Magnetic moments of neutrinos: Particle and astrophysical aspects

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We discuss some observational consequences of finite values of neutrino magnetic moments and flavor-changing transition moments (which are expected in some extensions of the standard electroweak model). These include effects on solar and supernova neutrino emission, and contributions to the electromagnetic background in galaxies and clusters of galaxies. To avoid conflict with observational data, transition moments (in units of the Bohr magneton) cannot exceed 3×10^{-15} for neutrino masses m_{ν_i} above 16 eV, and 6×10^{-14} for $4\le m_{\nu_i}\le16$ eV.

I. INTRODUCTION

Neutrinos can have electric charge radii and anomalous electric dipole moments

$$\mu(\nu_i)\equiv\chi(\nu_i\nu_i)\mu_e^B , \tag{1a}$$

or transition moments

$$\mu(\nu_i\nu_j)\equiv\chi(\nu_i\nu_j)\mu_e^B , \tag{1b}$$

where μ_e^B is the Bohr magneton. In renormalizable electromagnetic and weak gauge models, these parameters—representing nonminimal couplings—are calculable and finite. The fairly large $\langle r^2\rangle_{\nu_e}$ is part of the $O(\alpha)$ radiative corrections to $\nu_e e$ scattering.[1] The imminent measurement of such corrections will provide meaningful tests of the standard Glashow-Weinberg-Salam (GWS) model. This model predicts, however, miniscule magnetic moments for the light, stable neutrinos:[2]

$$\chi^{\rm st}(\nu_i\nu_i)\simeq\frac{m(\nu_i)G_F}{\sqrt{2}8\pi^2}\simeq3.2\times10^{-19} . \tag{2}$$

These values fall much below the sensitivity of direct $\nu_e e$ (or $\nu_\mu e$) scattering experiments, which is at the $10^{-10}-10^{-9}$ level and could conceivably be improved by an order of magnitude.[3]

The anomalous magnetic term

$$\frac{\chi(\nu_i\nu_i)}{2m_e}\Psi(\nu_i)\sigma^{\alpha\beta}\partial_\beta\Psi(\nu_i) \tag{3}$$

causes a $L\rightarrow R$ helicity flip. Since the right-handed neutrinos do not participate in the standard $V-A$ interactions, such flips could have astrophysical consequences.[4] Also, if $m(\nu_j)\neq0$ and the ν_j form (or are part of) the dark matter, then $\chi(\nu_i\nu_j)\neq0$ induce observable radiative $\nu_j\rightarrow\nu_i+\gamma$ decays. As will be discussed below, the broad range of $10^{-15}\le\chi(\nu_i\nu_i)$ [or $\chi(\nu_i\nu_j)$] $\le10^{-9}$ can be relevant to various astrophysical phenomena. Values of χ in the range $10^{-12}-10^{-11}$ can arise in extensions of the standard model. Interest in $\chi(\nu_e\nu_e)\neq0$ intensified recently due to its possible relevance to neutrino emission from supernovas. It is therefore useful to discuss the possibility that $\chi\neq0$ in a broader context, as we do here.

II. ASTROPHYSICAL CONSEQUENCES OF FINITE NEUTRINO MAGNETIC MOMENT

A. Effect on the solar-neutrino puzzle

The strong anticorrelation between solar ν_e counts and the sun-spot activity[5] led to the suggestion that $\chi(\nu_e\nu_e)\neq0$. Left-handed electron neutrinos can then precess while traversing the magnetic field B in the convective zone (of size L) into the sterile ν_e^R which is undetectable in the Davis[6] chlorine experiment. If $\chi(\nu_e\nu_e)\simeq10^{-10}$ is properly "tuned" to the strength $B\simeq10^4$ G and extent $L\simeq10^{10}$ cm of the solar field, then at solar maximum the precession angle is $\theta\simeq\mu_e B\sin\alpha L\simeq3\sin\alpha$ (where α is the angle between the directions of B and the neutrino motion) and the ν_e^L flux is strongly suppressed. This effect—unlike ν flavor mixings[7]—is energy independent, so that identical suppressions are expected in both the chlorine and (proposed) galium experiments. Also, the neutrino flux should show semiannual variation due to the dependence on $\sin\alpha$ and the angle between the solar rotation plane and the ecliptic.[5] If these predictions will be verified, and the correlation of suppressed neutrino counts with solar magnetic activity will repeat over the next 11-yr solar cycle, then $\chi(\nu_e\nu_e)\simeq10^{-11}$ would seem very likely.

