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Reflections on moving mirrors

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A field theory constructed on a space with a moving boundary illustrates many of the physical characteristics of Hawking radiation from black holes. We review this analogy with a number of new refinements. We construct a model in which the Bogoliubov transformation, which is induced by the moving boundary, can be computed explicitly. This model is used to discuss correlations in the final state of the quantum field. This discussion can, in turn, serve as the basis for an investigation into Hawking's proposal that black holes can induce the evolution of quantum-mechanical pure states into mixed states.

I. INTRODUCTION

In contemplating a quantum-mechanical theory of gravity one encounters a number of paradoxes which play upon our fundamental notions of spacetime and of quantum measurements. These paradoxes may be viewed as obstacles to the construction of a viable theory of quantum gravity or as opportunities for insight into the structure such a theory will ultimately have. This second viewpoint has been quite fruitful, leading, for example, to Hawking's discovery[1] of a mechanism by which black holes can radiate away their associated mass. Hawking has even speculated[2] that black holes might disappear in consequence of this mechanism, with the net conversion of a pure quantum state into a thermal or, more generally, mixed quantum state.

We would like to establish a framework for investigating this speculation. To this end we will study in this paper a simple system which is in many ways analogous to the system considered by Hawking—that of a quantum field evolving in the classical background metric of a massive object undergoing gravitational collapse. The system which we choose to study is one of several which other authors[3] have used to exhibit certain physical aspects of Hawking radiation. It involves a quantum field in a two-dimensional flat spacetime bounded by a reflecting wall. Particles, or field quanta, are produced if this moving mirror undergoes an acceleration. And if the mirror trajectory is suitably chosen, the particle flux can mimic the particle flux expected in consequence of gravitational collapse.

Our development of the moving-mirror analog will parallel existing treatments in the literature, but with refinements at each stage. We will try to emphasize why we think that the model is particularly relevant to the question of pure states evolving into mixed states. And in a sequel to this paper[4] we will apply the model to precisely this question.

An outline of our approach is as follows. In Sec. II we review Hawking's arguments for particle production in the process of gravitational collapse, emphasizing those aspects which we feel to be physically most relevant. In Sec. III we formulate our moving-mirror analogy and explain why we reject other possible simplifications of the general problem of gravitational collapse. Choosing a particular trajectory for the moving mirror, we are able to exhibit the final state of the quantum field with no further approximations. Our approach makes it easy to examine stimulated-emission processes, as we do in Sec. IV. These processes emphasize the quantum nature of the problem and underscore the existence of correlations that are induced by the moving boundary. These correlations are studied more explicitly in Sec. V, where we compute correlation functions involving the stress-energy tensor of the quantum field. These correlations will form a key element in the sequel to this paper. In Sec. VI we remark upon some general properties of these correlations, including their relation to the Einstein-Podolsky-Rosen effect.[5]

An effort has been made to make this paper more or less self-contained for readers who are not familiar with the extensive literature on this subject. For those who are well versed in the literature, we should point out which aspects of our arguments are genuinely new. We choose a trajectory[6] which generates a constant flux of Hawking radiation and permits an exact computation of the associated Bogoliubov transformation. We are able to diagonalize this Bogoliubov transformation and provide an explicit construction of the quantum state at late times. This construction exhibits all final-state correlations and facilitates an examination of the process of stimulated emission. The treatment of stimulated emission includes a case previously treated by Wald.[7] But it also deals with a new physical process whose existence derives entirely from correlations in the quantum field. The computation of correlation functions of the stress-energy tensor is also a new result. Finally, there are our remarks about the Einstein-Podolsky-Rosen effect which emphasize the great physical importance that correlations play in our model. This emphasis will carry over to our sequel and will dominate our discussion of the possible evolution of pure states into mixed states.

II. HAWKING RADIATION

This section contains a brief review of Hawking's explanation of particle creation in the gravitational field of a collapsing mass. We emphasize the essential physical features of this process, which we will extract to construct an easily comprehended mechanical analog. In subsequent sections we will construct this analog system and pursue more detailed questions in the context of this analogy.

Hawking considered the evolution of a quantum field under the influence of the classical gravitational field of a collapsing massive object. For definiteness and simplicity we will assume that the quantum field describes massless scalar particles. Properties of the massive object need not be specified in any detail, save the fact that its mass M should be much larger than the Planck mass m_P. This assures that the Schwarzschild radius of the object should be much larger than its Compton wavelength so that its gravitational field may be described in classical terms.

Radially incoming null geodesics (or classical trajectories of our scalar particles) can be labeled by a coordinate v, outgoing null geodesics by a coordinate u. A typical particle might approach the collapsing object along a trajectory $v=\text{const}$. It would undergo a blue-shift as it approached the object and passed through its center. From there it would undergo a red-shift as it emerged from the object along a trajectory:

$$u=f(v) \ . \tag{2.1}$$

Since the massive object is collapsing, the red-shift along the outgoing trajectory must exceed the blue-shift along the incoming trajectory. This net red-shift is encoded in the function $f(v)$. More significantly, if the object collapses to form a black hole, there will be a final trajectory, $v=v_0$, that just escapes falling into the black hole. This implies a singularity in the function $f(v)$, with $f(v)\rightarrow\infty$ as $v\rightarrow v_0$. It is the nature of this singularity which determines the properties of Hawking radiation.

The singularity at $v=v_0$ divides the space of incoming trajectories into two distinct regions. This fact has profound consequences for the quantum field. In the distant past, modes of the quantum field can be described in terms of wave packets in the variable v. Continuity of the functions which describe these wave packets forces any given wave packet to have components in both of the regions $v>v_0$ and $v<v_0$. Obviously this effect is most pronounced for packets centered close to $v=v_0$. The component with $v<v_0$ corresponds to particles which can escape the gravitational pull of the black hole; the component with $v>v_0$ describes particles which fall into the hole.

Consider now a description of the quantum field at late times. A complete basis must account for both the escaping and trapped particles. Suppose we examine a wave packet which corresponds to some outgoing particle. The evolution of this packet can be traced back to the $v<v_0$ portion of some wave packet in the distant past. The $v>v_0$ portion of this same wave packet describes particles trapped by the black hole. Thus we can conclude that there is a correlation between escaping particles (or, more properly, modes of the quantum field which correspond to escaping particles) and particles which are trapped by the black hole (or, rather, the corresponding modes of the quantum field).

