where $\hat{C}$ is a numerical 3×3 matrix

$$\hat{C}=\mathrm{diag}(1,1,1+c)\ . \tag{29}$$

Then the term $ff_\pi\bar{\psi}\psi$ in Eq. (13) becomes

$$ff_\pi\bar{\psi}\hat{C}\psi=ff_\pi[\bar{u}u+\bar{d}d+(1+c)\bar{s}s]\ . \tag{30}$$

The kinetic-energy term $-\frac{1}{4}f_\pi{}^2\,\mathrm{Tr}(\partial_\mu U^\dagger\partial_\mu U)$ goes over to

$$-\tfrac{1}{4}f_\pi{}^2\,\mathrm{Tr}(\partial_\mu U'^\dagger\,\partial_\mu U')\approx-\tfrac{1}{2}f_\pi{}^2\left[\frac{1}{f_\pi{}^2}\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}+\frac{1}{f_K{}^2}\left[1+\frac{c}{2}\right]^2(2\partial_\mu\bar{K}^0\,\partial_\mu K^0+2\partial_\mu K^-\,\partial_\mu K^+)\right.$$
$$\left.+\left[\frac{1}{f_8{}^2}\partial_\mu\eta_8\,\partial_\mu\eta_8+\frac{1}{f_1{}^2}\partial_\mu\eta_1\,\partial_\mu\eta_1\right]+(c^2+2c)\left[-\frac{2}{\sqrt{3}f_8}\partial_\mu\eta_8\,\partial_\mu\eta_8+\frac{\sqrt{2}}{\sqrt{3}f_1}\partial_\mu\eta_1\right]^2\right]. \tag{31}$$

In the broken SU(3), when we identify the fields ξ with the pseudoscalar fields, we use the appropriate decay constants f_π, f_K, etc. The normalization of the kinetic-energy term gives

$$f_\pi{}^2\left[1+\frac{c}{2}\right]^2=f_K{}^2. \tag{32}$$

If we use

$$c=\frac{m_s-\bar{m}}{\bar{m}},\quad \bar{m}=\tfrac{1}{2}(m_u+m_d)\ , \tag{33}$$

as given by Eq. (30), we obtain

$$f_K/f_\pi=\left[1+\frac{m_s-\bar{m}}{2\bar{m}}\right]. \tag{34}$$

In the above relation, if we use current-quark masses $m_u=5.1$ MeV, $m_d=8.9$ MeV, and $m_s=175$ MeV, we obtain too high a value of f_K/f_π, which is not tenable. Although with constituent-quark masses $m_u\sim m_d\sim340$ MeV and $m_s\sim540$ MeV, we obtain $f_K/f_\pi\approx1.29$; however, it is not appropriate to use constituent-quark masses. We conclude that this approach to symmetry breaking is not tenable. This is not surprising, because this approach corresponds to spontaneous SU(3) breaking. But we know that SU(3) is not spontaneously broken; there is no Nambu-Goldstone scalar meson.

We now consider case (ii) in which SU(3) is explicitly broken in the kinetic-energy term. For quarks, we add to the term

$$-f(\bar{\psi}_L\Sigma\psi_R+\bar{\psi}_R\Sigma^\dagger\psi_L)$$

an additional term

$$-f(\bar{\psi}_L B\Sigma\psi_R+\bar{\psi}_R\Sigma^\dagger B\psi_L)\ ,$$

where B is a diagonal matrix

$$B=\mathrm{diag}(0,0,b)\ .$$

When we nonlinearize the Lagrangian, the extra term takes the form

$$-f(\bar{\psi}_L\Omega^\dagger B\Omega\psi_R+\bar{\psi}_R\Omega B\Omega^\dagger\psi_L)\ .$$

We will not discuss this term any further.

Taking into account explicit SU(3)-symmetry breaking in the U-dependent meson kinetic-energy term of the Lagrangian (13), we obtain this kinetic-energy term

$$-\tfrac{1}{4}f_\pi{}^2\,\mathrm{Tr}(\partial_\mu U^\dagger\,\partial_\mu U)-\tfrac{1}{4}f_\pi{}^2\kappa^2$$
$$\times[(\partial_\mu U^\dagger)_3^\alpha(\partial_\mu U)_\alpha^3+(\partial_\mu U^\dagger)_\alpha^3(\partial_\mu U)_3^\alpha]\ . \tag{35}$$

The extra term in Eq. (35) gives

$$-\tfrac{1}{2}f_\pi{}^2\kappa^2\left[\frac{1}{f_K{}^2}2(\partial_\mu\bar{K}^0\,\partial_\mu K^0+\partial_\mu K^-\,\partial_\mu K^+)\right.$$
$$\left.+\left[-\frac{2}{\sqrt{3}}\frac{1}{f_8}\partial_\mu\eta_8+\left[\frac{2}{3}\right]^{1/2}\frac{1}{f_1}\partial_\mu\eta_1\right]^2\right]. \tag{36}$$

The normalization of the kinetic-energy term (35) gives

$$\frac{f_\pi{}^2}{f_K{}^2}(1+\kappa^2)=1\ , \tag{37}$$

where κ is an arbitrary parameter and cannot be fixed from the theory in this approach.

B. Mass-mixing model of SU(3)-symmetry breaking

The other kind of SU(3)-breaking term which we introduce is in the mass term of vector bosons. We call this the mass-mixing model of symmetry breaking. Taking this into account, we obtain for the mass term,

$$-\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}[(V_\mu-\mathcal{V}_\mu)^2]-\tfrac{1}{2}\delta m^2[(V_\mu-\mathcal{V}_\mu)^\alpha_3(V_\mu-\mathcal{V}_\mu)^3_\alpha+(V_\mu-\mathcal{V}_\mu)^3_\alpha(V_\mu-\mathcal{V}_\mu)^\alpha_3]\ . \tag{38}$$

Here $\mathcal{V}_\mu$ is as given in Eq. (16). V_μ is a 3×3 matrix containing nine vector bosons. For vector bosons, nonet symmetry is good. Therefore, to simplify the calculation, we use this symmetry and identify ω and ϕ mesons as

$$\omega\approx\frac{1}{\sqrt{3}}\omega_8+(\tfrac{2}{3})^{1/2}\omega_1,\quad \phi\approx-(\tfrac{2}{3})^{1/2}\omega_8+(\tfrac{1}{3})^{1/2}\omega_1\ . \tag{39}$$

