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Statistical and dynamical aspects of hadronic clusters in high-energy collisions: Statistical methods

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Statistical aspects of production and decay of hadronic clusters in high-energy collisions are discussed. It is shown in particular that, by applying statistical methods, multiplicity distributions and multiplicity correlations of different kinds of hadrons observed in different rapidity windows can be used to obtain useful information on the properties of such clusters.

I. INTRODUCTION

There has been a persistent interest in studying hadronic cluster production in high-energy collisions since the early 1970s;[1] and it seems that there even has been an increase in interest recently.[2] There are a number of reasons why the hadronic clusters have attracted, and still are attracting, so much attention. Some of them are listed below. In fact, these are the ones which made us decide that a contribution in this field would be worthwhile and helpful.

(A) Short(-rapidity)-range correlations between the observed hadrons have been observed in multiparticle production processes in different energy ranges, and it has been seen[3] that the main properties of such correlations are independent of the energy. One simple, and indeed very attractive way of describing this fact[3] is to assume[1,2] that clusters are produced and subsequently decay into final-state hadrons. But, in spite of the long history of the cluster notion, not very much about its nature is known.

(B) Because of the difficulties associated with the measurements and the data analyses of three- and four-particle short-range correlations,[4] it seems difficult to find out experimentally whether clusters consisting of more than two charged hadrons exist, and if so, how good is the chance of observing them (compared, for example, to the chance of observing a two-charged-hadron cluster).

(C) Most of the properties of the clusters (for example, size, charge, distribution in rapidity space, etc.) are assumptions which are not directly[5] associated with measurable quantities. Perhaps this is also the reason why there has been little discussion on problems such as *why* such hadronic clusters exist in high-energy collisions and *why* they have the assumed properties which fit the correlation and multiplicity distribution data.

(D) Based on the analysis of their electron-positron annihilation experiment, Derrick *et al.*[6] have recently raised the question of whether the usual cluster picture is valid for e^+e^- annihilation processes. This implies that the clusters produced in different reactions may be different in nature. Can this question be checked experimentally and theoretically with other methods?

In order to answer the questions mentioned above, we think it would be helpful to establish more direct links between the assumed characteristics of the clusters and the experimentally measurable quantities. Such an attempt has been made and will be reported in this and in a subsequent paper. In this paper we propose a method with the help of which the characteristic properties of hadronic clusters can be studied in a systematic manner. Here, standard statistical concepts and methods are extensively used. This is because a systematic analysis[7] of the multiplicity and (low-E_T jet) transverse-energy distribution data[6,8–12] has led us to the conclusion[7] that such concepts and methods are useful in describing the data, and that in fact a considerable part of the observed phenomena in high-energy multiparticle production processes are nothing but pure statistical effects. We show in this part that, by applying statistical methods, useful information on the intrinsic properties (such as the size and the charge) as well as the (multiplicity and rapidity) distributions of such clusters can be obtained from multiplicity distributions and multiplicity correlations of different kinds of hadrons in different rapidity intervals. We make every effort to avoid theoretical bias when we carry out the statistical analysis in this part of our work. [For example, the ansatz for $\alpha_{W\max}(l)$, used in Sec. II to solve the equation system for $\alpha_W(l)$ and $P(N)$, is motivated by experimental facts. Its final value is found by trial and error in the successive approximation mentioned there.] Possible dynamical pictures will be discussed in a subsequent paper.

II. STATISTICAL ANALYSIS OF MULTIPLICITY DISTRIBUTIONS

Multiplicity distributions of charged hadrons in limited rapidity intervals (windows) have been measured by the UA5 Collaboration[8] in connection with critical tests of the Koba-Nielsen-Olesen (KNO) scaling at the present CERN $\bar{p}p$ collider energies ($\sqrt{s}$ = 200–900 GeV). Similar measurements have also been made for e^+e^- annihilation processes at $\sqrt{s}=29$ GeV by the High Resolution Spectrometer (HRS) group[6] at the SLAC storage ring PEP, for pp and πp reactions at $\sqrt{s}=22$ GeV by the NA22 Collaboration,[9] for μ^+p reactions for (total hadron final-state) energies up to 20 GeV by the European Muon Collaboration[10] (EMC), and for hadron-nucleus reactions at $p_{\text{lab}}=200$ GeV by Dengler *et al.*[11]

Inspired by the striking results obtained in the above-mentioned experiments[6,8–12] and the corresponding theoretical activities,[13,14] a systematic analysis[7] of these data was made which has led us to the following conclusions.

(i) Standard statistical methods, in particular, the binomial distribution law, can be used to describe the observed rapidity dependence of multiplicity in hadron-hadron,[8,9] electron-positron,[6] lepton-hadron,[10] as well as in hadron-nucleus[11] processes.