To make this correlation more explicit, and to exploit it more fully, we should exhibit the precise form of the singularity in $f(v)$. Hawking found that, as $v\rightarrow v_0$,

$$f(v)\sim -\kappa^{-1}\ln[\kappa(v_0-v)] \ , \tag{2.2}$$

where

$$\kappa=\frac{m_P{}^2}{4M} \tag{2.3}$$

denotes the surface gravity of the black hole (i.e, the gravitational acceleration at the Schwarzschild radius). It follows from Eqs. (2.1) and (2.2) that an outgoing wave packet, described at late times in terms of the variable u, will have the same functional form at early times in terms of a variable

$$V=-\kappa^{-1}\ln[\kappa(v_0-v)] \ . \tag{2.4}$$

In particular, if the width of the packet in the variable u is Δu, then its width in the variable V is

$$\Delta V=\Delta u \ . \tag{2.5}$$

Hence from Eq. (2.4) the width of the packet in the variable v is

$$\Delta v=e^{\kappa V}\Delta V\simeq e^{-\kappa u}\Delta u \ . \tag{2.6}$$

For late u (and $v\sim v_0$) it is apparent that

$$\Delta v \ll \Delta u \ . \tag{2.7}$$

The functional form (2.4) applies to the region $v<v_0$, but we can analytically continue past the singularity at $v=v_0$. In the region $v>v_0$ it is convenient to define

$$W=\kappa^{-1}\ln[\kappa(v-v_0)] \ . \tag{2.8}$$

Analytic continuation takes

$$V\rightarrow -W\mp i\pi\kappa^{-1} \ , \tag{2.9}$$

with the sign depending upon the sense in which the singularity is passed. The upper sign corresponds to an incoming wave packet of positive energy, the lower sign to an incident packet of negative energy. The fact that an outgoing mode of positive energy is linked to incoming modes of both positive and negative energies, tells us that there is a nontrivial Bogoliubov transformation linking the vacuum states of the quantum field at late and early times. This implies that particle creation takes place under the influence of the gravitational field of the collapsing object.

In what follows we will make the preceding arguments more precise and explicit. In the interest of simplicity and clarity we would first like to isolate those features of the problem which are essential to the physical results that we can extract. Note that our discussion has so far dealt only with radial trajectories. Although angular coordinates are necessary to describe the modes of the quantum field in the gravitational field of a collapsing three-dimensional object, and the mode functions have a nontrivial dependence upon angular momentum, the essential physics of particle production seems to involve the radial coordinate more directly and more fundamentally. Therefore, it should be possible to extract the physics of particle production from some two-dimensional (one space plus one time dimension) model. Note further that the crucial element for describing particle production and correlations among the produced particles is the Bogoliubov transformation linking the early and late time vacua of the quantum field. This means that we can concentrate on Bogoliubov transformations in two-dimensional quantum systems. The relevant Bogoliubov transformation should be based on the function $f(v)$ as in Eq. (2.1), but we need not involve ourselves with details of the matter distribution which generated this function in the original four-dimensional (three space plus one time dimension) problem.

There is a class of two-dimensional models which have been studied extensively in this context. These involve a reflecting wall which is accelerated away from a distant observer. The observer sees a flux of particles generated in consequence of the accelerating boundary. If the acceleration is constant, there results a thermal flux exactly analogous to the flux deduced in Hawking's original work. We will develop this model in the following section.

III. MOVING MIRRORS

The moving-mirror analogy is only one of several pathways to simplicity that have been followed in the literature of the Hawking effect. We will comment briefly on possible alternatives in order to emphasize the salient physical features of the moving-mirror approach. Let us begin with Rindler models,[3] which compare the observations of a fixed observer with those of an observer moving at a constant acceleration. There is a Bogoliubov transformation which connects the observations of the two observers. But this transformation correlates particles seen by the accelerating observer with particles in the "second Rindler wedge," a region of spacetime which is inaccessible to the accelerating observer. We reject this correlation as an inappropriate analogy to the case of gravitational collapse, where the correlation involves emitted particles and particles falling into the hole—both of which can interact with an observer outside the black hole. In the following section we will exhibit the nature of this interaction in a study of certain processes involving stimulated emission.

The existence of unphysical correlations in the Rindler model is related to the existence in that model of a "white-hole" horizon in past times which appears symmetrically with the expected black-hole horizon to the future. Another model which simplifies the dynamics of the Hawking process and which also exhibits a white-hole horizon is the so-called eternal black hole,[3] in which the gravitational field is represented by the Schwarzschild solution for a static distribution of matter. Although by a clever choice[8] of boundary conditions along the white-hole horizon, one can mimic the particle production which occurs in gravitational collapse, this model is flawed by unphysical correlations, as in the Rindler model. Therefore, we feel that this model is inadequate for investigating such questions as the possible evolution of pure states into mixed states or even the physical correlations which are induced by gravitational collapse.

We would like to select a model which exhibits the essential physical features of the Hawking process but which is as simple as possible for practical computations. To this end we will construct a moving-mirror trajectory for which particle emission occurs at a constant rate. Here the Bogoliubov transformation is so simple that it can be diagonalized explicitly, permitting us to construct an explicit representation of the state of the quantum field at late times. This example will be useful in establishing the nature of the correlations that will persist in more complicated examples. Our simplest model corresponds to an emitter of fixed temperature, rather like the eternal black hole; but we can easily modify the model to incorporate a temperature which increases as the effective mass of the black hole decreases to match Hawking's general arguments. The Bogoliubov transformation cannot be explicitly constructed for this model, but one can discuss[4] the structure of correlations which occur.