B. Effect on neutrino emission from supernova

A nonvanishing ν_e magnetic moment could have dramatic effects on neutrino emission in supernova collapse.[8] The issue is of particular interest due to the recent detections[9,10] of ν bursts from the supernova 1987A in the Large Magellanic Cloud (LMC). The squeezing of flux in the progenitor star during the collapse can produce very intense magnetic fields, $B=O(10^{12}$ G), in the $R\simeq10^6$ cm core and in its immediate vicinity. This could then produce a sizable L-R precession:

$$\theta\simeq\mu(\nu_e\nu_e)RB\simeq3\times10^{14}\chi(\nu_e\nu_e)B_{12}R_6 , \tag{4}$$

where B_{12} and R_6 are the values of B and R in units of 10^{12} G and 10^6 cm, respectively. This precession occurs near the dense core but not within it, where it is quenched by the large interaction energy difference of ν_e^L and ν_e^R with nucleons and electrons.[8] Since for $B=O(10^{12}\text{ G})$ and $\chi(\nu_e\nu_e)\geq 10^{-14}$ we will have large precessions (which may vary with B over the surface of the core), we expect an equal admixture of L and R neutrinos to emerge from the star.

Thus, tiny neutrino magnetic moments may lower by a factor of 2 the ν_e and $\bar{\nu}_e$ flux. The observed neutrino flux from SN1987A is somewhat on the high side compared to what is allowed by the total gravitational energy released[11] in collapse to a standard neutron star. Also, a particularly high flux of neutronization neutrinos is indicated by the first two directional events observed at Kamiokande.[9] It has been argued that the high flux may suggest that the SN1987A collapse was into a black hole.[12] Nonetheless, if a remnant neutron star will be found, e.g., through its x-ray emission, then from the above consideration it will follow that

$$\chi(\nu_e\nu_e)\leq 3\times 10^{-15}B_{12}{}^{-1}\ , \tag{5}$$

so as to avoid the need for doubling the intrinsic ν_e and $\bar{\nu}_e$ luminosity.

Dar has proposed[8] that the ν_e and $\bar{\nu}_e$ signals may be enhanced by the interaction of the neutrinos with the core magnetic field: An initial ν_e^L is flipped into ν_e^R via the incoherent scattering $\nu_e^L+e\to\nu_e^R+e$ in the center of the core. The ν_e^R escape immediately, but about half of them are back flipped into ν_e^L when traversing the strong magnetic field outside the core. This allows energetic ν_e's, which are more easily detectable, to escape early (within 0.1 sec) before they thermalize. The cross section for the flip scattering is

$$\sigma_f(\nu_e^Le\to\nu_e^Re)\simeq\frac{\pi\alpha^2\chi^2(\nu_e\nu_e)}{m_e{}^2}\ , \tag{6}$$

so that $\sigma_f\simeq 1.6\times 10^{-47}[10^{11}\chi(\nu_e\nu_e)]^2\text{ cm}^2$. The electron density in the core, about $3\times 10^{37}\text{ cm}^{-3}$, determines a mean free time between successive flip collisions:

$$t_f=\frac{l_f}{c}\simeq 6\times 10^{-2}[10^{11}\chi(\nu_e\nu_e)]^{-2}\text{ sec}\ . \tag{7}$$

Demanding $t_f\leq 0.1-10$ sec typical thermalization (trapping) times, and $t_f\geq 3\times 10^{-5}$ sec, the escape time of ν^R from the core (to avoid back flips $\nu^R\to\nu^L$ within the core) fixes then the range for the Dar effect $1\times 10^{-12}\leq\chi(\nu_e\nu_e)\leq 4\times 10^{-10}$. This is also the range of values of $\chi(\nu_e\nu_e)$ required to explain the solar-neutrino problem.