(ii) The concept of hadronic clusters is very useful. In fact, it turns out that simple and naive assumptions (such as that there are only neutral clusters which decay into two charged hadrons; the rapidity distributions of such clusters are flat, and the "decay width" of the clusters in rapidity space can be neglected) are already sufficient to give a very good description of the multiplicity distributions for the charged hadrons. (Such assumptions have for example been used in the first two papers of Ref. 7.)

(iii) In accordance with the facts mentioned in (i) and (ii), the multiplicity distribution $P_W(n_W)$ for charged hadrons produced at a given total center-of-mass-system (c.m.s.) energy $\sqrt{s}$ (we omit s in P_W as well as in other s-dependent quantities in this paper) in a given rapidity window W, can be written as[7]

$$P_W(n_W=2N_W)=\sum_{N=0,1,\ldots} P(N)N!/[N_W!(N-N_W)!](1-q_W)^{N-N_W}q_W^{N_W} , \tag{1}$$

provided that there is only one emitting system which contributes to this window. Here $P(N)$ is the multiplicity distribution of the neutral clusters which decay into one positively and one negatively charged hadron, $q_W=\langle N_W\rangle/\langle N\rangle=\langle n_W\rangle/\langle n\rangle$, where $\langle n_W\rangle$ is the average multiplicity of charged hadrons observed in the rapidity window W, and $\langle n\rangle$ is the average number of charged hadrons produced by the emitting system.

(iv) A general formula for processes in which more than one independent emitting system exists and more than one kind of cluster is produced can be explicitly given. For sake of completeness, the formula is included in Appendix A. Details can be found in the second paper of Ref. 7, where it is also shown that the experimental results given in Refs. 6 and 8–12 can be well described by this formula.

We now propose a method to study the cluster properties in this framework. In order to see how it works in practice, we explain the method by discussing at the same time a concrete example.

For a given reaction (for example, e^+e^- annihilation) at a given energy (for example, $\sqrt{s}=29$ GeV), consider a rapidity window W (for example, $0<y<0.5$), where the hadrons observed in W are contributions from one emitting system (in this case, one jet). Let us consider an event in which N clusters—each of them is assumed to decay at most into c charged hadrons—are produced by the emitting system. Let $N_W(l\mid N)$ be the number of clusters which contribute l charged hadrons to the rapidity window W in such an N-cluster event. Then, n_W, the number of charged hadrons in W, is

$$n_W=\sum_{l=0}^{c} lN_W(l\mid N) , \tag{2}$$

while

$$N=\sum_{l=0}^{c} N_W(l\mid N) \tag{3}$$

is the number of clusters in that event. We note that n_W is a function of the rapidity window W, while N is independent of W.

Let $P(N)$ be the probability of the emitting system to produce N clusters, and $\alpha_W(l)$ be the average probability for any one of the $N_W(l\mid N)$ clusters to exist where $\alpha_W(l)$ is taken to be independent of N, then the probability for observing n_W charged hadrons in the rapidity window W is

$$P_W(n_W)=\sum_N \sum{}' P(N)\left[N!/\prod_{l=0}^{c} N_W(l\mid N)!\right] \times\prod_{l=0}^{c}\alpha_W(l)^{N_W(l\mid N)} , \tag{4}$$

where the sum is first taken over $N_W(l\mid N)$. The prime on the summation sign indicates that the condition given in Eqs. (2) and (3) should be satisfied.

As we shall see in this section, Eq. (4) is the basis of a successive approximation, with the help of which $\alpha_W(l)$ and $P(N)$ can be estimated from the experimental data for $P_W(n_W)$. In practice, the $P_W(n_W)$ data are always taken for a series of rapidity windows W, including that for the entire rapidity space and/or those for every one of the independent emitting systems [for example, $P_W(n_W)$ for the half-rapidity space with respect to the jet axis in e^+e^- annihilation processes, that is, $P_W(n_W)$ for each of the two jets]. One simple and convenient way to carry out the successive approximation is to start with an ansatz for $\alpha_{W_{\max}}(l)$, where the index $W_{\max}$ stands for the largest possible rapidity window (with respect to the relevant emitting system), and see whether the corresponding solution for $\alpha_W(l)$ and $P(N)$ is consistent with the experimental distributions $P_W(n_W)$ for all other rapidity windows. Before describing this approximation, the following points concerning Eq. (4) should be mentioned.

(a) Equation (4) is a natural generalization of Eq. (1), the validity of which has been shown in Ref. 7. In fact, this is the reason why we are confident in using Eq. (4) as the basis of our method of analysis.