Using Eq. (39), we obtain, from Eq. (38),

$$m_\rho{}^2=m_\omega{}^2=m_0{}^2,\quad m_{K^*}{}^2=m_0{}^2+\delta m^2,\quad m_\phi{}^2=m_0{}^2+2\delta m^2\ , \tag{40}$$

and the nonet mass formula

$$m_\phi{}^2-m_\omega{}^2=2\delta m^2=2(m_{K^*}{}^2-m_\rho{}^2)\ . \tag{41}$$

Using Eqs. (38), (16), and (40) the Lagrangian for VPP ($V=\rho$, ω, K^*, and ϕ and $P=\pi$ and K) coupling is given by

$$\begin{aligned}\mathcal{L}_{VPP}=-\frac{1}{2}\Bigg[&\frac{m_\rho{}^2}{gf_\pi{}^2}\left[\rho_\mu\cdot(\pi\times\partial_\mu\pi)+\frac{i}{2}\frac{f_\pi{}^2}{f_K{}^2}[\rho_\mu\cdot(\bar{K}\tau\,\partial_\mu K+\bar{K}^c\tau\,\partial_\mu K^c)-\omega_\mu(\bar{K}^c\,\partial_\mu K^c-\bar{K}\,\partial_\mu K)]\right]\\&+\frac{i}{2}\frac{m_{K^*}{}^2}{gf_\pi f_K}[K^*_\mu(-\bar{K}\tau\cdot\partial_\mu\pi+\mathrm{H.c.})+K^{*c}_\mu(-\bar{K}^c\tau\cdot\partial_\mu\pi+\mathrm{H.c.})]\\&+i(m_\phi{}^2/gf_K{}^2)\left[\phi_\mu\frac{1}{\sqrt{2}}(-\bar{K}\,\partial_\mu K+\bar{K}^c\,\partial_\mu K^c)\right]\Bigg]\ ,\end{aligned} \tag{42}$$

where

$$K=\begin{bmatrix}K^+\\K^0\end{bmatrix},\quad K^c=-i\tau_2\bar{K}\ . \tag{43}$$

From Eq. (42), we obtain

$$g_{\rho\pi\pi}=m_\rho{}^2/2gf_\pi{}^2,\quad g_{K^*K\pi}=m_{K^*}{}^2/2gf_Kf_\pi,\quad g_{\phi K\bar{K}}=-m_\phi{}^2/2gf_K{}^2\ . \tag{44}$$

In order to introduce electromagnetic interaction, we replace[11] V_μ by $V_\mu+(eQ/g)\mathcal{A}_\mu$ in the kinetic-energy term $-\frac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})$ and $\partial_\mu\xi$ by

$$\partial_\mu\xi+\frac{ie}{2}[\xi,Q]\mathcal{A}_\mu$$

in the kinetic-energy term

$$-\tfrac{1}{4}f_\pi{}^2(\partial_\mu U^\dagger\,\partial_\mu U)\sim-\tfrac{1}{4}-4f_\pi{}^2\,\mathrm{Tr}(\partial_\mu\xi\,\partial_\mu\xi)$$

in the meson part of the Lagrangian (13). Here Q is a 3×3 diagonal matrix

$$Q=\sqrt{2}\,\mathrm{diag}(\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3})\ . \tag{45}$$

It is understood that in the Lagrangian (13), the vector-boson mass term is to be replaced by Eq. (38). In the Lagrangian that we obtain, we set $V_\mu=V'_\mu-(e/g)Q\mathcal{A}_\mu$. Dropping the prime, we obtain the Lagrangian

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\,\partial_\mu M)\\&+\{-ie[(\pi^-\partial_\mu\pi^+-\pi^+\partial_\mu\pi^-)+(K^-\partial_\mu K^+-K^+\partial_\mu K^-)]\mathcal{A}_\mu+O(e^2)\}\\&-\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}\left[\left[V_\mu-\frac{e}{g}Q\mathcal{A}_\mu-\mathcal{V}_\mu\right]\left[V_\mu-\frac{e}{g}Q\mathcal{A}_\mu-\mathcal{V}_\mu\right]\right]-\tfrac{1}{2}\delta m^2\left[2\left[V_\mu-\frac{e}{g}Q\mathcal{A}_\mu-\mathcal{V}_\mu\right]^\alpha_3\left[V_\mu-\frac{e}{g}Q\mathcal{A}_\mu-\mathcal{V}_\mu\right]^3_\alpha\right].\end{aligned} \tag{46}$$

Here M is a 3×3 matrix containing the well-known pseudoscalar mesons (π,K,η_8,η_1), and $\mathcal{V}_\mu$ is given by Eq. (16). From (46) we obtain our final result

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}\,\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu M\,\partial_\mu M)+\mathcal{L}_{VPP}-\tfrac{1}{2}m_\rho{}^2\rho_\mu{}^2-\tfrac{1}{2}m_\omega{}^2\omega_\mu{}^2-\tfrac{1}{2}m_\phi{}^2\phi_\mu{}^2$$
$$+\frac{e}{g}\left[m_\rho{}^2\rho_\mu{}^0+\frac{m_\omega{}^2}{3}\omega_\mu-\frac{\sqrt{2}}{3}m_\phi{}^2\phi_\mu\right]\mathcal{A}_\mu-\frac{1}{2}\frac{e^2}{g^2}(m_\rho{}^2+\tfrac{1}{9}m_\omega{}^2+\tfrac{2}{9}m_\phi{}^2)\mathcal{A}_\mu{}^2$$
$$+ie\left[\left[\frac{m_\rho{}^2}{2g^2f_\pi{}^2}-1\right](\pi^-\partial_\mu\pi^+-\pi^+\partial_\mu\pi^-)+\frac{2}{3}\frac{m_\phi{}^2-m_{K^*}{}^2}{2g^2f_K{}^2}(\bar{K}^0\partial_\mu K^0-K^0\partial_\mu\bar{K}^0)\right.$$
$$\left.+\left[\frac{m_\rho{}^2+\frac{2}{3}(m_\phi{}^2-m_{K^*}{}^2)}{2g^2f_\pi{}^2}-1\right](K^-\partial_\mu K^+-K^+\partial_\mu K^-)\right]\mathcal{A}_\mu\ . \tag{47}$$