C. Effect on nucleosynthesis

The flip reaction, Eq. (6), keeps the right-handed neutrinos coupled to the rest of the primordial radiation as long as the reaction rate $\sigma_f n_e c$, where $n_e\simeq 0.36T^3$, exceeds the expansion rate (m_P is the Planck mass)

$$H=\left[\frac{8\pi G_N\rho}{3}\right]^{1/2}=\left[\frac{8\pi N_fT^4}{90m_P{}^2}\right]^{1/2}. \tag{8}$$

Taking $N_f\simeq 30$ for the number of degrees of freedom, we find, from Eq. (6),

$$T_d\simeq 2\times 10^4[10^{11}\chi(\nu_e\nu_e)]^{-2}\text{ MeV}\ . \tag{9}$$

Since $T_d\geq 200$ MeV for $\chi(\nu_e\nu_e)\leq 10^{-10}$, the number density of the various right-handed neutrino species is small at the time of nucleosynthesis when $T\simeq 1$ MeV. In the 200–1 MeV era the s-, u-, d-quark and μ lepton species annihilate into ν_1^L, $\bar{\nu}_1^L$, e^-, e^+, and γ, so we expect that $n(\nu_i^L)/n(\nu_i^R)\simeq 4$, so all the ν_1^R together will not affect helium synthesis more than one additional light neutrino.[13]

The effect of $\nu^L\to\nu^R$ could, however, be much stronger if there were primordial magnetic fields. Such fields could have induced $\nu_i^R\leftrightarrow\nu_i^L$ precessions

$$\theta=B_p\mu(\nu_i\nu_i)1\simeq B_p\mu(\nu_i\nu_i)ct\ , \tag{10}$$

where we took the coherence scale 1 of B_p to be the horizon size, $ct\simeq 3\times 10^{12}$ cm at 1 MeV. If $B_p\geq 3\times 10^3$ G, then $\chi(\nu_i\nu_i)\geq 10^{-12}$ suffices to keep ν_i^R in thermal equilibrium, increasing by 1 the number of light species, and $\chi(\nu_i\nu_i)\geq 10^{-12}$ for all species would then be ruled out. Note that $B_p\simeq 3\times 10^3$ G at $T=1$ MeV would adiabatically decrease to $B\simeq B_p(T_0/T)^2\simeq 3\times 10^{-14}$ G at the present epoch (T_0), a value which is certainly consistent with the established bounds on the value of the intergalactic field.[14] There is, however, no compelling argument for large scale fields in the early Universe, and the origin and evolution of such fields are unknown. In fact, knowledge of the neutrino magnetic moment can perhaps shed some light on the issue.

III. OFF-DIAGONAL MAGNETIC MOMENTS

If the new interactions giving rise to $\chi(\nu_i\nu_i)\neq 0$ do not conserve separately e, μ, and τ lepton numbers, then off-diagonal $\chi(\nu_i\nu_j)\neq 0$ are expected. For self-charge-conjugate Majorana neutrinos, all flavor-diagonal matrix elements of the electromagnetic current vanish, $\langle\nu_i\,|\,J_\mu^{\text{em}}\,|\,\nu_i\rangle\equiv 0$. Only some of the off-diagonal transition moments could be finite. Some indication for the need to utilize flavor-nonconserving new interaction comes from the large value, $\chi(\nu_e\nu_e)\simeq 10^{-10}-10^{-11}$, which may be required for resolution of the solar-neutrino problem. A simple approach to avoid the neutrino mass factor in Eq. (2), from the GWS model, is to consider a L-R-symmetric extension[15] so that the $L\to R$ flip can arise by utilizing the mass of the intermediate charge lepton. One then finds

$$\chi(\nu_e\nu_e)=\frac{G_Fem_e\sin(2\xi)}{2\sqrt{2}\pi^2}\ , \tag{11}$$

with $\sin(2\xi)\leq 0.1$ being W_L-W_R mixing. Equation (11) still yields far too small electron-neutrino magnetic moment, $\chi(\nu_e\nu_e)\leq 10^{-14}$.