We will now begin to examine the evolution of a massless scalar field ϕ in a two-dimensional space bounded by a movable wall whose position is given by

$$x=z(t)\ . \tag{3.1}$$

The time and position coordinates t and x are related to null coordinates of the previous section by

$$v=t+x\ , \tag{3.2}$$

$$u=t-x\ . \tag{3.3}$$

The boundary condition at the wall is

$$\phi(t-z(t),t+z(t))=0\ . \tag{3.4}$$

Modes of the field which satisfy this boundary condition can be expressed as

$$\phi_\omega(u,v)=e^{-i\omega v}-e^{-i\omega p(u)}\ . \tag{3.5}$$

The function $p(u)$ can be written as

$$p(u)=2\tau_u-u\ , \tag{3.6}$$

where τ_u denotes the time at which the wall reaches the null coordinate u:

$$\tau_u-z(\tau_u)=u\ . \tag{3.7}$$

The function $p(u)$ is simply the inverse of the function $f(v)$ which appears in Eq. (2.1). For our purposes $p(u)$ will be more convenient to use than $f(v)$, since $p(u)$ is real for all values of u, while $f(v)$, as emphasized previously, is singular at $v=v_0$. From Eqs. (3.6) and (3.7) it follows that the first derivative of $p(u)$ is directly related to the red-shift suffered by a particle which is reflected from the wall:

$$p'(u)=\frac{1+\dot{z}(t)}{1-\dot{z}(t)} \ . \tag{3.8}$$

The energy-momentum tensor, which describes the flux of particles produced by the accelerating wall, can be given entirely in terms of $p'(u)$ and its derivatives:

$$\langle T_{uu}\rangle=\frac{1}{12\pi}(p')^{1/2}\partial_u{}^2(p')^{-1/2} \ , \tag{3.9}$$

with other components vanishing. The integrated energy flux is given by

$$\int du\,\langle T_{uu}\rangle=\frac{1}{48\pi}\int du\,[\partial_u \ln p'(u)]^2 \ , \tag{3.10}$$

where we have dropped surface terms in the integral on u.

A constant energy flux is obtained if

$$p(u)=-\kappa^{-1}e^{-\kappa u} \ . \tag{3.11}$$

The corresponding mirror trajectory is given by the hyperbolic equation

$$t+z(t)=-\kappa^{-1}e^{-\kappa t+\kappa z(t)} \ , \tag{3.12}$$

whose solution is plotted in Fig. 1. Note that this trajectory exhibits a future horizon at $v=v_0=0$, in that

$$t+z(t)\le 0, \quad \forall u \ . \tag{3.13}$$

There is no past horizon, meaning that the trajectory covers all real values of $u=t-z(t)$. In this regard our trajectory differs from a Rindler trajectory:

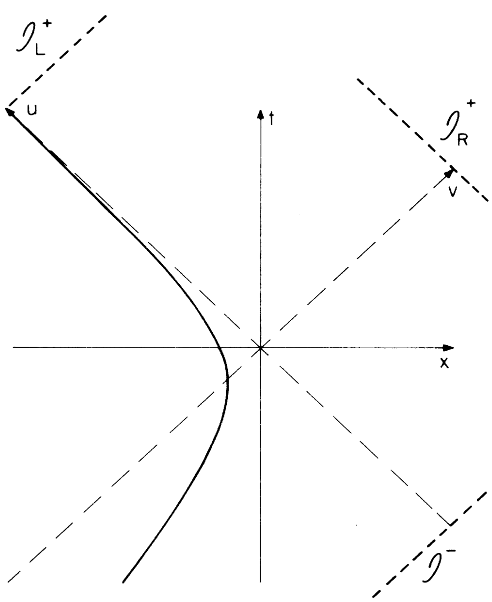

FIG. 1. Mirror trajectory with a future horizon at $v=0$. Rays incident from $\mathcal{J}^-$ with $v<0$ reflect from the mirror and travel outward to $\mathcal{J}_R^+$. Rays with $v>0$ never strike the mirror and pass on to $\mathcal{J}_L^+$.

$$z=-\kappa^{-1}\ln[2\cosh(\kappa t)] \ , \tag{3.14}$$

which coincides with our trajectory at late times, $\kappa t\gg 1$. The Rindler trajectory has

$$e^{\kappa[t+z(t)]}+e^{-\kappa[t-z(t)]}=1 \ , \tag{3.15}$$

and hence exhibits a future horizon at $v=0$ and a past horizon at $u=0$. There is, however, a superficial resemblance to our trajectory in that

$$\frac{1+\dot{z}}{1-\dot{z}}=e^{-2\kappa t} \ . \tag{3.16}$$

This has the same functional form as our Eq. (3.11) but involves the variable $2t$ in place of u.

We would now like to construct the Bogoliubov transformation induced by the moving boundary Eq. (3.12). To this end consider an expansion of the field ϕ in terms of the normal modes ϕ_ω of Eq. (3.5):

$$\phi(u,v)=\frac{1}{4\pi}\int_0^\infty\frac{d\omega}{\omega}[a_\omega\phi_\omega(u,v)+a_\omega^\dagger\phi_\omega^*(u,v)] \ . \tag{3.17}$$

Since the mode functions ϕ_ω have the structure of plane waves on $\mathcal{J}^-$, the creation operators $a_\omega^\dagger$ have a natural interpretation in terms of particles which leave $\mathcal{J}^-$ in the distant past. The mode functions ϕ_ω do not have a simple interpretation on $\mathcal{J}^+$. Therefore, we seek an alternative to Eq. (3.17), which will involve an expansion of ϕ in terms of creation and annihilation operators for particles which approach $\mathcal{J}^+$ in the distant future.

Owing to the horizon, $\mathcal{J}^+$ consists of two components, $\mathcal{J}_L^+$ and $\mathcal{J}_R^+$. In terms of our analogy, right-moving quanta, which escape to $\mathcal{J}_R^+$, correspond to particles which can escape from the black hole. Left-moving quanta, which asymptotically approach $\mathcal{J}_L^+$, correspond to trapped particles. Plane waves on $\mathcal{J}_R^+$ are represented by the functions $e^{-i\omega u}$. Therefore, the mode functions

$$\phi_\omega^R(u,v)=-e^{-i\omega u}+e^{-i\omega f(v)}\theta(-v) \ , \tag{3.18}$$

have a simple physical interpretation on $\mathcal{J}_R^+$. The function $f(v)$ is the inverse of $p(u)$, so that, given our expression for $p(u)$, Eq. (3.11), we have

$$\phi_\omega^R(u,v)=-e^{-i\omega u}+e^{-i\omega V}\theta(-v) \ , \tag{3.19}$$

with V given by Eq. (2.4).