In addition to the results given in Eq. (44), we obtain the following results:

$$g_\rho=(m_\rho{}^2/g)=2g_{\rho\pi\pi}f_\pi{}^2,\quad g_\omega=(m_\omega{}^2/3g)=\tfrac{2}{3}g_{\rho\pi\pi}f_\pi{}^2\ ,$$
$$g_\phi=-(\sqrt{2}\,m_\phi{}^2/3g)=-\frac{\sqrt{2}}{3}(m_\phi{}^2/m_\rho{}^2)g_{\rho\pi\pi}f_\pi{}^2\ . \tag{48}$$

The first part of Eq. (48) are the results as given by nonet symmetry. As we have noted in Sec. II, the assumption that the pion form factor is completely dominated by the ρ meson, viz., $m_\rho{}^2/2g^2f_\pi{}^2=1$ gives the KSRF relation and the universality. However, we see from Eq. (47) that direct interactions of $K^+(K^-)$ and $K^0(\bar{K}^0)$ with a photon do not vanish and are, respectively, given by

$$ie\left[\left[\frac{f_\pi{}^2}{f_K{}^2}\right]\left[1+\frac{2}{3}\frac{m_{K^*}{}^2-m_\rho{}^2}{m_\rho{}^2}\right]-1\right](K^-\partial_\mu K^+-K^+\partial_\mu K^-)\ ,$$
$$ie\left[\left[\frac{f_\pi{}^2}{f_K{}^2}\right]\frac{2}{3}\frac{m_{K^*}{}^2-m_\rho{}^2}{m_\rho{}^2}\right](\bar{K}^0\partial_\mu K^0-K^0\partial_\mu\bar{K}^0)\ , \tag{49}$$

where we have used the nonet mass formula (41). The above expressions vanish only in the SU(3) limit. In fact, when SU(3) is broken their presence is essential to give the correct normalizations $F_{K^+}(0)=1$ and $F_{K^0}(0)=0$. With the presence of (49), the electromagnetic form factors of K^+ and K^0 mesons with vector-meson dominance are

$$F_{K^+}(k^2)=1-\frac{f_\pi{}^2}{f_K{}^2}\left[1+\frac{2}{3}\frac{m_{K^*}{}^2-m_\rho{}^2}{m_\rho{}^2}\right]+\frac{f_\pi{}^2}{f_K{}^2}\frac{1}{m_\rho{}^2}\left[\frac{m_\rho{}^4}{2(k^2+m_\rho{}^2)}+\frac{m_\omega{}^4}{6(k^2+m_\omega{}^2)}+\frac{m_\phi{}^4}{3(k^2+m_\phi{}^2)}\right]\ , \tag{50a}$$

$$F_{K^0}(k^2)=-\frac{f_\pi{}^2}{f_K{}^2}\frac{2}{3}\frac{m_{K^*}{}^2-m_\rho{}^2}{m_\rho{}^2}+\frac{f_\pi{}^2}{f_K{}^2}\frac{1}{m_\rho{}^2}\left[-\frac{m_\rho{}^4}{2(k^2+m_\rho{}^2)}+\frac{m_\omega{}^4}{6(k^2+m_\omega{}^2)}+\frac{m_\phi{}^4}{3(k^2+m_\phi{}^2)}\right]\ , \tag{50b}$$

where we have used

$$g_{\rho K\bar{K}}=\frac{m_\rho{}^2}{2gf_K{}^2},\quad g_{\omega K\bar{K}}=\frac{m_\omega{}^2}{2gf_K{}^2}\ ,$$
$$g_{\phi K\bar{K}}=-\frac{m_\phi{}^2}{2gf_K{}^2}\ , \tag{51}$$

as given by the Lagrangian (42) and the first of relations (48) and $m_\rho{}^2/2g^2f_\pi{}^2=1$. Equations (50) give the correct normalization and the charge mean-square radii

$$\langle r_{K^+}{}^2\rangle=\frac{f_\pi{}^2}{f_K{}^2}\frac{6}{m_\rho{}^2}=\frac{f_\pi{}^2}{f_K{}^2}\langle r_{\pi^+}{}^2\rangle\ ,$$
$$\langle r_{K^0}{}^2\rangle=0\ . \tag{52}$$

The numerical predictions for various values of f_K/f_π and the comparison with experimental values are given in Table I.

We now discuss our results for the decays $\rho\to\pi\pi$, $K^*\to K\pi$, and $\phi\to K\bar{K}$. If we write $\Gamma(\rho\to\pi\pi)=\Gamma$, we obtain in SU(3) the limit $\Gamma(K^*\to K\pi)=\frac{3}{4}\Gamma$,

$$\Gamma(\phi\to K^+K^-)=\tfrac{1}{2}\Gamma=\Gamma(\phi\to K^0\bar{K}^0)\ .$$

Using the results of Eq. (47), we obtain

$$\Gamma(K^*\to K\pi)=\frac{m_{K^*}{}^2}{m_\rho{}^2}\frac{f_\pi{}^2}{f_K{}^2}\frac{p_{K^*}{}^3}{p_\rho{}^3}(\tfrac{3}{4}\Gamma)\ ,$$
$$\Gamma(\phi\to K^+K^-)=\frac{m_\phi{}^2}{m_\rho{}^2}\frac{f_\pi{}^4}{f_K{}^4}\frac{p_\phi{}^3}{p_\rho{}^3}(\tfrac{1}{2}\Gamma)$$
$$\approx 1.55\Gamma(\phi\to K^0\bar{K}^0)\ . \tag{53}$$

The predictions of Eqs. (53) and the comparison of these results with experimental values are given in Table I. In Eqs. (53) we use the experimental value $\Gamma=153$ MeV for comparison with experimental results.

It is clear from Table I that the KSRF value for $g_{\rho\pi\pi}$ is in good agreement with the experimental value for $\rho\to\pi\pi$ decay, as is well known. It is also clear from this table that SU(3)-symmetry-breaking effects as exemplified by $f_K/f_\pi\neq 1$ (and >1) are definitely there. In fact, for $f_K/f_\pi\approx 1.3$, as also indicated by weak de-

TABLE I. Decay widths for $\rho\to\pi\pi$, $K^*\to K\pi$, and $\phi\to K\bar{K}$, as given by Eqs. (44) with $m_\rho^2/2g^2f_\pi^2=1$ and charge mean-square radii for π^+ and K^+ [Eq. (52)] in the mass-mixing model for various values of f_K/f_π, and their comparison with the experimental values.