(b) The average probability $\alpha_W(l)$ for having a cluster which contributes exactly l charged hadrons in a given rapidity window W is a useful quantity. On the one hand, it is closely related to the intrinsic properties of the clusters and, on the other hand, it reflects the relative importance of the different kinds of clusters contributing to the given rapidity window. For example, extraordinary large clusters and/or clusters with extremely wide rapidity distributions may or may not exist in the reaction discussed here, but the role such clusters play statistically is probably not an important one. Hence, these clusters can be neglected in the first-order approximation.

(c) This formula for $P_W(n_W)$ can be readily generalized to include cases in which more than one independent emitting system contributes to the given window. Obviously, it should be similar to that given in Eq. (A1) in Appendix A.

(d) Since $\langle n_W \rangle$, the average value of n_W (the multiplicity of charged hadrons observed inside the rapidity window W), and $\langle n_W{}^{\lambda} \rangle$ ($\lambda=2,3,\ldots$), the higher moments of n_W are the measured quantities in experiments, it is clear that direct links between these quantities and the unknowns will be very useful. As can be readily shown (details are given in Appendix B, the following relations between $\langle n_W{}^{\lambda} \rangle$ ($\lambda=1,2,3,\ldots$), $\langle N^{\sigma} \rangle$ ($\sigma=1,2,3,\ldots$), and $\alpha_W(l)$ ($l=0,1,2,\ldots$) are direct consequences of Eq. (4):

$$\langle n_W \rangle = \langle N \rangle \sum_l^c l\alpha_W(l) , \tag{5}$$

$$\langle n_W(n_W-1) \rangle = \langle N(N-1) \rangle \left[\sum_l^c l\alpha_W(l) \right]^2 + \langle N \rangle \sum_l^c l(l-1)\alpha_W(l) , \tag{6}$$

$$\begin{aligned}\langle n_W(n_W-1)(n_W-2) \rangle = {} & \langle N(N-1)(N-2) \rangle \left[\sum_l^c l\alpha_W(l) \right]^3 \\ & + 3\langle N(N-1) \rangle \left[\sum_l^c l\alpha_W(l) \right] \left[\sum_l^c l(l-1)\alpha_W(l) \right] + \langle N \rangle \sum_l^c l(l-1)(l-2)\alpha_W(l) .\end{aligned} \tag{7}$$

These, as well as similar equations which include higher moments of the n_W and those of the N distributions are extremely useful in carrying out the successive approximation mentioned above.

(e) For the purpose of carrying out this successive approximation it is also useful to consider the distributions $P_W(n_W=1)$ and $P_W(n_W=2)$. From Eq. (4) we have

$$P_W(1) = \sum_N P(N) N \alpha_W(0)^{N-1} \alpha_W(1) , \tag{8}$$

$$P_W(2) = \sum_N P(N) N \alpha_W(0)^{N-1} \alpha_W(2) + \tfrac{1}{2} \sum_N P(N) N(N-1) \alpha_W(0)^{N-2} \alpha_W(1)^2 . \tag{9}$$

Now, since neither $P(N)$ nor $\alpha_W(l)$ ($l=0$, 1, and 2) can be negative, it follows from Eqs. (8) and (9)

$$\frac{P_W(1)}{P_W(2)} < \frac{\alpha_W(1)}{\alpha_W(2)} \tag{10}$$

which is a lower bound for the ratio $\alpha_W(1)/\alpha_W(2)$, provided that $\alpha_W(2)$ does not vanish.

Based on the fact[7] that the measured multiplicity distribution $P_W(n_W)$, for different rapidity windows W, in different reactions (including the e^+e^- annihilation processes we consider here) can be fairly well reproduced by assuming that all the observed charged hadrons are produced via neutral clusters which decay into a pair of oppositely charged hadrons, we start our successive approximation by setting, for full rapidity space W(max),

$$\alpha_{W\,\mathrm{max}}(2) = 1 . \tag{11}$$

Inserting this into Eqs. (5) and (6), we obtain

$$\langle n_{W\,\mathrm{max}} \rangle = 2\langle N \rangle = 2\langle N \rangle^{(0)} , \tag{12}$$

$$\langle (n_{W\,\mathrm{max}})^2 \rangle = 4\langle N^2 \rangle = 4\langle N^2 \rangle^{(0)} , \tag{13}$$

where we denote the moments $\langle N^k \rangle$ ($k=1,2,\ldots$) for the cluster distribution $P(N)$ in this zeroth-order approximation by $\langle N^k \rangle^{(0)}$ ($k=1,2,\ldots$). In fact, we can write in this case

$$P_{W\,\mathrm{max}}(n_{W\,\mathrm{max}}) = P(N = n_{W\,\mathrm{max}}/2) . \tag{14}$$