There is more arbitrariness in the choice of mode functions appropriate to $\mathcal{J}_L^+$, since there will be no actual observer stationed at the analogous position in the case of gravitational collapse. (This would involve an observer stationed along the horizon.) It proves convenient to select a set of modes complementary to those of Eq. (3.19) in the form

$$\phi_\omega^L(u,v)=e^{-i\omega W}\theta(v) \ , \tag{3.20}$$

where W is given by Eq. (2.8). Normalization of the modes ϕ_ω and ϕ_ω^I $(I=R,L)$ is discussed in the Appendix.

Suppose now that we expand ϕ in terms of the modes ϕ_ω^I:

$$\phi(u,v)=\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}[a_\omega^I\phi_\omega^I(u,v)+a_\omega^{I\dagger}\phi_\omega^{I*}(u,v)] \ . \tag{3.21}$$

The creation operators $a_\omega^{I\dagger}$ have an obvious interpretation in terms of outgoing particles propagating toward $\mathcal{I}_R^+$ or $\mathcal{I}_L^+$ (for $I=R$ and $I=L$, respectively). The Bogoliubov transformation describes the relationship between the operators a_ω and a_ω^I:

$$a_{\omega'}=\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}(\alpha^I_{\omega'\omega}a^I_\omega+\beta^I_{\omega'\omega}a^{I\dagger}_\omega)\ . \tag{3.22}$$

An explicit expression for the Bogoliubov coefficients $\alpha^I_{\omega'\omega}$ and $\beta^I_{\omega'\omega}$ may be obtained from the overlap of the mode functions ϕ_ω with the functions ϕ^I_ω, as described in the Appendix. There results the explicit expressions

$$\alpha^R_{\omega'\omega}=\frac{2\omega}{\kappa}e^{\pi\omega/2\kappa}\left[\frac{\omega'}{\kappa}\right]^{-i\omega/\kappa}\Gamma(i\omega/\kappa)\ , \tag{3.23}$$

$$\beta^R_{\omega'\omega}=-\frac{2\omega}{\kappa}e^{-\pi\omega/2\kappa}\left[\frac{\omega'}{\kappa}\right]^{i\omega/\kappa}\Gamma(-i\omega/\kappa)\ , \tag{3.24}$$

$$\alpha^L_{\omega'\omega}=\frac{2\omega}{\kappa}e^{\pi\omega/2\kappa}\left[\frac{\omega'}{\kappa}\right]^{i\omega/\kappa}\Gamma(-i\omega/\kappa)\ , \tag{3.25}$$

$$\beta^L_{\omega'\omega}=-\frac{2\omega}{\kappa}e^{-\pi\omega/2\kappa}\left[\frac{\omega'}{\kappa}\right]^{-i\omega/\kappa}\Gamma(i\omega/\kappa)\ . \tag{3.26}$$

The orthogonality and completeness relations appropriate to these Bogoliubov coefficients are also described in the Appendix.

The Bogoliubov coefficients (3.23)–(3.26) have appeared previously in the literature,[3] but in a subtly different context. Following Hawking, other authors have obtained these expressions as the approximate Bogoliubov coefficients for a trajectory which differs somewhat from our Eq. (3.12). There is, however, no difference in the two trajectories at late times, so the approximate Bogoliubov coefficients could legitimately be used to deduce the late-time properties of Hawking radiation. With our particular trajectory, the coefficients (3.23)–(3.26) are physically meaningful at all times.

The Bogoliubov transformation can be used to express the zero-particle state on $\mathcal{I}^-$, which we define to be the vacuum state $|\text{vac}\rangle$ of the scalar field ϕ, in terms of operators $a_\omega^{I\dagger}$ acting upon the zero-particle state on $\mathcal{I}^+$: $|0_R,0_L\rangle$. It is apparent from the form of Eq. (3.22), which mixes creation and annihilation operators, that $|\text{vac}\rangle$ and $|0_R,0_L\rangle$ cannot be identical. An explicit expression of $|\text{vac}\rangle$ in terms of $|0_R,0_L\rangle$ can be obtained if we can explicitly diagonalize the Bogoliubov transformation, Eqs. (3.23)–(3.26).

Considering the derivation of these equations, one realizes that this can easily be achieved. Suppose that instead of the mode functions (3.5), one were to expand ϕ in terms of functions

$$\Phi^R_\omega(u,v)=\frac{(-\kappa v+i\epsilon)^{i\omega/\kappa}-e^{-i\omega u}}{(1-e^{-2\pi\omega/\kappa})^{1/2}} \tag{3.27}$$

and

$$\Phi^L_\omega(u,v)=\frac{(-\kappa v+i\epsilon)^{-i\omega/\kappa}-e^{-i\omega u}}{(e^{2\pi\omega/\kappa}-1)^{1/2}}\ . \tag{3.28}$$

These modes are normalized in the same way as the ϕ_ω. Like the ϕ_ω, they describe positive-energy solutions of the Klein-Gordon equation. One can easily verify that, owing to the $i\epsilon$ prescription for passing the singularity at $v=0$, the Φ^I_ω are orthogonal to all negative-energy solutions $\phi^*_{\omega'}$, so the corresponding Bogoliubov transformation does not mix creation and annihilation operators. Let us now expand

$$\phi(u,v)=\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}[A^L_\omega\Phi^L_\omega(u,v)+A^{I\dagger}_\omega\Phi^{I*}_\omega(u,v)]\ . \tag{3.29}$$

The creation operators $A_\omega^{I\dagger}$ do not create simple plane waves upon $\mathcal{I}^-$, but they do create particles in the same sense as the $a^\dagger_\omega$, and we have

$$A_\omega\,|\,\text{vac}\rangle=0\ . \tag{3.30}$$

The overlap between the mode functions ϕ^I_ω and the functions $\Phi^I_{\omega'}$ is simple to evaluate and is diagonal in frequency labels ω and ω'. One finds that

$$A^R_\omega=\cosh(\theta)a^R_\omega-\sinh(\theta)a^{L\dagger}_\omega\ , \tag{3.31}$$

$$A^L_\omega=\cosh(\theta)a^L_\omega-\sinh(\theta)a^{R\dagger}_\omega\ , \tag{3.32}$$

where

$$\tanh\theta=e^{-\pi\omega/\kappa}\ . \tag{3.33}$$

Given this simple result, it is easy to construct a unitary transformation U such that

$$A=UaU^\dagger\ . \tag{3.34}$$

The explicit form of U is

$$U=\exp\left[\frac{\theta}{4\pi}\int\frac{d\omega}{\omega}(a^{R\dagger}_\omega a^{L\dagger}_\omega-a^R_\omega a^L_\omega)\right]\ . \tag{3.35}$$