Decay	Decay width (MeV) (Theory)			Decay width (MeV) (Experimental) (Ref. 16)
$\rho\to\pi\pi$		141		153 ± 2
	$f_K/f_\pi=1.0$	1.2	1.3	
$K^*\to K\pi$	80.0	55.6	47.4	51.01 ± 0.8
$\phi\to K^+K^-$	8.0	3.9	2.8	2.09 ± 0.13
Charge mean-square radii	Theory (fm^2)			Experiment (Ref. 17) (fm^2)
$\langle r_\pi^2\rangle$		0.39		(i) 0.46 ± 0.01
				(ii) 0.31 ± 0.04
	$f_K/f_\pi=1.0$	1.2	1.3	
$\langle r_{K^+}^2\rangle$	0.30	0.27	0.23	0.28 ± 0.05

cays, an agreement for $K^*\to K\pi$ width and $\langle r_{K^+}^2\rangle$ with corresponding experimental values is obtained within a few percent and that for $\phi\to K\bar{K}$ width within 25%.

C. Current-mixing model of SU(3)-symmetry breaking

We now consider another case in which SU(3) breaking is in the V-dependent kinetic-energy term of vector bosons. This case we call the current mixing model.[12] Taking this into account, we obtain for the vector-bosons kinetic-energy term

$$-\tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})-\tfrac{1}{4}\lambda(V^{\alpha}_{\mu\nu3}V^{3}_{\mu\nu\alpha}+V^{3}_{\mu\nu\alpha}V^{\alpha}_{\mu\nu3})\ . \quad (54)$$

To simplify the calculation, we will again use the nonet mixing between ω_8 and ω_1 as given by Eq. (39). Using Eq. (54), we have, for the kinetic-energy and mass terms of vector bosons,

$$-\tfrac{1}{4}(1+\lambda)\,\mathrm{Tr}(V_{u\nu}V_{\mu\nu})-\tfrac{1}{2}m_0^2\,\mathrm{Tr}[(V_\mu-\mathcal{V}_\mu)^2]\ , \quad (55)$$

where $\lambda=0$ for ρ and ω mesons, $\lambda=\epsilon$ for K^* mesons, and $\lambda=2\epsilon$ for ϕ mesons. Redefining the vector field

$$V^r_\mu=\sqrt{1+\lambda}\,V_\mu\ , \quad (56)$$

we obtain, from Eq. (55),

$$-\tfrac{1}{4}\mathrm{Tr}(V^r_{\mu\nu}V^r_{\mu\nu})-\tfrac{1}{4}m_0^2\,\mathrm{Tr}\left[\left[\frac{V^r_\mu}{\sqrt{1+\lambda}}-\mathcal{V}_\mu\right]^2\right]\ . \quad (57)$$

Dropping r, we obtain the results

$$m_\rho^2=m_\omega^2=m_0^2,\quad m_{K^*}^2=m_0^2/(1+\epsilon)\ ,$$
$$m_\phi^2=m_0^2/(1+2\epsilon)\ . \quad (58)$$

Equation (58) gives the mass formula

$$\frac{1}{m_\phi^2}-\frac{1}{m_\omega^2}=2\left[\frac{1}{m_{K^*}^2}-\frac{1}{m_\rho^2}\right]\ . \quad (59)$$

Using Eqs. (57), (59), and (16), we obtain the Lagrangian (42), but with

$$g_{K^*K\pi}=\frac{m_\rho m_{K^*}}{2gf_\pi f_K},\quad g_{\phi K\bar{K}}=-\frac{m_\rho m_\phi}{2gf_K^2}\ . \quad (60)$$

Introducing electromagnetism in the usual way, we have, using Eqs. (57), (58), and (60), the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu})-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\,\partial_\mu M)+\mathcal{L}'_{VPP}-\tfrac{1}{2}m_\rho^2\rho_\mu^2-\tfrac{1}{2}m_\omega^2\omega_\mu^2-\tfrac{1}{2}m_\phi^2\phi_\mu^2$$
$$+\frac{e}{g}\left[m_\rho^2\rho^0_\mu+\tfrac{1}{3}m_\omega^2\omega_\mu-\frac{\sqrt{2}}{3}m_\rho m_\phi\phi_\mu\right]\mathcal{A}_\mu-\frac{1}{2}\frac{e^2}{g^2}\frac{4}{3}m_\rho^2\mathcal{A}_\mu^2+ie\left[\left[\frac{m_\rho^2}{2g^2f_K^2}-1\right](K^-\partial_\mu K^+-K^+\partial_\mu K^-)\right]\mathcal{A}_\mu\ , \quad (61)$$

where $\mathcal{L}'_{VPP}$ is given in Eq. (42), with $K^*K\pi$ and $\phi K\bar{K}$ couplings given in Eq. (60). Thus, for the current-mixing model, we obtain the results

$$g_\rho/m_\rho^2=3g_\omega/m_\omega m_\rho=-3g_\phi/\sqrt{2}\,m_\rho m_\phi=1/g\ , \quad (62)$$

instead of Eqs. (48).

With the presence of an additional term

$$\left[\frac{m_\rho^2}{2g^2f_K^2}-1\right]=\left[\frac{f_\pi^2}{f_K^2}-1\right]$$

in the $K^{\pm}$ current, the electromagnetic form factor of the K^+ meson with vector meson dominance in this case is given by

$$F_{K^+}(k^2)=1-\frac{f_\pi{}^2}{f_K{}^2}+\frac{f_\pi{}^2}{f_K{}^2 m_\rho{}^2}\left[\frac{m_\rho{}^4}{2(k^2+m_\rho{}^2)}+\frac{m_\rho{}^2 m_\omega{}^2}{6(k^2+m_\omega{}^2)}+\frac{m_\rho{}^2 m_\phi{}^2}{3(k^2+m_\phi{}^2)}\right], \quad (63)$$

where we have used

$$g_{\rho K\bar{K}}=m_\rho{}^2/2gf_K{}^2,\quad g_{\omega K\bar{K}}=m_\omega m_\rho/2gf_K{}^2,\quad g_{\phi K\bar{K}}=-m_\rho m_\phi/2gf_K{}^2\ , \quad (64)$$

and (62) and $(m_\rho{}^2/2g^2f_\pi{}^2)=1$. Equation (63) gives

$$F_{K^+}(0)=1,\quad \langle r_{K^+}{}^2\rangle=\frac{f_\pi{}^2}{f_K{}^2}\left[1-\frac{1}{3}\left[1-\frac{m_\rho{}^2}{m_\phi{}^2}\right]\right]\langle r_{\pi^+}{}^2\rangle,\quad \langle r_{\pi^+}{}^2\rangle=6/m_\rho{}^2\ . \quad (65)$$

For the decays $K^*\to K\pi$ and $\phi\to K^+K^-$ and $\langle r_{K^+}{}^2\rangle$ the results are given in Table II for various values of f_K/f_π. A good agreement with the corresponding experimental values is obtained for $f_K/f_\pi\approx 1.1$, somewhat smaller than the value needed for mass-mixing model and that obtained from weak decays ($f_K/f_\pi\approx 1.25$).