For rapidity windows W other than the full rapidity space, the above-mentioned clusters and/or their decay products may or may not "fall into W." In fact, if we completely neglect the cluster decay we have[15]

$$\begin{aligned} &\alpha_W(l) \neq 0 \quad \text{for } l=2 \text{ and } 0 , \\ &\alpha_W(l) = 0 \quad \text{otherwise} . \end{aligned} \tag{15}$$

But, in general, we should have

$$\begin{aligned} &\alpha_W(l) \neq 0 \quad \text{for } l=2,\ 1, \text{ and } 0 , \\ &\alpha_W(l) = 0 \quad \text{otherwise} . \end{aligned} \tag{16}$$

This is because there should be an appreciable chance also for one of the two charged hadrons to enter W after the cluster decay, provided that the hadrons from the same cluster exhibit the usual type of short-range-rapidity correlations. In fact, we can calculate, for every given W, the quantities $\alpha_W(2)$ and $\alpha_W(1)$ by inserting into Eqs. (5) and (6) the experimental values of $\langle n_W \rangle$ and $\langle n_W{}^2 \rangle$, and the values $\langle N \rangle^{(0)}$ and $\langle N^2 \rangle^{(0)}$ which

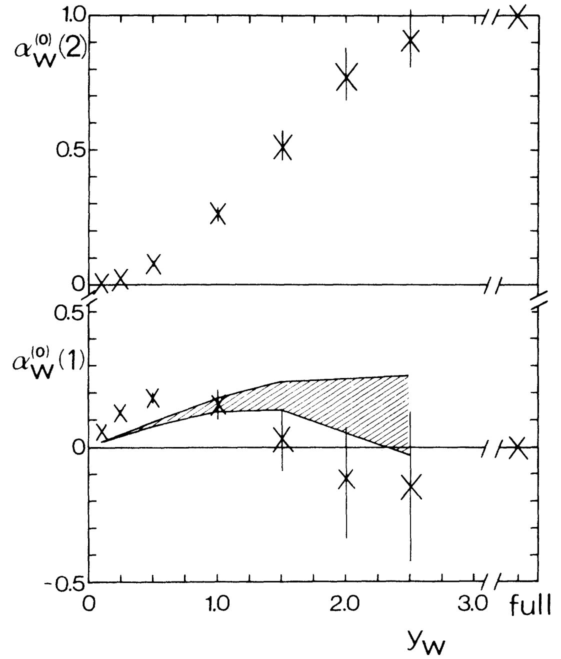

FIG. 1. The calculated result of $\alpha_W^{(0)}(1)$ and $\alpha_W^{(0)}(2)$ as functions of the size of the rapidity window y_W ($|y| \le y_W$). The values and the corresponding error bars are calculated from the data given in Ref. 6. The value for the full rapidity space is used as input. Here, the corresponding y_W is indicated by "full." The lower limit for $\alpha_W^{(0)}(1)$ is obtained from the inequality given in (14) taken together with the above-mentioned data. The hatched area shows the possible values for the lower limit. The boundaries are determined by the error bars of $P_W(1)$ and $P_W(2)$ given in the data.

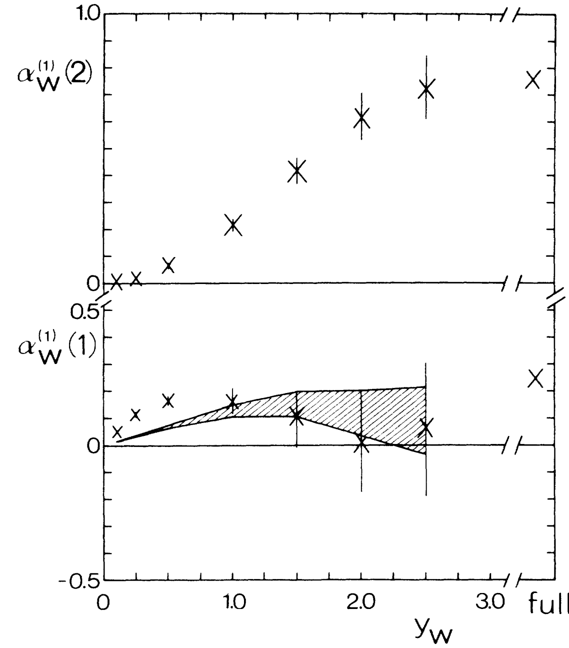

FIG. 2. The calculated result of $\alpha_W^{(1)}(1)$ and $\alpha_W^{(1)}(2)$ as functions of y_W. The set of data as well as the notations are the same as those in Fig. 1.