If, however, we had a new interaction—as in the Fukugita-Yanagida (FY) model[16]—with strong lepton flavor violation, we could utilize m_τ instead of m_e, thereby enhancing $\chi(\nu_e\nu_e)$ by about 3.5 orders of magnitude. For all the above reasons, and for its own right, we will explore in the following some of the possible effects of $\chi(\nu_i\nu_j)\neq 0$. These include, in particular, radiative decays of the heavier neutrino species $\nu_i \to \nu_j + \gamma$ at a rate

$$\Gamma = \frac{\alpha \chi^2(\nu_i\nu_j)}{8m_e^2}\left[\frac{m_{\nu_i}{}^2 - m_{\nu_j}{}^2}{m_{\nu_i}}\right]^3 , \tag{12}$$

so that $\Gamma \propto \chi^2(\nu_i\nu_j) m_{\nu_i}{}^3$ for $m_{\nu_i} \gg m_{\nu_j}$.

We now obtain astrophysical bounds on $\chi(\nu_i\nu_j)$ using Eq. (12) for the neutrino radiative decay rate. We first distinguish between heavy neutrinos of mass higher than ~ 1 MeV that decouple in the early Universe, when they are nonrelativistic, and lighter neutrinos that decouple when still relativistic. Because of the m^3 dependence in the expression for Γ, the decay time is predicted to be very short for the former. The decay photons would have thermalized, causing some heating of the thermal radiation field, but with no other characteristic consequences. We concentrate, therefore, on the more interesting possibility that the surviving neutrino species are light, $m_{\nu_i} \ll 1$ MeV. The present cosmological density n of an unclustered neutrino species is[17] 110 cm^{-3}. If neutrinos contribute appreciably to the mass density of the Universe, then they are bound in clusters of galaxies, perhaps even within galaxies. The mass density in the Universe (in units of the closure density) $\Omega \leq 1$; therefore, the well-known Cowsik-McClelland[18] bound yields $\sum m_{\nu_i} \leq 100$ eV for the combined masses of all neutrino species. Analysis of the time of arrivals of neutrino signals from the recent SN1987A suggest $m_{\nu_e} \leq 10$ eV.[19] No direct limits are set on the masses of the u and τ neutrinos.

Radiative decays of cosmological neutrinos could have significantly affected the electromagnetic background, as well as the emissivity of galaxies and clusters in various wavelength regions. Observations set relatively tight limits on the radiative decay rates, which we now translate to bounds on the value of $\chi(\nu_i\nu_j)$. Consider first neutrinos of mass ≥ 27.2 eV whose decay photons can ionize hydrogen. Measurements of the 21-cm (hyperfine) radiation from neutral hydrogen 30 kpc from the center of the nearby galaxy M31 set a limit on the photoionizing flux there.[20] This limit was then used by Rephaeli and Szalay[21] to set a lower limit on the neutrino lifetime. Adopting their analysis, we find (taking $H_0 = 50$ km sec^{-1}Mpc^{-1}) that $\Gamma \leq (0.4-1)\times 10^{-24}$ sec^{-1}, for $m_{\nu_i} = 30-200$ eV, respectively. From Eq. (12), we then have $\chi(\nu_i\nu_j) \leq 1.7\times 10^{-15}$ for $m_{\nu_i} = 30$ eV, and 1.5×10^{-16} for $m_{\nu_i} = 200$ eV. A somewhat stronger limit results if we take account of the fact that neutrinos in this mass range can easily cluster in the local supercluster.