IV. GAUGED CHIRAL LAGRANGIAN

In this section, we consider the gauged[2,3] σ model. The Lagrangian invariant under local chiral $SU(2)_L\times SU(2)_R$ group (except for the vector-boson mass term) is given by

$$\mathcal{L}=-\bar{\psi}_L\gamma_\mu\left[\partial_\mu-\frac{1}{\sqrt{2}}igV_{L\mu}\right]\psi_L-\bar{\psi}_R\gamma_\mu\left[\partial_\mu-\frac{1}{\sqrt{2}}igV_{R\mu}\right]\psi_R-\sqrt{2}f(\bar{\psi}_L\Sigma\psi_R+\bar{\psi}_R\Sigma\psi_L)$$
$$-\tfrac{1}{2}\mathrm{Tr}D_\mu\Sigma^\dagger D_\mu\Sigma-\tfrac{1}{4}m_0{}^2\mathrm{Tr}(V_{L\mu}{}^2+V_{R\mu}{}^2)-\tfrac{1}{8}\mathrm{Tr}(V_{L\mu\nu}V_{L\mu\nu}+V_{R\mu\nu}V_{R\mu\nu})\ , \quad (66)$$

where

$$D_\mu\Sigma=\partial_\mu\Sigma-\frac{i}{\sqrt{2}}gV_{L\mu}\Sigma+\frac{i}{\sqrt{2}}g\Sigma V_{R\mu}\ , \quad (67)$$

$$V_{L\mu}=V_\mu+A_\mu,\quad V_{R\mu}=V_\mu-A_\mu,\quad V_{L\mu\nu}V_{L\mu\nu}+V_{R\mu\nu}V_{R\mu\nu}=2(V_{\mu\nu}V_{\mu\nu}+A_{\mu\nu}A_{\mu\nu})\ , \quad (68)$$

V_μ and A_μ are triplets of vector and axial-vector mesons.

We nonlinearize the Lagrangian (66) by means of field-dependent gauge transformations given in Eqs. (5a) and (5b) [with $\frac{1}{2}f_\pi$ replaced by F in Eq. (5b)]. $V_{L\mu}$ and $V_{R\mu}$ transform as follows under these gauge transformations:[2]

$$V_{L\mu}=\Omega_L\hat{V}_{L\mu}\Omega_L^\dagger+\frac{\sqrt{2}i}{g}\Omega_L\,\partial_\mu\Omega_L^\dagger\ ,$$
$$V_{R\mu}=\Omega_R\hat{V}_{R\mu}\Omega_R^\dagger+\frac{\sqrt{2}i}{g}\Omega_R\,\partial_\mu\Omega_R^\dagger\ , \quad (69)$$
$$V_{L\mu\nu}=\Omega_L\hat{V}_{L\mu\nu}\Omega_L^\dagger,\quad V_{R\mu\nu}=\Omega_R\hat{V}_{R\mu\nu}\Omega_R^\dagger\ .$$

We note that, under the above transformation,

$$D_\mu\Sigma=-F\frac{i}{\sqrt{2}}g\Omega_L(\hat{V}_{L\mu}-\hat{V}_{R\mu})\Omega_R^\dagger\ . \quad (70)$$

Hence, we have the Lagrangian, using the gauge given in Eq. (15) (removing the caret),

TABLE II. Decay width for $K^*\to K\pi$ and $\phi\to K\bar{K}$ [Eqs. (60) with $m_\rho{}^2/2g^2f_\pi{}^2=1$] and $\langle r_{K^+}{}^2\rangle$ [Eq. (65)] in the current-mixing model for various values of f_K/f_π.

f_K/f_π	1.0	1.1	1.2
$\Gamma(K^*\to K\pi)$ (MeV)	59.7	49.3	41.4
$\Gamma(\phi\to K^+K^-)$ (MeV)	3.44	2.53	1.70
$\langle r_{K^+}{}^2\rangle$ (fm)	0.33	0.28	0.23

$$\mathcal{L}=-\bar{\psi}\gamma_\mu\left[\partial_\mu-i\frac{g}{\sqrt{2}}(V_\mu+\gamma_5 A_\mu)\right]\psi-\sqrt{2}fF\bar{\psi}\psi-\tfrac{1}{2}(2F^2g^2)\,\mathrm{Tr}(A_\mu{}^2)$$
$$-\tfrac{1}{2}m_0{}^2\left[V_\mu{}^2+A_\mu{}^2-\frac{\sqrt{2}i}{g}(V_\mu\mathcal{V}_\mu+A_\mu b_\mu)+\frac{2}{g^2}(\partial_\mu\Omega\,\partial_\mu\Omega^\dagger)\right]-\tfrac{1}{4}\,\mathrm{Tr}(V_{\mu\nu}V_{\mu\nu}+A_{\mu\nu}A_{\mu\nu})\;. \tag{71}$$

We note that axial-vector mesons acquire mass partially due to the Higgs mechanism. If the explicit gauge-symmetry-breaking term, viz., the term with $m_0{}^2$ is absent, the π meson is completely eliminated by the Higgs mechanism. Pion fields reappear due to the presence of the explicit gauge symmetry-breaking term.