Since, by definition,

$$a^L_\omega\,|\,0_R,0_L\rangle=0\ , \tag{3.36}$$

it follows from Eqs. (3.30) and (3.34) that we must have

$$|\,\text{vac}\rangle=U\,|\,0_R,0_L\rangle\ . \tag{3.37}$$

This exhibits the structure of the quantum state on $\mathcal{I}^+$, which has evolved from the initial state $|\text{vac}\rangle$ on $\mathcal{I}^-$. The transformation U describes a superposition of correlated left- and right-moving quanta. In the next section we will examine the structure of these correlations in more detail. We will also show that if one constructs a reduced density matrix, by tracing over the coordinates of the left-moving quanta, one obtains a simple thermal density matrix for the right-moving quanta.

IV. CORRELATIONS AND STIMULATED EMISSION

The structure of the quantum field on $\mathcal{I}^+$, as exhibited in Eq. (3.37), contains correlated left- and right-moving quanta. In this section we will explore some physical aspects of these correlations. The right-moving quanta are uncorrelated among themselves and have the apparent structure of a thermal ensemble. The quantum nature of these particles is underscored by the possibility of stimulated emission processes involving these particles. The correlations imply further that one can stimulate the emission of right-moving quanta through the addition to the system of left-moving quanta. In terms of the black-hole analogy, this means that one can stimulate Hawking radiation by dropping particles across the horizon. Neither type of stimulated emission process proves to be of great practical importance (as a means of extracting energy from the hole), but their existence is a necessity in any quantum model of black-hole decay.

Let us now consider the detailed structure of the state specified by Eq. (3.37). The operators a_ω^I and $a_{\omega'}^{I\dagger}$ which appear in Eq. (3.35), commute for $\omega'\neq\omega$. It follows that the state $U\,|\,0_R,0_L\,\rangle$ has the structure of a direct product of terms of states of the general form

$$|\,\Omega\,\rangle=\exp[\theta(a_R^\dagger a_L^\dagger-a_R a_L)]\,|\,00\,\rangle\ , \tag{4.1}$$

where the operators a_R and a_L correspond to operators a_ω^R and a_ω^L, respectively, averaged over some small frequency interval, and the state $|00\rangle$ designates a state annihilated by the operators a_R and a_L. The precise structure of the state (4.1) may be obtained by algebraic means. Let

$$X^+=a_R^\dagger a_L^\dagger \tag{4.2}$$

and

$$X^-=(X^+)^\dagger=a_R a_L\ . \tag{4.3}$$

Then

$$[X^+,X^-]=-2X_3\ , \tag{4.4}$$

where

$$X_3=\tfrac{1}{2}(a_L^\dagger a_L+a_R a_R^\dagger)=\tfrac{1}{2}(a_L^\dagger a_L+a_R^\dagger a_R+1)\ . \tag{4.5}$$

The commutators of the various X's close, with

$$[X_3,X^\pm]=\pm X^\pm\ . \tag{4.6}$$

One recognizes that Eqs. (4.4) and (4.6) define the Lie algebra of the group SU(1,1). Therefore, the transformation in Eq. (4.1) is simply an element of the group SU(1,1), and an explicit representation of the transformation can be displayed in terms of the representation matrices of this group.

By the identifications

$$X^+\sim iJ^+,\quad X^-\sim iJ^-,\quad X_3\sim J_3\ , \tag{4.7}$$

we can associate representations of SU(1,1) with infinite-dimensional representations of the group SU(2). This correspondence has

$$\begin{aligned}J^2&=\tfrac{1}{2}(J^+J^-+J^-J^+)+J_3{}^2\\&\sim-\tfrac{1}{2}(X^+X^-+X^-X^+)+X_3^2\\&=\tfrac{1}{4}(a_R^\dagger a_R+a_L^\dagger a_L)^2-a_R^\dagger a_L^\dagger a_R a_L-\tfrac{1}{4}\ .\end{aligned} \tag{4.8}$$

For the state $|00\rangle$, in particular, we have $J^2=j(j+1)=-\frac{1}{4}$, or $j=-\frac{1}{2}$.

Exploiting this correspondence further we can write

$$|\,\Omega\,\rangle=\sum_n C_n\,|\,nn\,\rangle\ , \tag{4.9}$$

with

$$C_n=\langle\,-\tfrac{1}{2},n+\tfrac{1}{2}\,|\,e^{2\theta J_2}\,|\,-\tfrac{1}{2},\tfrac{1}{2}\,\rangle\ , \tag{4.10}$$

in the language of SU(2), where $|\,jm\,\rangle$ denotes a state with $J^2=j(j+1)$ and $J_3=m$. Explicit evaluation of the C_n yields

$$C_n=\frac{(\tanh\theta)^n}{\cosh\theta}=(1-e^{-2\pi\omega/\kappa})^{1/2}e^{-\pi n\omega/\kappa}\ . \tag{4.11}$$

The density matrix $|\text{vac}\rangle\langle\text{vac}|$ can thus be written as a product of terms of the form

$$\rho=|\,\Omega\,\rangle\langle\,\Omega\,|=\sum_{mn}C_m C_n^*\,|\,mm\,\rangle\langle\,nn\,|\ . \tag{4.12}$$

An observer on $\mathscr{I}_R^+$ who can detect only right-moving quanta would describe his measurements in terms of a reduced density matrix:

$$\rho_R=\text{tr}_L\,\rho=\sum_m|\,C_m\,|^2\,|\,m\,\rangle\langle\,m\,|\ . \tag{4.13}$$

This reduced density matrix has the structure of a thermal density matrix with a temperature

$$T=\frac{\kappa}{2\pi}\ . \tag{4.14}$$

The absence of any correlations in ρ_R—and the absence of any off-diagonal terms—is a consequence of extracting the trace on left-moving coordinates in the construction of ρ_R. The density matrix ρ describes a pure state. The reduced matrix ρ_R describes only a portion of this state—precisely that portion of the state which is accessible to inclusive measurements carried out on $\mathscr{I}_R^+$.