We next consider limits on Γ from observations of clusters of galaxies. These are based on the assumption that massive neutrinos constitute the dynamically deduced mass in clusters. The flux of red-shifted photons from decay of neutrinos in the mass range 20–30 eV is constrained[22] by ultraviolet observations of Virgo and the Coma clusters of galaxies.[23,24] A rough lower limit is[23] $\Gamma \leq 1\times 10^{-23}$ sec^{-1}, so that $\chi(\nu_i\nu_j) \leq 2\times 10^{-14}$. The ultraviolet measurements of Virgo by Henry and Feldman directly yield $\Gamma \leq 2\times 10^{-25}$ sec^{-1} for $m_{\nu_i} = 16-20$ eV (Ref. 24) leading to $\chi(\nu_i\nu_j) \leq 3\times 10^{-15}$. A set of optical and ultraviolet observations analyzed by Shipman and Cowsik[23] covers the approximate range 4–16 eV. Radiative decays of neutrinos of mass in the range 8–16 eV will conflict with observations of the Virgo cluster unless $\Gamma \leq 1\times 10^{-23}$ sec^{-1} (taking 20 Mpc for the distance to Virgo), giving $\chi(\nu_i\nu_j) \leq 6\times 10^{-14}$. In the mass range 4–8 eV, the upper limit on radiation from Coma (mass $\simeq 3\times 10^{15} M_\odot$) requires $\Gamma \leq 2.5\times 10^{-25}$ sec^{-1}, and thus $\chi(\nu_i\nu_j) \leq 2.7\times 10^{-14}$.

If neutrino masses are lower than a few eV, neutrinos do not dominate the mass density of the Universe, and in any case are not expected to be bound in clusters of galaxies. Thus, radiative bounds necessarily involve diffuse cosmic background radiations at energies of 1 eV or lower. The infrared background is very poorly known; consequently, only weak limits can be set on the neutrino decay lifetime, $\sim 10^{20}$ sec or lower.[22] Because $\Gamma \propto m_{\nu_i}{}^3$, the corresponding bounds on $\chi(\nu_i\nu_j)$ are at best $O(10^{-10})$ or lower. One of the goals of NASA's COBE mission[25] (scheduled to be launched in 1989) is the measurement of the diffuse infrared background. Significantly better limits on Γ and $\chi(\nu_i\nu_j)$ may then be obtained.

Nonvanishing neutrino transition moments can affect supernova neutrinos and, in general, will limit the effectiveness of the mechanism proposed by Dar[8] for the early escape of energetic ν_e^L out of the core. Finite values of $\chi(\nu_e\nu_\mu)$ and $\chi(\nu_e\nu_\tau)$ can lead to the flip reactions $\nu_e^L \to \nu_\mu^R$ or ν_τ^R, just as $\chi(\nu_e\nu_e)\neq 0$ induces $\nu_e^L \to \nu_e^R$ [see Eq. (6)]. However, unless the mass differences $\Delta m^2(\nu_\mu\nu_e) = m_{\nu_\mu}{}^2 - m_{\nu_e}{}^2$ or $\Delta m^2(\nu_\tau\nu_e)$ are very small, the slowly varying magnetic field outside the core will simply rotate $\nu_\mu^R \to \nu_\mu^L$ (or $\nu_\tau^R \to \nu_\tau^L$), but will not induce[26] the energy changing ν_μ^R (or ν_τ^R)$\to \nu_e^L$. But since initially all the neutrinos are ν_e^L, the flavor and helicity changes result in a reduction of the ν_e^L flux by the factor

$$r = \frac{\chi^2(\nu_e\nu_e)}{\chi^2(\nu_e\nu_e) + \chi^2(\nu_e\nu_\mu) + \chi^2(\nu_e\nu_\tau)} . \tag{13}$$

Therefore, if $\chi(\nu_e\nu_e) \ll \chi(\nu_e\nu_\mu)$ [or $\chi(\nu_e\nu_\tau)$], then the Dar mechanism will suppress, rather than enhance, the initial ν_e flux from neutronization of the core. As we have already mentioned, the detected neutrino flux from SN1987A may be too large to be consistent with collapse into a neutron star. If a neutron star will eventually be found, the $r \simeq 1$ will be required, and transition moments larger than $\chi(\nu_e\nu_e)$ would be excluded. Transition moments do not strongly affect the escape of ν_e (or $\bar{\nu}_e$) from a thermalized neutrino distribution (with roughly equal populations of all neutrino species) which is established at a later time. However, if we wish to optimize the flux of thermal ν_e or ν_e (rather than the other species) via the Dar mechanism, then the values of the diagonal moments should satisfy

$$\chi^2(\nu_e\nu_e) > \chi^2(\nu_\mu\nu_\mu) \quad \text{or} \quad \chi^2(\nu_\tau\nu_\tau) , \tag{14}$$

so that the core cools mainly by ν_e emission.