We now use the Lagrangian (71) for low-energy phenomenonology. Using Eqs. (16), we obtain

$$\mathcal{L}=-\bar{\psi}\gamma_\mu\left[\partial_\mu-i\frac{g}{2}\boldsymbol{\tau}\cdot(\mathbf{V}_\mu+\gamma_5\mathbf{A}_\mu)\right]\psi-\sqrt{2}fF\bar{\psi}\psi-\tfrac{1}{2}(2F^2g^2+m_0{}^2)\,\mathbf{A}_\mu{}^2$$
$$-\tfrac{1}{2}m_0{}^2\mathbf{V}_\mu{}^2-\tfrac{1}{2}m_0{}^2\frac{1}{gf_\pi{}^2}\mathbf{V}_\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})-\tfrac{1}{2}m_0{}^2\frac{2}{gf_\pi}\mathbf{A}_\mu\cdot\partial_\mu\boldsymbol{\pi}-\tfrac{1}{2}m_0{}^2\frac{1}{g^2f_\pi{}^2}\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}-\tfrac{1}{4}(\mathbf{V}_{\mu\nu}\cdot\mathbf{V}_{\mu\nu}+\mathbf{A}_{\mu\nu}\cdot\mathbf{A}_{\mu\nu})\;. \tag{72}$$

In order to eliminate the $\mathbf{A}_\mu\cdot\partial_\mu\boldsymbol{\pi}$ term, we define new fields

$$\mathbf{A}_\mu=\mathbf{A}_\mu^r+bz^{1/2}\partial_\mu\boldsymbol{\pi}^r,\quad \boldsymbol{\pi}=z^{1/2}\boldsymbol{\pi}^r,\quad f_\pi=z^{1/2}f_{\pi^r}\;. \tag{73}$$

We have two conditions from the requirements that the coefficient of the $\mathbf{A}_\mu^r\cdot\partial_\mu\boldsymbol{\pi}^r$ term should be zero and that the coefficient of the $\partial_\mu\boldsymbol{\pi}^r\cdot\partial_\mu\boldsymbol{\pi}^r$ term should be $-\frac{1}{2}$. These conditions give (identifying V_μ with ρ_μ and $m_0=m_\rho$, $\mathbf{A}_\mu^r$ with the axial-vector bosons A_1, and $\boldsymbol{\pi}^r$ with pion fields, and then dropping r)

$$bz^{1/2}=-m_\rho{}^2/gf_\pi m_A{}^2,\quad (m_A{}^2-m_\rho{}^2)=(g^2f_\pi{}^2/m_\rho{}^2)m_A{}^2\;. \tag{74}$$

From Eqs. (72)–(74), we obtain our final Lagrangian:

$$\mathcal{L}=-\bar{\psi}\gamma_\mu\left[\partial_\mu-i\frac{g}{2}\boldsymbol{\tau}\cdot(\boldsymbol{\rho}_\mu+\gamma_5\mathbf{A}_\mu)+\frac{i}{2f_\pi}(1-g^2f_\pi{}^2/m_\rho{}^2)\gamma_5\boldsymbol{\tau}\cdot\gamma_\mu\boldsymbol{\pi}\right]\psi$$
$$-\sqrt{2}f[(m_A{}^2-m_\rho{}^2)/2g^2]\bar{\psi}\psi-\tfrac{1}{2}m_A{}^2\,\mathbf{A}_\mu{}^2-\tfrac{1}{2}m_\rho{}^2\boldsymbol{\rho}_\mu{}^2-\tfrac{1}{2}(m_\rho{}^2/gf_\pi{}^2)\boldsymbol{\rho}_\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})$$
$$-\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}-\tfrac{1}{4}[\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}_{\mu\nu}-\mathbf{A}_{\mu\nu}\cdot\mathbf{A}_{\mu\nu}]\;, \tag{75}$$

where

$$\boldsymbol{\rho}_{\mu\nu}=\partial_\nu\boldsymbol{\rho}_\mu-\partial_\mu\boldsymbol{\rho}_\nu-g\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu-g(\mathbf{A}_\mu+bz^{1/2}\partial_\mu\boldsymbol{\pi})\times(\mathbf{A}_\nu+bz^{1/2}\partial_\nu\boldsymbol{\pi})\;,$$
$$\mathbf{A}_{\mu\nu}=\partial_\nu\mathbf{A}_\mu-\partial_\mu\mathbf{A}_\nu-g\boldsymbol{\rho}_\mu\times(\mathbf{A}_\nu+bz^{1/2}\partial_\nu\boldsymbol{\pi})+g\boldsymbol{\rho}_\nu\times(\mathbf{A}_\mu+bz^{1/2}\partial_\mu\boldsymbol{\pi})\;. \tag{76}$$

From Eqs. (75), (76), and (79), we obtain

$$g_{\rho\pi\pi}(p^2)=\frac{m_\rho{}^2}{2gf_\pi{}^2}\left[1+\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right]\frac{p^2}{m_\rho{}^2}\right]\;, \tag{77}$$

$$f_{A\rho\pi}(p^2,k^2)=-\frac{1}{f\pi}\left[\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right](p^2-k^2)\right]\Big/(m_A{}^2-m_\rho{}^2),\quad g_{A\rho\pi}=0\;, \tag{78}$$

where we have defined $A_1\rho\pi$ couplings as

$$[(m_A{}^2-m_\rho{}^2)f_{A\rho\pi}\eta\cdot\epsilon+2(k\cdot\eta)(p\cdot\epsilon)g_{A\rho\pi}]\;. \tag{79}$$

η_μ and ϵ_μ are polarization vectors of A_1 and ρ, respectively, and p and k are their four-momenta.

From Eqs. (77) and (78), we obtain

$$g_{\rho\pi\pi}(0)=m_\rho{}^2/2gf_\pi{}^2\;,$$
$$g_{\rho\pi\pi}(m_\rho{}^2)\equiv g_{\rho\pi\pi}=g\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right]\;, \tag{80}$$
$$f_{A\rho\pi}(m_A{}^2,m_\rho{}^2)\equiv f_{A\rho\pi}=\frac{1}{f_\pi}\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right]\;.$$

If we set

$$g=\frac{m_\rho}{\sqrt{2}f_\pi}(1-\delta), \quad -1<\delta<1 , \tag{81}$$

we obtain

$$g_{\rho\pi\pi}\approx\frac{m_\rho}{\sqrt{2}f_\pi}\left[\frac{3-\delta}{4}\right](1-\delta^2) ,$$
$$m_A{}^2/m_\rho{}^2\approx 2/(1+2\delta-\delta^2) , \tag{82}$$
$$f_{A\rho\pi}\approx\frac{1}{2f_\pi}(1+2\delta-\delta^2) .$$

First we note that the value $\delta=0$ is of special interest since it gives the gauge coupling constant $g=m_\rho/\sqrt{2}f_\pi$ and the Weinberg relation $m_A=\sqrt{2}m_\rho$. This is the value that also gives $g_{\rho\pi\pi}=m_\rho/\sqrt{2}f_\pi$ if one ignores the momentum-dependent term in Eq. (77). However, if we take into account the momentum dependence of $g_{\rho\pi\pi}$, then, on the mass shell, $g_{\rho\pi\pi}=\frac{3}{4}m_\rho/\sqrt{2}f_\pi$ for $\delta=0$. In fact, as is clear from Eq. (82), $\delta=0$ gives the results obtained by Schwinger.[13] The value of $g_{\rho\pi\pi}=m_\rho/\sqrt{2}f_\pi$ is in good agreement with experimental result, but Eq. (82) shows that this value is not attainable for any value of δ. In fact, the maximum value of $g_{\rho\pi\pi}$ is $(0.79)m_\rho/\sqrt{2}f_\pi$, which one gets for $\delta=-0.15$. It is interesting to note that for this value one obtains $m_A=1.3$ GeV to be compared with the experimental value of 1.27 GeV.