Up to this point we have concentrated on how the initial state $|\text{vac}\rangle$ evolves in consequence of a moving boundary. What if the initial state were more complicated and contained additional quanta of the field ϕ? If Hawking radiation is viewed as spontaneously emitted quanta, then additional quanta in the initial state should be able to induce the stimulated emission of further quanta. This is indeed what happens, as was first noted by Wald.[7] Consider the state

$$\begin{aligned}|\,1_R\,\rangle&=\mathcal{N}a_R^\dagger\,|\,\Omega\,\rangle\\&=\mathcal{N}\sum_n C_n\sqrt{n+1}\,|\,n+1\ n\,\rangle\ .\end{aligned} \tag{4.15}$$

The normalization factor $\mathcal{N}$ is given by

$$\mathcal{N}^{-2}=\sum_n(n+1)C_n{}^2=(1-e^{-2\pi\omega/\kappa})^{-1}\ . \tag{4.16}$$

The average number of right-moving quanta in this state is

$$\begin{aligned}N(1_R)=\mathcal{N}^2\sum_n(n+1)^2C_n{}^2&=\frac{1+e^{2\pi\omega/\kappa}}{e^{2\pi\omega/\kappa}-1}\\&=1+\frac{2}{e^{2\pi\omega/\kappa}-1}\ .\end{aligned} \tag{4.17}$$

This result may be compared with the average number of right-movers in the state $|\Omega\rangle$:

$$N(\Omega)=\sum_n nC_n{}^2=\frac{1}{e^{2\pi\omega/\kappa}-1} \ . \tag{4.18}$$

This corresponds to the usual thermal population of quanta. The difference

$$N(1_R)-N(\Omega)=1+\frac{1}{e^{2\pi\omega/\kappa}-1} \ , \tag{4.19}$$

involves two terms. The first represents the quantum added explicitly in Eq. (4.15) to form the state $|1_R\rangle$. The second term is proportional to $N(\Omega)$ itself and represents the effect of stimulated emission.

Owing to the correlations in the state $|\Omega\rangle$, it is also possible to stimulate the emission of R quanta by adding L quanta to the state. Consider

$$|1_L\rangle=\mathcal{N}a_L^\dagger|\Omega\rangle=\mathcal{N}\sum_n C_n\sqrt{n+1}\,|n\ \ n+1\rangle \ . \tag{4.20}$$

The average number of R quanta in this state is

$$N(1_L)=\mathcal{N}^2\sum_n n(n+1)C_n{}^2=\frac{2}{e^{2\pi\omega/\kappa}-1} \ . \tag{4.21}$$

The difference

$$N(1_L)-N(\Omega)=\frac{1}{e^{2\pi\omega/\kappa}-1} \tag{4.22}$$

is again proportional to $N(\Omega)$ and can be interpreted, like the second term in Eq. (4.19), as the consequence of a stimulated emission process.

It would appear that these stimulated emission processes might provide an efficient means of extracting energy from a black hole. The analog of left-moving quanta in the mirror system are quanta which cross the horizon and disappear into the black hole. Hence we have shown that by adding quanta to the hole one can actually increase the rate at which quanta are emitted from the hole. In Wald's analysis of stimulated emission, he showed that the quanta which must be used to stimulate late-time emission from the hole (late u in our model) must be infinitesimally close to v_0. Equation (2.6) shows that to stimulate the emission of a packet of width Δu, one must send in a packet of much narrower width Δv. The frequencies contained in the incident packet are thus much higher than those in the emitted packet. In this manner the red-shift which is induced by the black hole mitigates against stimulated emission as a viable means of energy extraction.

Wald considered only incident quanta which could themselves escape from the hole. In the preceding discussion we have pointed out a second type of stimulated emission process, which involves incident quanta which fall into the hole. Owing to the symmetry between L and R quanta in our model, we expect that in order to stimulate late-time (or large-u) emission, we will still require incident quanta which are close to v_0 (only with v less than v_0 in this case). Wald's argument about dominant frequencies should still apply, and our process of stimulated emission should be of no more practical value than the conventional one considered by Wald. We will demonstrate this in the following section, where we compute the spacetime correlation functions for certain physical observables. The structure of these correlation functions will help to reinforce some of the intuitive ideas that we have been trying to develop in this paper.

V. CORRELATION FUNCTIONS

Knowledge of the mode functions [Eq. (3.5)] for the moving mirror problem allows one to construct the two-point correlation functions, or propagators, for the quantum field ϕ. Since ϕ is a free field, albeit on a dynamical background, these propagators are sufficient to determine the correlation functions for any operators constructed from ϕ. In this section we use this observation to evaluate correlations of the stress-energy tensor at different spacetime points. These correlations underscore the correlations between left-moving and right-moving quanta which were discussed in the previous section.

The stress-energy tensor for the field ϕ is defined by

$$T_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-\tfrac{1}{2}g_{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi \ . \tag{5.1}$$

We would like to evaluate the correlation function

$$C_{\mu\nu,\mu'\nu'}=\langle T_{\mu\nu}(u,v)T_{\mu'\nu'}(u'v')\rangle - \langle T_{\mu\nu}(u,v)\rangle\langle T_{\mu'\nu'}(u',v')\rangle \ . \tag{5.2}$$

The expectation values $\langle T_{\mu\nu}(u,v)\rangle$ have been calculated previously,[3] with the result (3.9). Given a trajectory specified by the function (3.11), Eq. (3.9) becomes

$$\langle T_{uu}(u,v)\rangle=\frac{\kappa^2}{48\pi} \ , \tag{5.3}$$

which corresponds to a constant flux of particles leaving the surface of the accelerated mirror. These are the right-moving quanta of the previous section. Note that

$$\langle T_{vv}\rangle=0 \ ,$$

so there is no physical energy-flux associated with our left-moving quanta. They correspond only to vacuum fluctuations, which are subtracted in defining the expectation value $\langle T_{\mu\nu}\rangle$. This is not to say that the left-moving quanta do not exist, and it is the purpose of our calculation of $C_{\mu\nu,\mu'\nu'}$ to help illustrate the physical role that they play in the model.