IV. DISCUSSION

A. Neutrino masses and magnetic moments

The predicted radiative decays and astrophysical bounds on $\chi(\nu_i\nu_j)$ depend on neutrino masses, whereas the $L \to R$ precession in a magnetic field is mass independent. Could we then assume very small m_{ν_i} and disregard the limits on $\chi(\nu_i\nu_j)$ which in models with strong flavor mixings and $\chi(\nu_i\nu_j) \simeq \chi(\nu_i\nu_i)$ would also limit the diagonal moments? This approach is questionable at best, since in the standard model the same underlying chiral symmetry of neutrino dynamics prevents both $m_{\nu_i} \neq 0$ and $\chi(\nu_i\nu_i) \neq 0$. Once we assume Dirac neutrinos so that $\chi(\nu_i\nu_i)$ could differ from zero, we allow the GWS Higgs couplings $g_\nu \bar{\nu}_i^L \nu_i^R \phi_{\rm GWS}$ or Dirac masses $m_i^D \bar{\nu}_i^L \nu_i^R$. The natural choice $g_{\nu_i} \simeq g_{L_i}$, with g_{L_i} the GWS Higgs Yukawa couplings to the corresponding charged lepton, yields substantial Dirac masses $m^D_{\nu_e} \simeq m_e$, $m^D_{\nu_\mu} \simeq m_\mu$, and $m^D_{\nu_\tau} \simeq m_\tau$. We can still obtain light physical masses of neutrinos by utilizing the "seesaw" mechanism[27] with a very large Majorana mass, m^R, for the right-handed neutrino. The diagonalization of the mass matrix

$$\begin{bmatrix} 0 & m^D \\ m^D & m^R \end{bmatrix}$$

yields then a very light neutrino of mass $m_\nu = (m^D)^2/m^R$. This neutrino is, up to a small $O(m^D/m^R)$ admixture of ν^R, a pure left-handed Majorana particle. Diagonal $\chi(\nu_i\nu_i) \neq 0$ are forbidden for such neutrinos, so we expect m^D/m^R suppression of $\chi(\nu_i\nu_i)$. Thus, we cannot utilize such models to obtain the large values of $\chi(\nu_e\nu_e)$ required for the (above-mentioned) resolution of the solar-neutrino puzzle.

Even if at the tree level the Dirac neutrino masses vanish, the same exchanges which generate the neutrino magnetic moment will radiatively generate a Dirac mass

$$m^D_{\nu_i} \simeq \sum m_{L_j} g^2(\eta,\nu_j,L_j) \ln \left[\frac{\Lambda}{m^L} \right] , \tag{15}$$

where η is the isosinglet scalar in the FY model[16] which couples the ν_i to the various charged leptons L_j. The logarithmic divergence indicates the need for an uncalculable counterterm—the Yukawa g_{ν_i} coupling. Ignoring the logarithmic divergence and taking $\ln(\Lambda/m^L) \simeq 1$, we have $m_{\nu_i} \simeq m_{L_i} g^2(\eta,\nu_j,L_j)$, and since the same factors appear in the expression for $\chi(\nu_i\nu_i)$, we again have $\chi(\nu_i\nu_i) \simeq m_{\nu_i}$. In models (such as that of FY) with new interactions which strongly mix the leptonic flavors, the radiatively induced neutrino mass matrix is nondiagonal in neutrino flavors. Strong neutrino mixings, and consequently neutrino mass differences $\Delta m^2 \equiv m_{\nu_i}{}^2 - m_{\nu_j}{}^2$, are tightly constrained by accelerator and reactor oscillation experiments.[28] Nevertheless, it may well be possible to introduce appropriate bare $g^D_{\nu_i}$ so as to have the desired small neutrino masses along with "large" magnetic moments, $\chi(\nu_e\nu_e) \simeq 10^{-11}$.