In our model $A_1\rho\pi$ decay is a pure S wave. The D-wave component of $A_1\rho\pi$ decay is zero. This is the natural value that one gets in the spirit of minimal gauge-coupling of vector and axial-vector bosons to hadrons. However, one can generate D-wave coupling for $A_1\rho\pi$ decay by introducing nonminimal coupling involving an arbitrary parameter.[3,4]

A simple way is to add a gauge invariant term

$$\mathcal{L}_D=-\tfrac{1}{4}C\,\mathrm{Tr}(D_\mu\Sigma D_\nu\Sigma^\dagger V_{L\mu\nu}+D_\mu\Sigma^\dagger D_\nu\Sigma V_{R\mu\nu})$$

to the Lagrangian (66). Using Eqs. (69) and (70) and nonlinearizing it, we obtain

$$\mathcal{L}_D=\tfrac{1}{2}\beta g\mathbf{V}_{\mu\nu}\cdot(\mathbf{A}_\mu\times\mathbf{A}_\nu) ,$$

where we have set $CF^2g/\sqrt{2}=-\frac{1}{2}\beta$. Finally, using Eqs. (73), we obtain

$$\mathcal{L}_D=\tfrac{1}{2}\beta g\rho_{\mu\nu}\cdot[(\mathbf{A}_\mu+bz^{1/2}\partial_\mu\boldsymbol{\pi})\times(\mathbf{A}_\nu+bz^{1/2}\partial_\nu\boldsymbol{\pi})] .$$

Taking this term into account, we obtain, instead of Eqs. (80),

$$g_{\rho\pi\pi}=g\left[\left[1-\frac{g^2f_\pi{}^2}{2m_\rho{}^2}\right]-\beta\frac{m_\rho{}^2}{2g^2f_\pi{}^2}\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right]^2\right] ,$$
$$f_{A\rho\pi}=\frac{1}{2f_\pi}\left[1-\frac{g^2f_\pi{}^2}{m_\rho{}^2}\right](2+\beta) , \tag{80'}$$
$$g_{A\rho\pi}=\frac{1}{2f_\pi}\left[1-\frac{g^2f_\pi{}^2}{m_0{}^2}\right]\beta .$$

As we have argued above, δ is expected to be small. Neglecting terms of order δ^2, we obtain

$$m_A{}^2=2m_\rho{}^2(1-2\delta) ,$$
$$g_{\rho\pi\pi}=\frac{m_\rho}{\sqrt{2}f_\pi}[(1-\gamma)+\tfrac{1}{2}\delta(1-6\gamma)] ,$$
$$f_{A\rho\pi}=\frac{1}{4f_\pi}(1+4\gamma)(1+2\delta) , \tag{82'}$$
$$g_{A\rho\pi}=-\frac{1}{4f_\pi}(1-4\gamma)(1+2\delta) ,$$

where we have set $\beta=-(1-4\gamma)$. With $\delta=0$, Eqs. (82′) give precisely the same results as given in the current algebra approach.[14]

We now consider the case in which A_1 meson fields are eliminated in the nonlinear version of the Lagrangian.[3,4] This is an attractive idea as A_1 mesons, being p-wave bound states in the quark model, should be transformed away by gauge transformation similar to the elimination of the σ field. This has been successfully done in Ref. 4. In our case, the procedure of Ref. 4 is equivalent to adding a term

$$-\tfrac{1}{2}R^2\,\mathrm{Tr}(\Sigma^\dagger V_{L\mu}\Sigma V_{R\mu}) \tag{83}$$

in the Lagrangian (66). Like the mass term, this term violates gauge symmetry but respects global chiral symmetry. Instead of transformations given in Eqs. (69), we consider the transformations

$$V_{L\mu}=\Omega\hat{V}_\mu\Omega^\dagger+\frac{\sqrt{2}i}{g}\Omega\,\partial_\mu\Omega^\dagger ,$$
$$V_{R\mu}=\Omega^\dagger\hat{V}_\mu\Omega+\frac{\sqrt{2}i}{g}\Omega^\dagger\partial_\mu\Omega , \tag{84}$$

so that $\hat{A}_\mu=0$. The transformations (84) are possible subject to the following constraint

$$V_{L\mu}=UV_{R\mu}U^\dagger+\frac{\sqrt{2}i}{g}U\,\partial_\mu U^\dagger . \tag{85}$$

Then following the procedure similar to that in this section (dropping the caret and setting $V_\mu=\rho_\mu$), we set the Lagrangian

$$\mathcal{L}=-\bar{\psi}\gamma_\mu\left[\partial_\mu-i\frac{g}{\sqrt{2}}\rho_\mu\right]\psi-\sqrt{2}fF\bar{\psi}\psi-\tfrac{1}{4}\mathrm{Tr}(\rho_{\mu\nu}\rho_{\mu\nu})$$
$$-\tfrac{1}{2}(m_0{}^2+R^2F^2)\,\mathrm{Tr}\left[\rho_\mu{}^2-\frac{i}{\sqrt{2}gf_\pi{}^2}\rho_\mu\times[\pi,\partial_\mu\pi]\right]$$
$$-\tfrac{1}{2}(m_0{}^2-R^2F^2)\frac{1}{g^2f_\pi{}^2}\mathrm{Tr}(\partial_\mu\pi\,\partial_\mu\pi) . \tag{86}$$

From Eq. (86), we obtain

$$m_\rho{}^2=m_0{}^2+R^2F^2 , \tag{87}$$

and the normalization of the pion kinetic-energy term gives

$$\frac{m_0{}^2-R^2F^2}{g^2f_\pi{}^2}=1 . \tag{88}$$

Hence, the Lagrangian (86) together with Eqs. (87) and (88) gives

$$g_{\rho\pi\pi}=\frac{m_\rho}{2f_\pi}\left[1-\frac{2R^2F^2}{m_\rho^2}\right]^{-1/2} . \quad (89)$$

If we set $R=0$, then we obtain[3]

$$g_{\rho\pi\pi}=\frac{m_\rho}{2f_\pi} ,$$

in complete disagreement with experimental result. We obtain the KSRF value for $g_{\rho\pi\pi}$ if

$$R^2F^2=\tfrac{1}{4}m_\rho^2 . \quad (90)$$

For this value we also obtain, from Eqs. (87) and (88),

$$g=m_\rho/\sqrt{2}f_\pi=g_{\rho\pi\pi} . \quad (91)$$

The result (91) agrees with the results obtained in Sec. II. As is clear from Eq. (86), pions have no coupling to quarks. This coupling has to be introduced by hand, which is a major defect of this approach.