are the zeroth-order approximation of $\langle N \rangle$ and $\langle N^2 \rangle$ obtained in Eqs. (12) and (13), respectively. We denote the calculated values in this approximation by $\alpha_W^{(0)}(2)$ and $\alpha_W^{(0)}(1)$, respectively. The results are given in Fig. 1. As we can see, some of the calculated values of $\alpha_W^{(0)}(1)$ are lower than the lower limit obtained from (10). The most probable reason for this inconsistency is that the input, the zeroth-order approximation based on the assumption given in Eq. (14), is not sufficient. In Fig. 2 we show the first-order approximation for $\alpha_W(2)$ and $\alpha_W(1)$ which we obtain from the ansatz

$$\alpha_{W\,\mathrm{max}}(2)=0.75 \ ,$$
$$\alpha_{W\,\mathrm{max}}(1)=0.25 \ . \tag{17}$$

We note that Eq. (17) implies that 25% of the produced "hadronic clusters" are nothing else but charged hadrons—a result which supports the observation of Derrick *et al.*[6] in their e^+e^- annihilation experiment.

The quality of the approximation can be tested by inserting into Eq. (7) the experimental data for $\langle n_W \rangle$, $\langle n_W{}^2 \rangle$, $\langle n_W{}^3 \rangle$, and the first-order approximation for $\alpha_W(l)$ given in Fig. 2 and that for $\langle N \rangle$, $\langle N^2 \rangle$, and $\langle N^3 \rangle$. [The latter are denoted by $\langle N \rangle^{(1)}$, $\langle N^2 \rangle^{(1)}$, and $\langle N^3 \rangle^{(1)}$, respectively. They are determined by inserting into Eqs. (5), (6), and (7) the values for $\alpha_{W\,\mathrm{max}}(2)$ and $\alpha_{W\,\mathrm{max}}(1)$ given in Eq. (17) and the experimental data for $P_W(n_W)$ for $W=W(\mathrm{max})$.] In Fig. 3 we plot the difference Δ_W between the expression on the left-hand side and that on the right-hand side of Eq. (7). The error bars are obtained from the corresponding directly and/or indirectly measured quantities. It should be mentioned that in this example (e^+e^- annihilation processes) only the presently available $P_W(n_W)$ data,[6] that

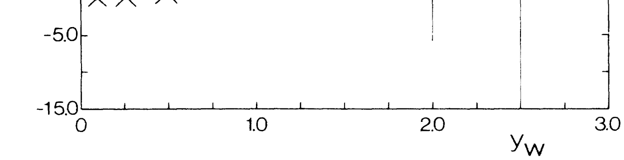

FIG. 3. Δ_W, the difference between the left-hand side and the right-hand side of Eq. (11), as a function of the size of the rapidity window y_W. The values and the error bars of the left-hand side are obtained from the data of Ref. 6 for $\langle n_W \rangle$, $\langle n_W{}^2 \rangle$, and $\langle n_W{}^3 \rangle$. The right-hand side is obtained from the calculated values for $\langle N \rangle^{(1)}$, $\langle N^2 \rangle^{(1)}$, $\langle N^3 \rangle^{(1)}$, $\alpha_W^{(1)}(1)$, and $\alpha_W^{(1)}(2)$. As we can see ΔW is not always zero. This would be the case if the first-order approximation were already sufficiently good.

is, the $P_W(n_W)$ data for *symmetric* rapidity windows ($y_W = 0.1$, 0.25, etc.), have been used. We recall (see Ref. 7 for details) that for symmetric rapidity windows the result for one emitting system and that for two emitting systems are the same.

If the error bars shown in Fig. 3 were so small such that we could conclude that this approximation is definitely insufficient, we would proceed one step further.[16] That is, we would try to determine $\alpha_{W\max}(l=1,2,3)$ from Eqs. (5), (6), and (7) in the second-order approximation and check the quality of this approximation by inserting in the corresponding equation for the moment $\langle n_W(n_W-1)(n_W-2)(n_W-3)\rangle$, etc. In this sense, we can carry out a successive approximation to find $\alpha_W(l)$, $l=0,1,2,\ldots$, and $P(N)$ from the experimentally measured $P_W(n_W)$ for different rapidity windows W. It is clear that such a method can be applied to different reactions at different energies, provided that there are precise measurements of multiplicity distributions as those mentioned above.

III. STATISTICAL ANALYSIS OF MULTIPLICITY CORRELATIONS

As we have shown in the second paper of Ref. 7, a possible way to learn more about the clusters is to study the charge-multiplicity correlations defined as

$$C_W(n_{W+},n_{W-}) = F_W(n_{W+},n_{W-}) - P_{W+}(n_{W+})P_{W-}(n_{W-}) . \tag{18}$$

Here, $F_W(n_{W+},n_{W-})$ is the (normalized) probability of finding n_{W+} positively and n_{W-} negatively charged hadrons in rapidity window W [at a given total center-of-mass-system (c.m.s.) energy $\sqrt{s}$.] It is simply the product of the corresponding probabilities $P_{W+}(n_{W+})$ and $P_{W-}(n_{W-})$ for finding n_{W+} positively and n_{W-} negatively charged hadrons, respectively, if these two kinds of hadrons are produced independently. (We note, also in this section that all s in the, in general, s-dependent quantities such as F_W, P_{W+}, and P_{W-} are omitted.)