B. Can $\chi(\nu_e\nu_e)$ and $\chi(\nu_e\nu_\mu)$ be constrained by $\chi(ee)$ and $\chi(e\mu)$?

New mechanisms generating $\chi(\nu_e\nu_\mu) \neq 0$ [or $\chi(\nu_e\nu_\tau) \neq 0$] are constrained by the good agreement between the measured[29] and calculated[30] (through α^4) values of the electron magnetic moment,

$$\left[\frac{\mu_e}{\mu_e^B} \right]^{\rm expt} = 1.001\,159\,652\,209 \pm 3.1 \times 10^{-11} ,$$

$$\left[\frac{\mu_e}{\mu_e^B} \right]^{\rm th} = 1.001\,159\,652\,200 \pm 1.5 \times 10^{-10} ,$$

and the bound[28] $B_r(\mu \to e\gamma) \leq 10^{-10}$. One can verify that these bounds are satisfied in any given model. But, perhaps, the following consideration may serve as a more general guideline for relating $\chi(\nu_e\nu_e)$ to $\delta(ee)$ is the difference between the (above) experimental and theoretical values, or for relating $B_r(\mu \to e\gamma)$ to $\chi(\nu_e\nu_\mu)$: Assume that whatever the new source for $\delta(ee)$ and $\chi(\nu_e\nu_e)$ is (compositeness, new interactions, etc.), it respects the weak isospin under which e and ν_e are in an isodoublet. The e and ν_e could also be in an isodoublet of a global "custodial" SU(2) symmetry which may be relevant in technicolor or composite W,Z theories. If so, we expect, essentially on symmetry grounds, similar new contributions to the magnetic moments of the two particles

$$\delta(\nu_e\nu_e) \simeq \delta(ee) \quad \text{or} \quad \delta(\nu_e\nu_e) \simeq \chi(\nu_e\nu_e) \simeq \delta(ee) . \tag{16}$$

To borrow a simple hadronic analogue, relation (16) is like $|\mu(p)| \simeq |\mu(n)|$, between the anomalous moments of the proton and neutron, which is very well satisfied. It is intriguing that at the present time the direct experimental limits on $\chi(\nu_e\nu_e)$ are very close to the estimate in Eq. (16). An improved experimental value of μ_e (and new measurements of α via the quantum Hall effect), together with new calculations, may determine δ to be below 10^{-11} and also $\chi(\nu_e\nu_e) \leq 10^{-11}$. Similar weak isospin arguments, if applicable to other lepton generations, would suggest $\chi(\nu_e\nu_\mu) \simeq (e\mu)$. The latter relation controls the $\mu \to e\gamma$ decay

$$B_r(\mu \to e\gamma) = \frac{\alpha^2(\chi(\mu e)) m_\mu{}^3}{8 m_e{}^2} \simeq 10^{-10} , \tag{17}$$

from which we deduce the very stringent bound $\chi(\nu\mu) \simeq \chi(\nu_e\nu_\mu) \leq 10^{-15}$.

C. Summary

The motivation, from a possible resolution to the solar-neutrino puzzle, and some further implications of finite neutrino magnetic moment have been discussed. Present bounds do not rule out the ν_e magnetic moment (in units of the Bohr magneton) of $\sim 10^{-11}$ or lower. Such values may arise in certain extensions of the standard model. In particular, nondiagonal transition moments of comparable magnitude are then likely as well. Since in such models neutrinos are likely to have finite masses, radiative flavor-changing decays of neutrinos are expected. Comparing the prediced radiative decay rates, Eq. (12), with radio, optical, and ultraviolet observations of galaxies and clusters of galaxies, we deduced that $\chi(\nu_i\nu_j)\leq 3\times10^{-15}$ for $m_{\nu_i}\geq 16$ eV. This upper limit is a factor $\simeq 20$ larger for $4\leq m_{\nu_i}\leq 16$ eV; the observational data do not yield useful bounds for $m_{\nu_i}\leq 4$ eV.

We have generalized the discussion of the possible role of the neutrino magnetic moment in supernovas by including the effects of transition moments, which may reduce the ν_e^L flux. Finally, a possible connection between precise measurements of the electron magnetic moment and bounds on the rare decay $\mu\rightarrow e+\gamma$ has been proposed.