The computation of $C_{\mu\nu,\mu'\nu'}$ is actually simpler than that of $\langle T_{\mu\nu}\rangle$ since there is no vacuum subtraction required to define the correlation function. For an arbitrary mirror trajectory the two-point function

$$D^+(u,v;u',v')=\langle \mathrm{vac}\,|\,\phi(u,v)\phi(u',v')\,|\,\mathrm{vac}\rangle \tag{5.4}$$

has the form[3]

$$D^+(u,v;u',v') = -\frac{1}{4\pi}\ln\frac{[p(u)-p(u')-i\epsilon][v-v'-i\epsilon]}{[v-p(u')-i\epsilon][p(u)-v'-i\epsilon]} \ . \tag{5.5}$$

The correlation function $C_{\mu\nu,\mu'\nu'}$ is given by the Feynman diagram illustrated in Fig. 2. Disconnected dia-

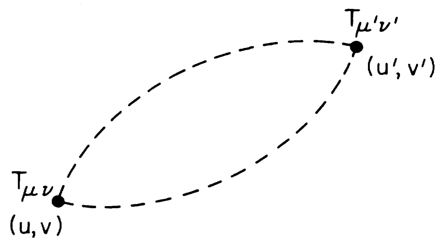

FIG. 2. Feynman diagram for the correlation function $C_{\mu\nu,\mu'\nu'}(u,v;u',v')$.

grams, which would require regularization, are eliminated by the subtraction of $\langle T_{\mu\nu}\rangle\langle T_{\mu'\nu'}\rangle$ in the definition of $C_{\mu\nu,\mu'\nu'}$ [Eq. (5.2)].

Consider first the quantity $C_{uu,uu}$. Given the trajectory (3.12) one readily finds that

$$C_{uu,uu}(u,v;u',v')=\frac{\kappa^4}{16\pi^2}(e^{\kappa(u-u')/2}-e^{\kappa(u'-u)/2})^{-4}\ . \tag{5.6}$$

This is precisely the correlation that is found in a two-dimensional thermal field theory. At short distances $(u\to u')$ $C_{uu,uu}$ displays the typical singularity of a free field theory. This strong correlation is damped out over a distance

$$\Delta u\sim 1/\kappa \tag{5.7}$$

characteristic of the dominant wavelengths in a heat bath of temperature $\kappa/2\pi$.

We have emphasized that our moving mirror model mimics a system of temperature $\kappa/2\pi$ only insofar as we restrict attention to measurements of right-moving quanta. The result (5.6) is consistent with this viewpoint. To delve into left-right correlations we will now examine $C_{uu,vv}$, which has the structure

$$C_{uu,vv}=\frac{\kappa^2}{16\pi^2 v'^2}(e^{\kappa(V'-u)/2}-e^{\kappa(u-V')/2})^{-4}\ , \tag{5.8}$$

in the region $v'<0$. This expression has a singularity of the free field type at $u=V'$ which corresponds to particles emitted at (u',v') which can reflect from the moving boundary to reach the point (u,v). There are also significant correlations for $v'>0$. Indeed, if we analytically continue Eq. (5.8) to the region $v'>0$, we obtain

$$C_{uu,vv}=\frac{\kappa^2}{16\pi^2 v'^2}(e^{\kappa(W'+u)/2}+e^{-\kappa(W'+u)/2})^{-4}\ , \tag{5.9}$$

which shows that $C_{uu,vv}$ is maximal at $u=-W'$. These correlations are a direct consequence of the correlations between left- and right-moving quanta which were emphasized in the previous section. Indeed one could have derived them directly from the structure of Eq. (3.37).

Note that the correlations in Eqs. (5.8) and (5.9) extend over a range in V' or W' of order κ^{-1}. This implies that quanta emitted at late times (or large u) are correlated with incident quanta very close to $v=v_0=0$. This substantiates the argument of the previous section to the effect that the stimulated emission of a quantum at large u requires incident quanta of very large frequency in a packet very close to $v=0$. As u increases, the incident packet (as a function of v) narrows by a factor $e^{-\kappa u}$ and the requisite frequencies increase by a factor $e^{\kappa u}$ [see Eq. (2.6)].

VI. CONCLUSION

In the preceding sections we have reviewed the structure of a two-dimensional field theory formulated on a flat spacetime with a moving boundary. This theory models many aspects of the Hawking radiation which accompanies gravitational collapse. We have emphasized the nature of the correlations induced by the moving boundary (or dynamical gravitational field). This feature of the quantum field will form the basis of a sequel[4] in which we discuss the possible evolution of pure states into mixed states under the influence of a black hole. It should be clear that in the present paper the state of the quantum field remains a pure state—even though right-moving quanta of the field do exhibit many features of a thermal state. The right movers are correlated not among themselves, but each right mover is correlated with some left-moving quantum.

These correlations are in some ways paradoxical. We found that $C_{uu,vv}$ is nonzero, even though $\langle T_{vv}\rangle$ itself vanishes. The nonvanishing of $C_{uu,vv}$ for $v'>0$ can be presented as an example of the Einstein-Podolsky-Rosen effect. The correlation persists between spacetime points (u,v) and (u',v') which need not themselves be causally connected. Both points, however, do contain the mirror trajectory in their past light cones; and it is through this common influence that the mutual correlation results.

In this paper we have restricted our attention to the trajectory (3.12) which results from the ray-tracing function (3.11). This restriction was made in the interest of simplicity, but, given the results of the previous section, it can easily be relaxed. The width in u of the correlation exhibited in Eqs. (5.6) and (5.9) is of order κ^{-1}, where κ is the acceleration of the moving mirror. This means that if $\partial_u \ln p'(u)$ varies on a time scale which is long compared to κ^{-1}, then it is still reasonable to approximate the local mirror trajectory in the form of Eq. (3.12), and to apply the intuition developed in this paper. This approach is valid for a more realistic model of the Hawking process, where the Hawking temperature increases as the mass of the black hole decreases. It can even be applied to models for the final stages of decay of a black hole. We reserve these extensions, and our speculations on this interesting topic, for a separate paper.