Similar to the general expression given in Eq. (4) for the probability $P_W(n_W)$ for observing n_W charged hadrons in the rapidity window W, we can express $F_W(n_{W+},n_{W-})$ as

$$F_W(n_{W+},n_{W-}) = \sum_N \sum{}' P(N) \left[N! \Big/ \prod_{j=0}^{p} \prod_{k=0}^{n} N_W(j,k \mid N)! \right] \prod_{j=0}^{p} \prod_{k=0}^{n} \alpha_W(j,k)^{N_W(j,k \mid N)} . \tag{19}$$

Here $P(N)$ is the probability for the emitting system to produce N clusters—each of them is assumed to decay at most into p positively and n negatively charged hadrons. $N_W(j,k \mid N)$ is the number of clusters which contribute j positively and k negatively charged hadrons to the given rapidity window W in a N-cluster event. $\alpha_W(j,k)$ is the average probability for any one of the $N_W(j,k \mid N)$ clusters to exist. Here, $\alpha_W(j,k)$ is taken to be independent of N. The prime on the summation sign indicates that the summation over $N_W(j,k \mid N)$ has to be taken under the conditions

$$n_{W+} = \sum_{k=0}^{p} \sum_{j=0}^{n} j N_W(j,k \mid N) , \tag{20}$$

$$n_{W-} = \sum_{j=0}^{p} \sum_{k=0}^{n} k N_W(j,k \mid N) , \tag{21}$$

$$N = \sum_{j=0}^{p} \sum_{k=0}^{n} N_W(j,k \mid N) . \tag{22}$$

In order to show how Eq. (19) can be used in practice, we consider the sum

$$n_W = n_{W+} + n_{W-} \tag{23}$$

and the difference

$$e_W = n_{W+} - n_{W-} \tag{24}$$

of n_{W+} and n_{W-}, as well as their distributions $P_W(n_W)$, $Q_W(e_W)$, and moments $\langle n_W{}^{\lambda}\rangle$, $\langle e_W{}^{\sigma}\rangle$, $\lambda,\sigma = 1,2,3,\ldots$. It can be shown (see Appendix C) that these measurable quantities are also closely connected with the cluster distribution $P(N)$ and the average probabilities $\alpha_W(k,l)$ mentioned above. While $P_W(n_W)$ and $\langle n_W{}^{\lambda}\rangle$ are obviously nothing else but those discussed in the preceding section, it is expected that $Q_W(e_W)$ and/or $\langle e_W{}^{\lambda}\rangle$ should yield additional information on the clusters. As an illustrative example, we consider the case in which there is symmetry between the positively and the negatively charged hadrons in the rapidity window W in the sense that

$$\alpha_W(j,k) = \alpha_W(k,j) . \tag{25}$$

It follows from the general expression for $\langle e_W{}^{\lambda}\rangle$ [see Appendix C, Eq. (C8)] and Eq. (25) that

$$\langle e_W{}^{2\sigma+1}\rangle = 0 \quad \text{for } \sigma = 0,1,2,\ldots \tag{26}$$

and

$$\langle e_W{}^2\rangle = \langle N\rangle \sum_j^p \sum_k^n (j-k)^2 \alpha_W(j,k) . \tag{27}$$

Taken together with the corresponding expression for $\langle n_W{}^{\lambda}\rangle$ [see Appendix C, Eq. (C7)], we obtain

$$\frac{\langle e_W{}^2\rangle}{\langle n_W\rangle} = \frac{\sum_j^p \sum_k^n (j-k)^2 \alpha_W(j,k)}{\sum_j^p \sum_k^n (j+k) \alpha_W(j,k)} . \tag{28}$$

Hence, in particular, if all clusters are charge neutral,

$$\frac{\langle e_W{}^2\rangle}{\langle n_W\rangle} \to 0 \quad \text{for sufficiently large } W . \tag{29}$$

Further examples will be discussed in connection with dynamical models in a subsequent paper.