[1]S. Sarantakos, A. Sirlin, and W. J. Marciano, Nucl. Phys. **B217**, 84 (1983).

[2]J. E. Kim, Phys. Rev. D **14**, 300 (1976); B. W. Lee and R. E. Shrock, *ibid.* **16**, 1444 (1977); W. J. Marciano and A. I. Sanda, Phys. Lett. **67B**, 303 (1977).

[3]D. B. Cline, University of Wisconsin Report No. EX-86 283, 1986 (unpublished).

[4]M. A. B. Beg, W. J. Marciano, and M. Ruderman, Phys. Rev. D **17**, 1395 (1978). For a review of other astrophysical consequences of finite neutrino magnetic moment, see A. D. Dolgov and Ya. B. Zeldovich, Rev. Mod. Phys. **53**, 1 (1981).

[5]M. B. Voloshin, M. I. Vysotskiĭ, and L. B. Okun', Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)].

[6]J. N. Bahcall, B. T. Cleveland, R. Davis, and J. K. Rowley, Astrophys. Lett. **292**, L179 (1985).

[7]S. M. Bilenky and B. Pontecorvo, Phys. Rep **4C**, 41 (1978).

[8]A. Dar, IAS report, February, 1987 (unpublished).

[9]Kamiokande II Collaboration, K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987).

[10]IMB Collaboration, R. M. Bionta *et al.*, Phys. Rev. Lett. **58**, 1494 (1987).

[11]I. Goldman, Y. Aharonov, G. Alexander, and S. Nussinov, TAUP Report No. 1543-87, 1987 (unpublished).

[12]S. Nussinov, I. Goldman, G. Alexander, and Y. Aharonov, Nature (London) (to be published).

[13]In this regard, see J. Yang, M. Turner, G. Steigman, D. N. Schramm, and K. Olive, Astrophys. J. **281**, 493 (1984).

[14]K. Brecher and G. Blumenthal, Astrophys. Lett. **6**, 169 (1970); M. J. Rees and M. Reinhardt, Astron. Astrophys. **19**, 189 (1972).

[15]M. J. Duncan, J. A. Grifols, A. Mendez, and S. Uma Sankar, University of Pennsylvania Report No. UPR-0138-T/UAB-FT-163, 1986 (unpublished).

[16]M. Fukugita and T. Yanagida, University of Kyoto report, 1987 (unpublished); M. Fukugita (private communication).

[17]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[18]R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972).

[19]See, e.g., J. N. Bahcall and S. L. Glashow, Nature (London) **326**, 476 (1987); R. Cowsik, Washington University report, St. Louis, 1987 (unpublished); W. D. Arnett and J. L. Rosner, Phys. Rev. Lett. **58**, 1906 (1987); C. W. Kim and W. K. Sze, Johns Hopkins University report, 1987 (unpublished); A. Dar and S. Dado, Technion report, 1987 (unpublished).

[20]J. Silk and R. A. Sunyaev, Nature (London) **260**, 508 (1976).

[21]Y. Rephaeli and A. S. Szalay, Phys. Lett. **106B**, 73 (1981).

[22]F. W. Stecker and R. W. Brown, Astrophys. J. **257**, 1 (1982).

[23]H. L. Shipman and R. Cowsik, Astrophys. J. Lett. **247**, L111 (1981).

[24]R. C. Henry and P. D. Feldman, Phys. Rev. Lett. **47**, 618 (1981).

[25]J. C. Mather, Opt. Eng. **21**, 769 (1982).

[26]The condition that the time variation of B on a time scale of R/c—when transformed to the neutrino rest frame—will have associated frequencies of order $\Delta m=m_i-m_j$, implies $\Delta m_{1j}{}^2/2E\leq R^{-1}$, or $\Delta m_{1j}{}^2\equiv m_1{}^2-m_j{}^2\leq 10^{-6}$ eV^2.

[27]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980).

[28]K. E. Bergkvist, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies,* Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1986), pp. 560–580.

[29]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[30]T. Kinoshita and W. B. Lindquist, Phys. Rev. Lett. **47**, 1573 (1981).