V. SUMMARY AND DISCUSSION

By extending Weinberg's formulation of the chiral Lagrangian (in which ρ mesons are gauge bosons of hidden local symmetry) so as to include electromagnetic interaction, we have obtained the result $g_\rho\equiv m_\rho^2/g=2g_{\rho\pi\pi}f_\pi^2$. By assuming that the pion form factor is completely dominated by the ρ meson, we obtain the result $g_{\rho\pi\pi}=g=m_\rho/\sqrt{2}f_\pi$, i.e., both the universality and the KSRF relation.

By extending the same approach to broken SU(3), we obtain the following results.

Case (i) mass-mixing model:

$$g_{K^*K\pi}/g_{\rho\pi\pi}=(m_{K^*}^2/m_\rho^2)(f_\pi/f_K) ,$$

$$g_{\phi K\bar{K}}/g_{\rho\pi\pi}=-(m_\phi^2/m_\rho^2)(f_\pi^2/f_K^2) ,$$

$$g_\rho/m_\rho^2=3g_\omega/m_\omega^2 =-3g_\phi/\sqrt{2}m_\phi^2=1/g=\sqrt{2}f_\pi/m_\rho ,$$

$$\langle r_{K^+}^2\rangle=(f_\pi^2/f_K^2)\langle r_{\pi^+}^2\rangle ,$$

$$\langle r_{\pi^+}^2\rangle=6/m_\rho^2 .$$

The third equation has been derived in Ref. 6. These results give reasonable values for the decay widths of $K^*\to K\pi$ and $\phi K\bar{K}$ and for $\langle r_{K^+}^2\rangle$, provided that $f_K/f_\pi\approx1.2-1.3$, consistent with the value one obtains from weak decays. Moreover, we obtain the results

$$\frac{1}{m_\rho}\Gamma(\rho\to e^+e^-)=\frac{9}{m_\omega}\Gamma(\omega\to e^+e^-) =\frac{9}{2m_\phi}\Gamma(\phi\to e^+e^-) =(4\pi\alpha^2/3)\frac{1}{g^2} , \quad (92)$$

so that we have the sum rule

$$\frac{1}{3m_\rho}\Gamma(\rho\to e^+e^-)=\frac{1}{m_\omega}\Gamma(\omega\to e^+e^-) +\frac{1}{m_\phi}\Gamma(\phi\to e^+e^-) . \quad (93)$$

The results obtained from Eqs. (92) and their comparison with experimental results are given in Table III.

Case (ii) current-mixing model:

$$g_{K^*K\pi}/g_{\rho\pi\pi}=(m_{K^*}/m_\rho)(f_\pi/f_K) ,$$

$$g_{\rho K\bar{K}}/g_{\rho\pi\pi}=-(m_\phi/m_\rho)(f_\pi^2/f_K^2) ,$$

$$g_\rho/m_\rho^2=3g_\omega/m_\rho m_\omega=-\sqrt{3}g_\phi/2m_\phi m_\rho =1/g=\sqrt{2}f_\pi/m_\rho ,$$

$$\langle r_{K^+}^2\rangle=(f_\pi^2/f_K^2)[1-\tfrac{1}{3}(1-m_\rho^2/m_\phi^2)]\langle r_{\pi^+}^2\rangle .$$

These results give good agreement between the theoretical and experimental values for the decay widths of $K^*\to K\pi$ and $\phi\to K\bar{K}$ and for $\langle r_{K^+}^2\rangle$ for $f_K/f_\pi\approx1.1$. In this case we get the following results for the leptonic widths:

$$m_\rho\Gamma(\rho\to e^+e^-)=9m_\omega\Gamma(\omega\to e^+e^-) =\tfrac{9}{2}m_\phi\Gamma(\phi\to e^+e^-) =(4\pi\alpha^2/3)(m_\rho^2/g^2) , \quad (94)$$

which give the sum rule[15]

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to e^+e^-)=m_\omega\Gamma(\omega\to e^+e^-)+m_\phi\Gamma(\phi\to e^+e^-). \quad (95)$$

TABLE III. Leptonic widths for mass-mixing and current-mixing models and their comparison with the experimental values.

Decay	Theoretical width (keV)		Experimental width (Ref. 16) (keV)
$\rho\to e^+e^-$	5.0		(6.9±0.4)
	Mass mixing	Current mixing	
$\omega\to e^+e^-$	0.61	0.59	0.66±0.05
$\phi\to e^+e^-$	(i) 1.48	(i) 0.84	1.31±0.06
	(ii) 2.04 with $\Gamma(\rho\to e^+e^-)=6.9$ keV as input	(ii) 1.16	

The results obtained from Eqs. (94) are summarized in Table III.

Both of the sum rules (93) and (94) have been derived here using nonet symmetry, but the sum rule (94) is in better agreement with the experimental data. (For other evidence in support of current-mixing model, see Ref. 10.)

It may be noted that the chiral scale is $\Lambda \sim 1$ GeV. The mass of the ϕ meson, which is the heaviest member of the vector boson nonet, is 1.02 GeV—not much higher than 1 GeV. If nonet symmetry, which is experimentally well satisfied, is to make sense, then the ϕ meson has to be treated along with the other members of the nonet. It is therefore justified to consider the processes $\phi \rightarrow K\bar{K}$ in the effective-Lagrangian approach followed in this paper.

We conclude that our Lagrangian gives good results for $K^+ \rightarrow K\pi$, $\phi \rightarrow K\bar{K}$ decays, and $\langle r_{K^\pm}{}^2 \rangle$ and also for leptonic widths of ρ, ω, and ϕ. At present it seems difficult to distinguish between the two models of SU(3)-symmetry breaking considered.

However, the approach of Sec. IV, with the assumption $g = m_\rho/\sqrt{2} f_\pi$, gives precisely the same results as one obtains in the current algebra.

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