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APPENDIX A

It has been shown in the first and second papers of Ref. 7 that the multiplicity distribution $P_W(n_W;s)$ at a given (total c.m.s.) energy $\sqrt{s}$, for a given rapidity window W ($y_l < y < y_h$), which may or may not be symmetric with respect to $y=0$, can be written in the general form

$$P_W(n_W;s)=\sum{}' \prod_i P_{iW}(n_{iW};s) \ . \tag{A1}$$

The right-hand side of this equation is a sum of products of $P_{iW}(n_{iW};s)$. Here, $P_{iW}(n_{iW};s)$ is the probability of observing n_{iW} charged hadrons from the ith emitting system (which is independent from all other emitting systems) in the rapidity window W. The prime on the summation sign means that the sum over all possible n_{iW}'s (which contribute to n_W) should be such that the condition

$$\sum_i n_{iW}=n_W \tag{A2}$$

is satisfied. $P_{iW}(n_{iW};s)$ is a weighted sum of binomial distributions:

$$P_{iW}(n_{iW};s)=\sum_{n_i/c_i=1,2,\ldots} P_i(n_i/c_i;s)B(n_i/c_i,n_{iW}/c_i;q_{iW}) \ , \tag{A3}$$

where

$$B(N_i,N_{iW};q_{iW})=\begin{bmatrix} N_i \\ N_{iW} \end{bmatrix} q_{iW}{}^{N_{iW}}(1-q_{iW})^{N_i-N_{iW}} \ , \tag{A4}$$

$$q_{iW}(s)=\frac{\langle n_{iW}/c_i\rangle}{\langle n_i/c_i\rangle}=\frac{\langle n_{iW}\rangle}{\langle n_i\rangle} \ . \tag{A5}$$

The weights $P_i(n_i/c_i;s)$ in Eq. (A3) is the probability for the system i (at energy $\sqrt{s}$) to emit $n_i/c_i=N_i$ clusters, where each cluster decays into c_i charged hadrons. Obviously, n_i is the number of charged hadrons produced by the system i, $\langle n_i\rangle$ is the average value of n_i, n_{iW} is the number of charged hadrons observed inside the rapidity window W, and $\langle n_{iW}\rangle$ its average value. We note that the average values $\langle n_i\rangle$, $\langle n_{iW}\rangle$ are in general functions of the total c.m.s. energy $\sqrt{s}$.

That is to say, the right-hand side of Eq. (A1), taken together with Eqs. (A2)–(A5), is the general form for the multiplicity distribution at a given (total c.m.s.) energy $\sqrt{s}$ in a given rapidity window W. This means the probability $P_W(n_W;s)$ to find n_W produced particles at energy $\sqrt{s}$ and in window W can be obtained from Eqs. (A1)–(A5) provided that the number of independent emitting systems and the properties of the produced cluster in each system [that is, c_i, $P_i(n_i/c_i;s)$, and q_{iW}] are known. In particular, if there is only one system, or a fixed number of (in phase space clearly separated) systems such that c_i, $P_i(n_i/c_i;s)$, and q_{iW} can directly be taken from the experimental data, then $P_W(n_W;s)$ can be calculated from Eqs. (A1)–(A5) without introducing any free parameter.

It should be emphasized that the general formula for $P_W(n_W;s)$ given here is an idealization. For example, it is certainly oversimplified to assume that in a given emitting system i, there is one and only one c_i, which implies that all clusters in the system i are the same. It is of course also an oversimplification when we neglect the rapidity distribution due to cluster decay in our calculation.

APPENDIX B

In order to calculate the average of n_W, $\langle n_W\rangle$, as well as its higher moments $\langle n_W{}^{\lambda}\rangle$ ($\lambda=2,3,\ldots$), it is convenient to use the generating function

$$G_W(t)=\sum_{n_W=0,1,\ldots} P_W(n_W)t^{n_W} =\sum_N P(N)\left[\sum_{l=0}^{c}\alpha_W(l)t^l\right]^N \ , \tag{B1}$$

the normalization condition of which is

$$G_W(1)=1 \ , \tag{B2}$$

$$\langle n_W{}^{\lambda}\rangle=\left[t\frac{d}{dt}\right]^{\lambda} G_W(t)\,|_{t=1} \ , \tag{B3}$$

$$\langle n_W(n_W-1)\cdots(n_W-\lambda+1)\rangle=\left[\frac{d}{dt}\right]^{\lambda} G_W(t)\,|_{t=1} \ , \tag{B4}$$

where λ is a positive integer.