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APPENDIX: NOTATION AND CONVENTIONS

Throughout the text of this manuscript we deal with a scalar field $\phi(u,v)$ defined on a flat two-dimensional spacetime. The equation of motion for this field is the Klein-Gordon equation

$$\partial_u \partial_v \phi(u,v)=0 , \tag{A1}$$

whose solutions are arbitrary functions of the null coordinates u or v. The boundary of the two-dimensional space asserts itself by requiring ϕ to vanish along the moving wall. This forces us to use particular linear combinations of functions of u and functions of v, as in Eq. (3.5).

These mode functions are normalized so that on the Cauchy surface $\mathcal{J}^-$ we have

$$i \int_{-\infty}^{\infty} dv\, \phi^*_{\omega'}(u,v)\overleftrightarrow{\partial}_v \phi_\omega(u,v)=4\pi\omega\delta(\omega'-\omega) . \tag{A2}$$

If we expand the field ϕ in terms of these mode functions,

$$\phi(u,v)=\frac{1}{4\pi}\int_0^\infty \frac{d\omega}{\omega}[a_\omega\phi_\omega(u,v)+a^\dagger_\omega\phi^*_\omega(u,v)] , \tag{A3}$$

then the creation and annihilation operators $a^\dagger_\omega$ and a_ω will obey the standard commutation relations

$$[a_\omega,a^\dagger_{\omega'}]=4\pi\omega\delta(\omega'-\omega) , \tag{A4}$$

$$[a_\omega,a_{\omega'}]=[a^\dagger_\omega,a^\dagger_{\omega'}]=0 . \tag{A5}$$

The mode functions (3.5) were chosen to provide a simple plane-wave representation of the field on the Cauchy surface $\mathcal{J}^-$. On the surface $\mathcal{J}^+=\mathcal{J}^+_L\cup\mathcal{J}^+_R$, a simpler representation would involve the functions $\phi^L_\omega(u,v)$ and $\phi^R_\omega(u,v)$ given in Eqs. (3.19) and (3.20). These functions are normalized so that, on the surface $\mathcal{J}^+$,

$$i\int_0^\infty dv\, \phi^{I*}_{\omega'}(u,v)\overleftrightarrow{\partial}_v\phi^J_\omega(u,v)+i\int_{-\infty}^\infty du\,\phi^{I*}_{\omega'}(u,v)\overleftrightarrow{\partial}_u\phi^J_\omega(u,v)=4\pi\omega\delta(\omega'-\omega)\delta^{IJ} . \tag{A6}$$

The first integral in this expression refers to the surface $\mathcal{J}^+_L$ and the second to the surface $\mathcal{J}^+_R$. An expansion of the field ϕ in terms of the mode functions $\phi^I_\omega(u,v)$ gives

$$\phi(u,v)=\frac{1}{4\pi}\sum_I\int_0^\infty\frac{d\omega}{\omega}[a^I_\omega\phi^I_\omega(u,v)+a^{I\dagger}_\omega\phi^{I*}_\omega(u,v)] . \tag{A7}$$

The creation and annihilation operators which appear in this expansion obey the commutation relations

$$[a^I_\omega,a^J_{\omega'}]=4\pi\omega\delta(\omega-\omega')\delta^{IJ} , \tag{A8}$$

$$[a^I_\omega,a^J_{\omega'}]=[a^{I\dagger}_\omega,a^{J\dagger}_{\omega'}]=0 . \tag{A9}$$

The Bogoliubov coefficients $\alpha^I_{\omega'\omega}$ and $\beta^I_{\omega'\omega}$ relate the operators $a_{\omega'}$ and $a^\dagger_{\omega'}$ to the operators a^I_ω and $a^{I\dagger}_\omega$:

$$a_{\omega'}=\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}(a^I_{\omega'\omega}a^I_\omega+\beta^I_{\omega'\omega}a^{I\dagger}_\omega) . \tag{A10}$$

The expansions (A3) and (A7) and the orthogonality condition (A2) provide us with explicit expressions for the Bogoliubov coefficients:

$$\alpha^I_{\omega'\omega}=i\int_{-\infty}^\infty dv\,\phi^*_{\omega'}(u,v)\overleftrightarrow{\partial}_v\phi^I_\omega(u,v) , \tag{A11}$$

$$\beta^I_{\omega'\omega}=i\int_{-\infty}^\infty dv\,\phi^*_{\omega'}(u,v)\overleftrightarrow{\partial}_v\phi^{I*}_\omega(u,v) , \tag{A12}$$

with integrals taken along the surface $\mathcal{J}^-$. The orthogonality condition (A6) provides us with a means of inverting the Bogoliubov transformation. Equivalently, one deduces that

$$\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}(a^I_{\omega'\omega}\alpha^{I*}_{\omega''\omega}-\beta^I_{\omega'\omega}\beta^{I*}_{\omega''\omega})=4\pi\omega'\delta(\omega''-\omega') , \tag{A13}$$

$$\frac{1}{4\pi}\sum_I\int\frac{d\omega}{\omega}(\alpha^I_{\omega'\omega}\beta^I_{\omega''\omega}-\beta^I_{\omega'\omega}\alpha^I_{\omega''\omega})=0 \tag{A14}$$

and that

$$\frac{1}{4\pi}\int\frac{d\omega'}{\omega'}(\alpha^{I*}_{\omega'\omega}\alpha^J_{\omega'\omega''}-\beta^I_{\omega'\omega}\beta^{J*}_{\omega'\omega''})=4\pi\omega\delta(\omega-\omega'')\delta^{IJ} , \tag{A15}$$

$$\frac{1}{4\pi}\int\frac{d\omega'}{\omega'}(\alpha^{I*}_{\omega'\omega}\beta^J_{\omega\omega''}-\beta^I_{\omega'\omega}\alpha^{I*}_{\omega\omega''})=0 . \tag{A16}$$

These relations can be verified if one uses the explicit forms of $\alpha^I_{\omega'\omega}$ and $\beta^I_{\omega'\omega}$ given in Eqs. (3.23)–(3.26).

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