APPENDIX C

To calculate the moments $\langle n_{W+}{}^{\lambda}\rangle$, $\langle n_{W-}{}^{\lambda}\rangle$ ($\lambda=1,2,\ldots$), and/or their algebraic combinations, we introduce the generating function

$$H_W(x,y)=\sum_{n_{W+}}\sum_{n_{W-}} F(n_{W+},n_{W-})x^{n_{W+}}y^{n_{W-}} =\sum_N P(N)\left[\sum_j^p\sum_k^n \alpha_W(j,k)x^jy^k\right]^N \ , \tag{C1}$$

the normalization condition of which is

$$H_W(1,1)=1 \ . \tag{C2}$$

From Eqs. (C1) and (C2) we obtain

$$\langle n_{W+}{}^{\lambda}\rangle=(x\,\partial/\partial x)^{\lambda}H_W(x,y)\,|_{x=y=1} \ , \tag{C3}$$

$$\langle n_{W-}{}^{\lambda}\rangle=(y\,\partial/\partial y)^{\lambda}H_W(x,y)\,|_{x=y=1} \ . \tag{C4}$$

The generating function of the distribution $P_W(n_W)$ of the number of charged hadrons n_W (that is the sum of n_{W+} and n_{W-}) and the distribution $Q_W(e_W)$ of the net charge (that is the difference between n_{W+} and n_{W-}) in the given rapidity window W can be readily expressed in terms of the generating function $H_W(x,y)$:

$$H_W(x,x)=\sum_{n_W} P_W(n_W)x^{n_W}$$
$$=\sum_N P(N)\left[\sum_j^p\sum_k^n \alpha_W(j,k)x^{j+k}\right]^N , \quad \text{(C5)}$$
$$H_W(x,1/x)=\sum_{e_W} Q_W(e_W)x^{e_W}$$
$$=\sum_N P(N)\left[\sum_j^p\sum_k^n \alpha_W(j,k)x^{j-k}\right]^N . \quad \text{(C6)}$$

The corresponding moments $\langle n_W{}^\lambda\rangle$ and $\langle e_W{}^\lambda\rangle$ ($\lambda=1,2,\ldots$) are

$$\langle n_W{}^\lambda\rangle=(x\,\partial/\partial x)^\lambda H_W(x,x)\,|_{x=1} , \quad \text{(C7)}$$

$$\langle e_W{}^\lambda\rangle=(x\,\partial/\partial x)^\lambda H_W(x,1/x)\,|_{x=1} . \quad \text{(C8)}$$

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[1]See, e.g., the following review articles: A. Bialas, in *Proceedings of the IV International Symposium on Multiparticle Hadrodynamics,* edited by F. Dulmio *et al.* (La Gollardica Pavese, Pavia, Italy, 1973), p. 93; L. Foa, Phys. Rep. **22**, 1 (1975); J. Whitmore, *ibid.* **27**, 187 (1976); G. Giacomelli and M. Jacob, *ibid.* **55**, 1 (1979), and the papers cited therein.

[2]See, e.g., K. Böckmann and B. Eckart, in *Proceedings of the XVth International Symposium on Multiparticle Dynamics,* Lund, Sweden, 1984, edited by G. Gustafson and C. Peterson (World Scientific, Singapore, 1984); G. Ekspong, in *Mulitparticle Dynamics 1985,* proceedings of the International Symposium, Kiryat Anavim, Israel, 1985, edited by J. Grunhaus (Editions Frontiéres, Gif-sur-Yvette, France, 1985); L. K. Mangotra, I. Otterlund, and E. Stenlund, Z. Phys. C **31**, 199 (1986); W. Bell *et al., ibid.* **32**, 335 (1986); L. van Hove and A. Giovannini, *ibid.* **30**, 391 (1986); C. C. Shih, Phys. Rev. D **34**, 2710 (1986); **34**, 2720 (1986); S. Barshay and E. Eich, Aachen report, 1986 (unpublished), and the papers cited therein.

[3]See, e.g., the experimental review articles given in Refs. 1 and 2 and the references given therein.

[4]Data on three-particle distributions can, e.g., be found in K. Eggert *et al.,* Nucl. Phys. **B86**, 201 (1975); T. Kafka *et al.,* Phys. Rev. D **16**, 1261 (1977). Difficulties in obtaining precise information on three-particle correlations have been reported in these papers. It has, for example, been pointed out by Kafka *et al.* that an analysis of their data based on the conventional correlation formalism gives the following result: The three-particle densities can be given in terms of the one- and two-particle densities; and it is difficult to assess to what extent this fact is a reflection of a collision dynamics and to what extent it is influenced by phase-space limitations.

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[15]In this case, we have $\alpha_W(2)=\langle n_W\rangle/\langle n_{W\,\max}\rangle=\langle n_W\rangle/[2\langle N\rangle^{(0)}]$, $\alpha_W(0)=1-\alpha_W(2)$, and $\alpha_W(l)=0$ for $l\neq 0,2$.

[16]We asked ourselves the following question: Is it possible to make Δ_W zero by choosing a nonvanishing $\alpha_{W\,\max}(3)$, that is, by assuming that there is a small percentage of clusters which decay into three charged hadrons? The answer we found is yes. This, as well as other possible solutions will be discussed in detail in connection with dynamical models.